267ODEs-121019, Hour 17: High order constant coefficient homogeneous ODEs

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Riddle Along. on strike.

TT. Friday Oct. 26 9-10 @ GB404

... a partial sample test is on web!

Further comments on numerical methods:

1. A comparisson of Taylor & RK4 as on the right.

Pensieve header : Runge-Kutta - Taylor comparison.

$$\left\{\sum_{k=0}^{1000} \frac{(-100.)^k}{k!}, \frac{(-100.)^{1000}}{1000!}\right\}$$

$\{5.2633\times 10^{26}, 2.48516814326680\times 10^{-568}\}$

```
h = 0.1; yj = 1.;
Do[
  k1 = -yj; k2 = -(yj + h k1 / 2); k3 = -(yj + h k2 / 2);
  k4 = -(yj + h k3);
  yj = yj + h (k1 + 2 k2 + 2 k3 + k4) / 6,
  {1000}
 ];
yj
```

3.72041×10^{-44}

2. There's much more!

3. If $f(x,y)=f(x)$ RK4 becomes:

$$\int_{x_0}^{x} f(t)\,dt = $$

$\frac{1}{6}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{6}$
x_0	$h/2$	$+h$	$3h/2$	$+2h$	$5h/2$	$+3h$

This is "simpson's rule", and it is way better than rectangles/trapezoids!

Constant-coefficient homogeneous high-order ODEs.

$Ly = ay'' + by' + c = 0$

or $Ly = \sum a_n y^{(n)} = 0$

expect an n-dimensional v.s. of sol'ns.

<u>Example</u> $Ly = y'' + y' - 6y = 0$; guess $e^{\alpha x}$

In general, if $Ly = \sum a_n D^n y = p(D)\,y$, then

$$L(e^{\alpha x}) = p(D)\,e^{\alpha x} = p(\alpha)\,e^{\alpha x}$$

If p has n distinct real roots — done

If p has a complex root ....

e.g. $y'' - 4y' + 5y = 0$, $\alpha_{1,2} = 2 \pm i$

done line

If p has a multiple root; i.e.

$$y'' - 2y' + y = 0 \ ...$$

Differentiate $p(D)\,e^{\alpha x} = p(\alpha)\,e^{\alpha x}$ w.r.t. α:

$$p(D)(x e^{\alpha x}) = (p'(\alpha) + x\,p(\alpha))\,e^{\alpha x}$$

$$p(D)(x^2 e^{\alpha x}) = (p'' + 2x\,p' + x^2 p)\,e^{\alpha x}$$

$$\vdots$$

Later added "reduction of order" & "undetermined coeffs"

Even better, do systems: $y' = Ay \quad y(0) = y_0$

Sol'n $y(x) = e^{Ax} \cdot y_0$

What's e^{Ax}?