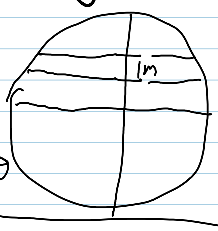
Read Along. The Gelfand-Fomin book.

Riddle Along

planet Earth



Counting area alone, which slice would you rather have?
Put a spherical bread through a bread slicer. Which slice will get the most crust?

Euler-Lagrange y "minimizes" $\int_a^b F(x,y,y')\,dx$

$$y(a)=A \quad y(b)=B$$

$$\Rightarrow \quad F_y - \frac{d}{dx}(F_{y'}) = 0 \quad \text{2nd order ODE!}$$

Special cases: 1. $F_{y'}=0 \Rightarrow F_y = 0$

2. $F_y = 0 \Rightarrow F_{y'} = C$ "conservation of momentum"

The $\frac{1}{2}m\dot{q}^2 - V(q)$ example.

3. $F_x = 0 \Rightarrow F_y - y'F_{y'y} - y''F_{y'y'} = 0 \;/\cdot y'$

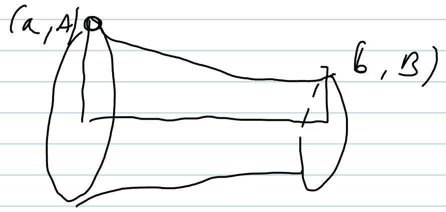
$$y'F_y - (y')^2F_{y'y} - y'y''F_{y'y'} = 0$$

$$\frac{d}{dx}\left(F - y'F_{y'}\right) = 0$$

$F - y'F_{y'} = C$ "conservation of energy".

Example: The Brachistochrone, $F = \sqrt{\frac{1+y'^2}{y}}$

Example: Find the rotational bread with the least crust



$$F = \pi y\sqrt{1+y'^2}$$

Who cares? Absolutely nobody, yet.....

$$\Rightarrow \quad y\sqrt{1+y'^2} - y'\frac{yy'}{\sqrt{1+y'^2}} = C$$

$$\Rightarrow y\frac{1}{\sqrt{1+y'^2}} = C \Rightarrow \frac{y^2}{C^2} = 1+y'^2 \quad y' = \sqrt{\left(\frac{y}{C}\right)^2 - 1}$$

$$y = C\cdot\cosh\frac{x - C'}{C}$$

Discuss the roach problem – "The hardest math I've ever really used"

done line