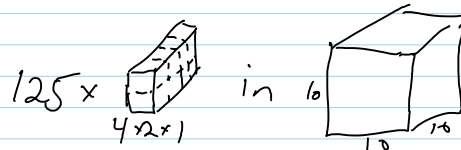


HW 3 on web!

Riddle Along.

On board:  $(x^x)' = x \cdot x^{x-1} + x^x \log x$ An aside on  $g(t)'$  where  $g(t) = F(x_1(t), x_2(t), x_3(t))$ When adding  $\epsilon$  to  $t$ ,  $g$  increases for 3 reasons; so

$$g'(t) = F_1 \cdot x_1' + F_2 \cdot x_2' + F_3 \cdot x_3'$$

Example  $(x^x)'$ 

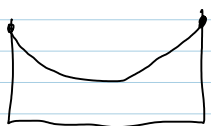
## Calculus of Variations

Problem Minimize  $J(y) = \int_a^b F(x, y, y') dx$  among

all sufficiently differentiable  $y(x)$  s.t.

$$y(a) = A, \quad y(b) = B$$

Examples 1. The power line problem



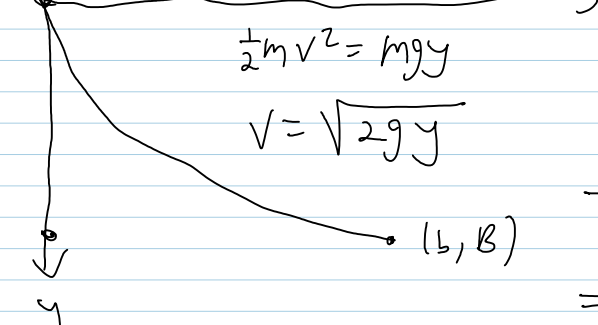
$$J(y) = \int y \cdot m ds = m \int y \cdot \sqrt{1+y'^2} dx$$

$$2. \mathcal{L}(q) = \int \left[ \frac{1}{2} m \dot{q}^2 - V(q(t)) \right] dt$$

(all of classical mechanics is hidden here)

3. The Brachistochrone.

(0,0)



$$\frac{1}{2} m v^2 = mgy$$

$$v = \sqrt{2gy}$$

$$ds = v dt = \sqrt{2gy} dt$$

$$ds = \sqrt{1+y'^2} dx$$

$$T(y) = \int dt =$$

$$= \int \frac{ds}{v} = \int \frac{\sqrt{1+y'^2}}{\sqrt{2gy}}$$

↓  
y

→ (b, B)

$$= \int \frac{ds}{\sqrt{2gy}} = \int_0^b \frac{\sqrt{1+y'^2}}{\sqrt{2gy}} dx$$

Euler-Lagrange:  $y$  is an extremum  $\Rightarrow$

$$\forall h \frac{d}{dh} J(y+eh)|_{e=0} = 0 \Leftrightarrow F_2 - \frac{d}{dx} F_3 = 0$$

That is, iff E-L eqn's hold:  $F_y - \frac{d}{dx} F_{y'} = 0$

Example. For  $F = \frac{1}{2}m\dot{q}^2 - V$  get  $m\ddot{q} = -V'$ .  
"Newton's eqn of motion"

Special cases: 1.  $F_{y'} = 0 \Rightarrow F_y = 0$

2.  $F_y = 0 \Rightarrow F_{y'} = C$  "conservation of momentum"

3.  $F_x = 0 \Rightarrow F_y - y'F_{y'y} - y''F_{y'y'} = 0 \cdot y'$

$$y'F_y - (y')^2 F_{y'y} - y'y''F_{y'y'} = 0$$

$$\frac{d}{dx} (F - y'F_{y'}) = 0$$

$F - y'F_{y'} = C$  "conservation of energy".

done  
line