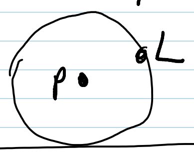
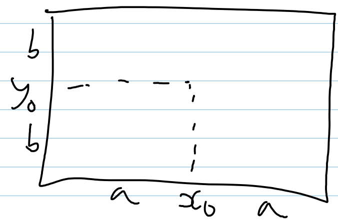
Id yourself in class photo!

Riddle Along.



$V_L = 4V_p$

Can prof. escape?



Thm Let $f : R = [x_0 - a, x_0 + a] \times [y_0 - b, y_0 + b] \to \mathbb{R}$ be cont. in x & uniformly Lipschitz in y: $|f(x, y_1) - f(x, y_2)| < k|y_1 - y_2|$

then the eqn $\phi' = f(x, \phi)$, $\phi(x_0) = y_0$, has a unique soln in the range $(x_0 - \delta, x_0 + \delta)$, where $\delta = \min(a, \frac{b}{M})$ & M is a bound on f in R.

Proof Rewrite the eqn as

$$\phi(x) = y_0 + \int_{x_0}^{x} f(t, \phi(t))\,dt$$

Idea: $\phi_0(x) = y_0 \qquad \phi_n(x) = y_0 + \int_{x_0}^{x} f(t, \phi_{n-1}(t))\,dt$

start line

Example Solve $y' = y$ w/ $y(0) = 1$.

$\phi_0(x) = 1 \qquad \phi_1(x) = 1 + \int_0^x \phi_0(t)\,dt = 1 + x$

$\ldots \quad \phi_n(x) = 1 + x + \ldots + \frac{x^n}{n!}$

Claims 1. ϕ_n is well defined.

2. For $n \geq 1$, $|\phi_n(x) - \phi_{n-1}(x)| < \frac{MK^{n-1}}{n!}|x - x_0|^n$

3. $\phi = \lim \phi_n$ converges uniformly.

done line

4. ϕ is a soln!

5. ϕ is unique; if ϕ, ψ both solve, then

$$|\phi(x) - \psi(x)| \leq A \cdot \int_0^x |\phi(t) - \psi(t)|\,dt$$

Set $U(x) = \int_0^x |\phi(t) - \psi(t)|\,dt$ then $U \geq 0$,

$V(0)=0$ & $\left(e^{-Ax}V(x)\right)' \leqslant 0$ so

$e^{-Ax}V(x) \leq 0$ so $V \equiv 0$.

$\square$