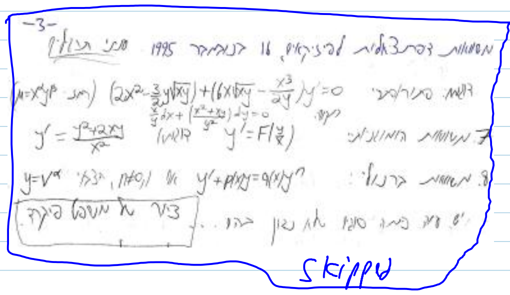
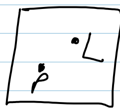
Class Photo at 9:55?
HW2 on web.
Read Along.
Riddle Along.



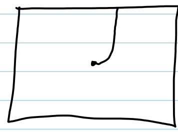


$V_L=V_P$
Hint: $\sum\frac{1}{k}=\infty$ but $\sum\frac{1}{k^2}<\infty$

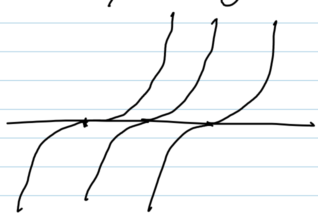
<u>Desired Thm</u> Given $f(x,y)$, $\phi'=f(x,\phi)$ with $\phi(x_0)=y_0$ has a solution, and it is unique.

<u>Problems</u> 1. This is hopeless unless f is at least continuous.

2. Even if f is smoothest, ϕ might exist only for a short time:



3. ϕ might not be unique:



$y=x^3 \quad y=(x+c)^3 \quad \sqrt[3]{y}=x+c$

$\sqrt[3]{y}-x=c$

$-dx+\frac{1}{3}y^{-\frac{2}{3}}dy=0$

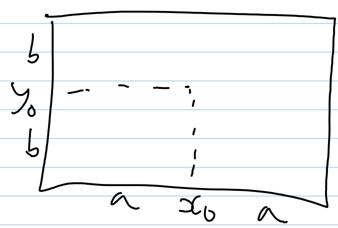
$y'=\frac{1}{\frac{1}{3}y^{-2/3}}=3y^{2/3}$ <u>Ex</u> check that for any c,

$$\phi_c(x)=\begin{cases}0 & x\le c\\ (x-c)^3 & x\ge c\end{cases}$$

is a solution.

<u>Def</u> We say that $f:\mathbb{R}_y\to\mathbb{R}$ is "Lipschitz" if $\exists K$ s.t. $|f(y_1)-f(y_2)|<K|y_1-y_2|$
"the Lipschitz constant of f"
(more than continuity, less than diff-ability)

<u>Thm</u> Let $f:R=[x_0-a,x_0+a]\times[y_0-b,y_0+b]\to\mathbb{R}$ be cont. in x & uniformly Lipschitz in y: $|f(x,y_1)-f(x,y_2)|<K|y_1-y_2|$



then the eqn $\phi'=f(x,\phi)$, $\phi(x_0)=y_0$, has a unique sol'n in the range $(x_0-\delta, x_0+\delta)$, where $\delta = \min(a, \frac{M}{b})$ & M is a bound on f in R.

Proof Rewrite the eqn' as

$$\phi(x) = y_0 + \int_{x_0}^{x} f(t, \phi(t))\,dt$$

Idea: $\phi_0(x) = y_0 \qquad \phi_n(x) = y_0 + \int_{x_0}^{x} f(t, \phi_{n-1}(t))\,dt$

done line

Claims 1. ϕ_n is well defined.

2. For $n \geq 1$, $|\phi_n(x) - \phi_{n-1}(x)| < \frac{MK^{n-1}}{n!}|x-x_0|^n$

3. $\phi = \lim \phi_n$ converges uniformly.

4. ϕ is a sol'n!

5. ϕ is unique; if ϕ, ψ both solve, then

$$|\phi(x) - \psi(x)| \leq A \cdot \int_0^x |\phi(t) - \psi(t)|\,dt$$

Set $U(x) = \int_0^x |\phi(t) - \psi(t)|\,dt$ then $U \geq 0$,

$U(0) = 0$ & $(e^{-Ax}U(x))' \leq 0$ so

$$e^{-Ax}U(x) \leq 0 \quad \text{so} \quad U \equiv 0.$$

$\square$