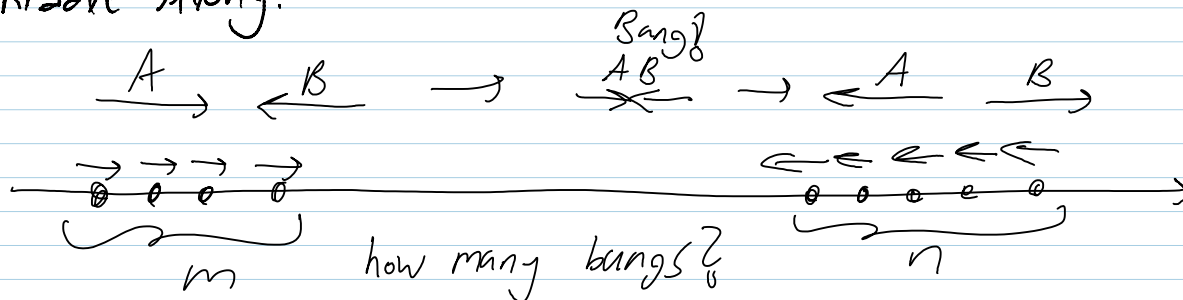


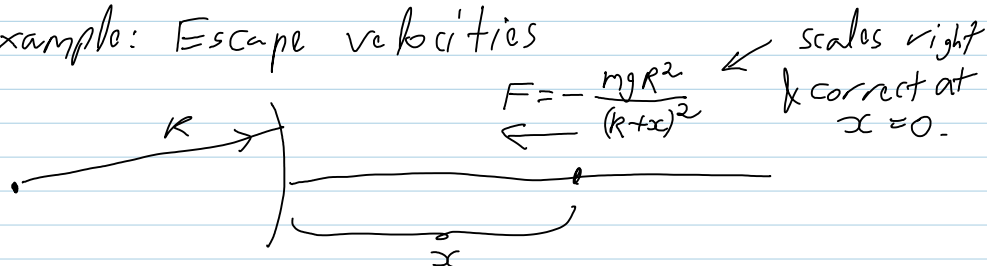
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Riddle Along.



on board

Example: Escape velocities



So $m \frac{dv}{dt} = -\frac{mgR^2}{(R+x)^2}$

$$\frac{dv}{dt} = \frac{dv}{dx} \frac{dx}{dt} = v \cdot \frac{dv}{dx} \Rightarrow v \frac{dv}{dx} = -\frac{gR^2}{(R+x)^2}$$

$$\frac{v^2}{2} = \frac{gR^2}{R+x} + C \quad v(0) = v_0 \Rightarrow C = \frac{v_0^2}{2} - gR$$

start line

So $\frac{v^2}{2} = \frac{gR^2}{R+x} - gR + \frac{v_0^2}{2}$

For which v_0 , $\lim_{x \rightarrow \infty} v(x) = 0$?

$$gR = \frac{v_c^2}{2} \quad v_c = \sqrt{2gR}$$

"Uncovering the tracks"

Autonomous equations. $y' = f(y)$

Homogeneous equations.

$y' = \frac{y^2 + 2xy}{x^2}$ and $y' = F\left(\frac{y}{x}\right)$: "הצורה הומוגנית"
 $v = \frac{y}{x}$: "הצורה הומוגנית"

$y = vx \quad y' = v'x + v$

$$xv' = F(v) - v \quad x \frac{dv}{dx} = F(v) - v \quad \frac{dx}{x} = \frac{dv}{F(v) - v}$$

Example: $y' = \frac{y^2 + 2xy}{x^2} \quad x \frac{dv}{dx} = v^2 + 2v - v$
 $= v(v+1)$

$$\frac{dv}{v(v+1)} = \frac{dx}{x} \quad \log|x| + C = \int \frac{dv}{v(v+1)} = \int \left(\frac{-1}{v+1} + \frac{1}{v} \right) dv$$
$$= -\log|v+1| + \log|v| = \log \left| \frac{v}{v+1} \right|$$

$$cx = \frac{v}{v+1} \quad (v+1)cx = v \quad v(cx-1) = -cx$$

$$v = \frac{-cx}{cx-1} \quad y = \frac{-cx^2}{cx-1}$$