$$y_0 = \frac{-c\, x^2}{c\, x - 1};$$

$$D[y_0, x] == \frac{y_0^2 + 2\, x\, y_0}{x^2}$$

$$\frac{c^2 x^2}{(-1 + c\, x)^2} - \frac{2\, c\, x}{-1 + c\, x} == \frac{\frac{c^2 x^4}{(-1+c\, x)^2} - \frac{2\, c\, x^3}{-1+c\, x}}{x^2}$$

$$D[y_0, x] == \frac{y_0^2 + 2\, x\, y_0}{x^2} \ // \ \text{Simplify}$$

True

$$D[y_0, x] == \frac{y_0^2 + 2\, x\, y_0}{x^2}$$

$$\frac{c^2 x^2}{(-1 + c\, x)^2} - \frac{2\, c\, x}{-1 + c\, x} == \frac{\frac{c^2 x^4}{(-1+c\, x)^2} - \frac{2\, c\, x^3}{-1+c\, x}}{x^2}$$

$$\text{DSolve}\left[D[y[x], x] == \frac{y[x]^2 + 2\, x\, y[x]}{x^2}, \ y[x], \ x\right]$$

$$\left\{\left\{y[x] \to -\frac{x^2}{x - C[1]}\right\}\right\}$$

$$\text{Integrate}\left[\frac{1}{v\, (v + 1)}, \ v\right]$$

$$\text{Log}[v] - \text{Log}[1 + v]$$