08-401

From Drorbn

Polynomial Equations and Fields

Department of Mathematics, University of Toronto, Spring 2008

Agenda: Follow Évariste Galois (http://en.wikipedia.org/wiki/Galois) to the top of mathematics' first mountain.

Classes: Wednesdays 6-9PM (OMG) at Sidney Smith 1086 (http://www.osm.utoronto.ca/cgi-bin/class_spec/spec03?bldg=SS&room=1086).

Instructor: Dror Bar-Natan (http://www.math.toronto.edu/~drorbn/), drorbn@math.toronto.edu, Bahen 6178, 416-946-5438. Office hours: by appointment.

Teaching Assistant: Yichao Zhang, zhangyichao2002@hotmail.com. Office hours: Tuesdays 1-3 at the Math Aid Centre, Sidney Smith 1071.

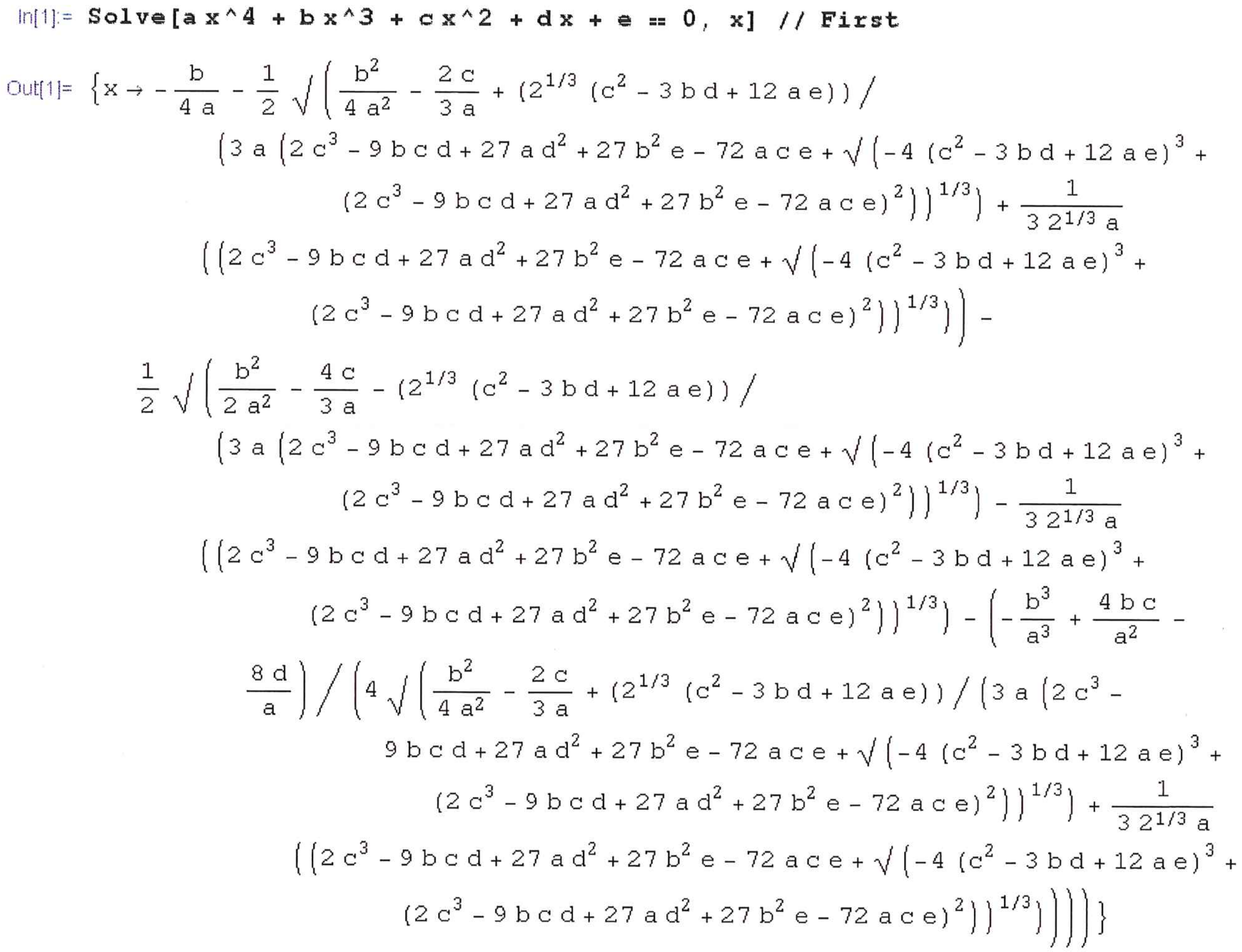
Grades. All grades will be on CCNet (http://ccnet.utoronto.ca/20081/mat401h1s/).

Further Resources

- J. Gallian's Algebra web site (http://www.d.umn.edu/~jgallian/).
- Undergraduate Information (http://www.math.toronto.edu/undergrad/) at the UofT Math Department (http://www.math.toronto.edu/)
- Undergraduate Course Descriptions (http://www.artsandscience.utoronto.ca/ofr/calendar/crs_mat.htm) at the Faculty of Arts and Science (http://www.artsci.utoronto.ca/).
- Last year's class: 07-401.
- 06-401 (http://www.math.toronto.edu/gor/mat401.html) with Julia Gordon.
- 05-302 (http://www.math.utoronto.ca/shub/mat302S.html) with Mike Shub.
- 06-301 (http://ccnet.utoronto.ca/20069/mat301h1f/) with Lindsey Shorser.

08-401/Navigation Panel

#	Week of...	Links
1	Jan 9	About, Notes, HW1
2	Jan 16	HW2, Notes
3	Jan 23	HW3, Photo, Notes
4	Jan 30	HW4, Notes
5	Feb 6	HW5, Notes
6	Feb 13	On TT, Notes
R	Feb 20	Reading week
7	Feb 27	Term Test (and solution)
8	Mar 5	HW6, Notes
9	Mar 12	HW7, Notes
10	Mar 19	HW8, Notes, RC (PDF)
11	Mar 26	HW9, Notes
12	Apr 2	FT, HW10, Notes
13	Apr 9	Notes
S	Apr 14-25	Study Period: blackboards (http://katlas.math.toronto.edu/drorbn/bbs/show?shot=08401-080425-142418.jpg)
F	Apr 28	Final
 Add your name / see who's in!		
Register of Good Deeds		



```
In[1]:= Solve[a x^4 + b x^3 + c x^2 + d x + e == 0, x] // First
```

Out[1]=

$$\{x \to -\frac{b}{4a} - \frac{1}{2}\sqrt{\left(\frac{b^2}{4a^2} - \frac{2c}{3a} + (2^{1/3}(c^2 - 3bd + 12ae)) / \left(3a\left(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace + \sqrt{(-4(c^2 - 3bd + 12ae)^3 + (2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace)^2)}\right)^{1/3}\right) + \frac{1}{3\,2^{1/3}a}\left(\left(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace + \sqrt{(-4(c^2 - 3bd + 12ae)^3 + (2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace)^2)}\right)^{1/3}\right)\right)} - \frac{1}{2}\sqrt{\left(\frac{b^2}{2a^2} - \frac{4c}{3a} - (2^{1/3}(c^2 - 3bd + 12ae)) / \left(3a\left(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace + \sqrt{(-4(c^2 - 3bd + 12ae)^3 + (2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace)^2)}\right)^{1/3}\right) - \frac{1}{3\,2^{1/3}a}\left(\left(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace + \sqrt{(-4(c^2 - 3bd + 12ae)^3 + (2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace)^2)}\right)^{1/3}\right) - \left(-\frac{b^3}{a^3} + \frac{4bc}{a^2} - \frac{8d}{a}\right) / \left(4\sqrt{\left(\frac{b^2}{4a^2} - \frac{2c}{3a} + (2^{1/3}(c^2 - 3bd + 12ae)) / \left(3a\left(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace + \sqrt{(-4(c^2 - 3bd + 12ae)^3 + (2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace)^2)}\right)^{1/3}\right) + \frac{1}{3\,2^{1/3}a}\left(\left(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace + \sqrt{(-4(c^2 - 3bd + 12ae)^3 + (2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace)^2)}\right)^{1/3}\right)\right)}\right)\right)}\}$$

Solving The Quartic With Mathematica. Read more! (http://en.wikipedia.org/wiki/Quartic_equation)

Retrieved from "https://drorbn.net/index.php?title=08-401&oldid=7037"

This page was last edited on 10 April 2008, at 13:23.

Dror Bar-Natan: Classes: 2007-08: Math 401 Polynomials, Equations, Fields:

Galois Theory Quick Reference

Goal. Some polynomials cannot be "solved" using $+$, $-$, $\times$, $\div$ and $\sqrt[n]{\ }$.
Galois Theory. Roughly, there is a correspondence

{field extensions}	$\xleftrightarrow[\text{Theorem}]{\text{The Fundamental}}$	{groups}
{extensions by roots}	$\longrightarrow$	{"solvable groups"}
splitting field of $3x^5-15x+5$	$\longrightarrow$	the non-solvable permutation group S_5

To do.

1. More on splitting fields.
2. Quick reminders on group theory.
3. Precise statement of the fundamental theorem.
4. Examples for the fundamental theorem.
5. On solvable groups: definition, basic properties, S_5 is not solvable.
6. "Extensions by radicals" correspond to solvable groups.
7. The splitting field of $3x^5-15x+5$ corresponds to S_5.
8. Proof of the fundamental theorem.

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8. Proof of the fundamental theorem.

The Fundamental Theorem of Galois Theory. Let F be a field of characteristic 0 and let E be a splitting field over F. Then there is a bijective correspondence between the set $\{K : E/K/F\}$ of intermediate field extensions K lying between F and E and the set $\{H : H < \operatorname{Gal}(E/F)\}$ of subgroups H of the Galois group $\operatorname{Gal}(E/F)$ of the original extension E/F:

$$\{K : E/K/F\} \quad \leftrightarrow \quad \{H : H < \operatorname{Gal}(E/F)\}.$$

The bijection is given by mapping every intermediate extension K to the subgroup $\operatorname{Gal}(E/K)$ of elements in $\operatorname{Gal}(E/F)$ that preserve K,

$$\Phi : \ K \mapsto \operatorname{Gal}(E/K) := \{g : E \to E : g|_K = I\},$$

and reversely, by mapping every subgroup H of $\operatorname{Gal}(E/F)$ to its fixed field E_H:

$$\Psi : \ H \mapsto E_H := \{x \in E : \forall h \in H, \ hx = x\}.$$

This correspondence has the following further properties:

- It is inclusion-reversing: if $H_1 \subset H_2$ then $E_{H_1} \supset E_{H_2}$ and if $K_1 \subset K_2$ then $\operatorname{Gal}(E/K_1) > \operatorname{Gal}(E/K_2)$.
- It is degree/index respecting: $[E : K] = |\operatorname{Gal}(E/K)|$ and $[K : F] = [\operatorname{Gal}(E/F) : \operatorname{Gal}(E/K)]$.
- Splitting fields correspond to normal subgroups: If K in $E/K/F$ is the splitting field of a polynomial in $F[x]$ then $\operatorname{Gal}(E/K)$ is normal in $\operatorname{Gal}(E/F)$ and $\operatorname{Gal}(K/F) \cong \operatorname{Gal}(E/F)/\operatorname{Gal}(E/K)$.

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UNIVERSITY OF TORONTO

Faculty of Arts and Sciences

FINAL EXAMINATIONS, APRIL-MAY 2008

Math 401H1S Polynomial Equations and Fields

Instructor: Dror Bar-Natan
Date: April 28, 2008

Duration. You have 3 hours to write this exam.

Allowed Material. Basic calculators, not capable of displaying text or sounding speech.

Solve 6 of the following 7 questions. Each question is worth 17 points, to a maximum possible total of 102. Different parts of the same question may be weighted differently. You will get 4 points total for any problem for which you will write explicitly "I don't know how to solve this problem" (whole problems only!).

Neatness counts! Language counts! The *ideal* written solution to a problem looks like a proof from the textbook; neat and clean and made of complete and grammatical sentences. Definitely phrases like "there exists" or "for every" cannot be skipped. Lectures are mostly made of spoken words, and so the blackboard part of proofs given during lectures often omits or shortens key phrases. The ideal written solution to a problem does not do that.

Good Luck!

Solve 6 of the following 7 problems. Neatness counts! Language counts!

Problem 1. Let R be a commutative ring with unity and let A be an ideal of R. Define "A is prime" and "a ring D is a domain" and prove that $D := R/A$ is a domain if and only if A is prime.

Tip. The phrase "if and only if" means that there are two things to prove.

define 5
⇒ 6
⇐ 6

−1 no definition of a "zero divisor".

Problem 2. Let $\mathbb{Q}$ be the ring of rational numbers and let $\mathbb{Z}$ be the ring of integers.

8.5 1. Is there a ring S and a ring homomorphism $\phi : \mathbb{Q} \to S$ so that $\ker \phi = \mathbb{Z}$?

8.5 2. Is there a ring S and a ring homomorphism $\psi : \mathbb{Q} \to S$ so that $\operatorname{im} \psi$ is isomorphic to $\mathbb{Z}$?

2 for each right answer w/ bogus justification.

Tip. These, of course, are not just yes/no questions. You are expected to fully justify your answers, whatever they are.

Problem 3. Let F be a field, A a non-zero ideal in $F[x]$, and $g \in F[x]$ a polynomial. Prove that $A = \langle g \rangle$ if and only if g is a non-zero polynomial of minimal degree in A.

Tip. As always in math exams, when proving a theorem you may freely assume anything that preceded it but you may not assume anything that followed it.

⇒ 7
⇐ 10

Problem 4. Is it always true that a splitting extension of a splitting extension is a splitting extension? In other words, let F be a field, $f \in F[x]$ be a polynomial with coefficients in F, K be a splitting field of f over F, $g \in K[x]$ be a polynomial with coefficients in K, and E be a splitting field of g over K. Is it always the case that there is a polynomial $h \in F[x]$ with coefficients in F so that E is a splitting field of h over F?

$$\begin{tikzcd} E \arrow[d, dash] \\ K \arrow[d, dash] \\ F \end{tikzcd}$$

Tip. This, of course, is also not just a yes/no question. Whatever you state, you have to prove, unless it is a known earlier result.

(17)+3: $\mathbb{Q}(\sqrt{2})$ isn't the splitting field of any polynomial.

+2 right answer, bogus justification.

cont'd...

Problem 5. Let E/F be a field extension, and let a_k (for $0 \leq k \leq n$) be elements of E that are algebraic over F. Let b be some solution in E of the equation $\sum_{k=0}^{n} a_k b^k = 0$. Prove that b is also algebraic over F.

$\frac{10}{17}$ 51

Problem 6. For any group A, recall that $[A, A]$, the commutator group of A, is the subgroup of A generated by all elements of the form $[x, y] := xyx^{-1}y^{-1}$, where $x, y \in A$.

7 1. Let G be a group. Define "G is solvable".

5 2. For any group H, prove that if $H' \lhd H$ is a normal subgroup, if H/H' is Abelian and if $A < H$ is some other subgroup, then $[A, A] < H'$.

5 3. Prove that if a group G contains a non-trivial subgroup A for which $[A, A] = A$, then G is not solvable.

[illegible]

Problem 7. Let F be the field $\mathbb{Q}(i)$ (note that F is not $\mathbb{Q}$!) and let E be the field $\mathbb{Q}(\sqrt[4]{2}, i)$.

6 1. Compute $G := \mathrm{Gal}(E/F)$.

5 2. Find all the subgroups H of G.

6 3. For exactly one non-trivial proper subgroup of G (that is, a subgroup that is neither $\{e\}$ nor G), describe the fixed field E_H.

Tip. The word "describe" here means "find $a \in E$ so that $E_H = F(a)$".

Good Luck!

Faculty of Arts & Science
University of Toronto
Survey Summary Results: SPR 08

Course: MAT 401H1S
Section: L5101
Instructor: D. BAR-NATAN

Enrolment: 34
Number of Forms Scanned: 15

STATEMENTS ABOUT THE INSTRUCTOR:

Quest.	% Resp. to Scale Rating 1	2	3	4	5	6	7	No.	Mean
2.	0	0	0	6	60	20	13	15	5.4
3.	0	0	20	26	26	20	6	15	4.7
4.	0	0	0	20	53	13	13	15	5.2
5.	0	0	6	26	33	20	13	15	5.1
6.	0	0	0	0	28	50	21	14	5.9
7.	0	0	0	21	35	28	14	14	5.4
8.	0	0	0	8	50	16	25	12	5.6
9.	0	0	7	35	28	21	7	14	4.9
10.	0	0	0	6	26	26	40	15	6.0
11.	0	0	0	7	42	21	28	14	5.7

STATEMENTS ABOUT THE COURSE:

Quest.	% Resp. to Scale Rating 1	2	3	4	5	6	7	No.	Mean
12.	0	0	0	33	26	26	13	15	5.2
13.	0	0	0	13	13	60	13	15	5.7
14.	0	0	7	23	30	38	0	13	5.0
15.	0	0	0	33	0	66	0	3	5.3
16.	0	0	0	33	0	66	0	3	5.3
17.	0	0	0	50	0	50	0	2	5.0
18.	0	0	0	40	20	40	0	5	5.0
19.	0	0	7	28	35	28	0	14	4.9

20. Yes: 50% No: 50% 14

OTHER QUESTIONS:

Quest.	% Resp. to Scale Rating 1	2	3	4	5	6	7	No.	Mean
25.	0	0	0	0	0	0	0	0	0.0
26.	0	0	0	0	0	0	0	0	0.0
27.	0	0	0	0	0	0	0	0	0.0
28.	0	0	0	0	0	0	0	0	0.0
29.	0	0	0	0	0	0	0	0	0.0
30.	0	0	0	0	0	0	0	0	0.0

Quest.	% Resp. to Scale Rating 1	2	3	4	5	6	7	No.	Mean
31.	0	0	0	0	0	0	0	0	0.0
32.	0	0	0	0	0	0	0	0	0.0
33.	0	0	0	0	0	0	0	0	0.0
34.	0	0	0	0	0	0	0	0	0.0
35.	0	0	0	0	0	0	0	0	0.0
36.	0	0	0	0	0	0	0	0	0.0

MEAN RATING ON QUESTION 11 (GLOBAL EVALUATION OF INSTRUCTOR) AS A FUNCTION OF STUDENT INFORMATION:

	No.	Mean Global Eval.
21. Number of full courses already completed:		
0- 4.5	0	-
5- 9.5	0	-
10-14.5	4	5.8
15-19.5	8	5.9
>=20	2	5.0
22. Status of the course for the student:		
Program Requirement	9	6.1
Selected from a required list in a program	3	4.7
Breadth requirement	0	-
Optional	0	-
23. Initial enthusiasm to take course:		
low	3	5.3
medium	6	5.7
high	3	6.0
24. Expected grade in course:		
<50	0	-
50-59	2	4.5
60-69	3	5.0
70-79	6	6.3
80-89	1	6.0
>=90	0	-

STUDENT SURVEY FORM

UNIVERSITY OF TORONTO

FACULTY OF
ARTS & SCIENCE

Note that survey results will be available to the instructor(s) only after final course marks have been submitted.

PART I: INSTRUCTIONS. PLEASE READ FIRST.

Using an HB pencil or a blue or black ball-point pen (but not a felt marking pen), fill completely the numbered oval corresponding to your response for each statement. If using a pen, do not alter original response by making another selection.

Part II *(on the reverse side)* requires a written answer.

Course Identification: Please print course and section you are evaluating

COURSE MAT401

SECTION L0101

INSTRUCTOR(S):

A: Bar Nan

B:

1. If evaluating only one instructor, write the name in the upper (A) box. If evaluating two instructors, write their names, one in box A and the other in box B.

DO NOT EVALUATE TEACHING ASSISTANTS ON THIS FORM

Statements about the instructor(s):

Respond to the statements below for instructor A (and instructor B) bearing in mind that there are wide variations in class size and subject matter in Arts and Science.

		extremely poor	very poor	poor	adequate	good	very good	outstanding
2. Communicates goals and requirements of the course clearly and explicitly.	A:	①	②	③	④	[✓ ⑤]	⑥	⑦
	B:	①	②	③	④	⑤	⑥	⑦
3. Uses methods of evaluation (e.g. papers, assignments, tests) that appropriately reflect the subject matter and provide a fair evaluation of student learning.	A:	①	②	③	④	[✓ ⑤]	⑥	⑦
	B:	①	②	③	④	⑤	⑥	⑦
4. Presents material in an organized, well-planned manner.	A:	①	②	③	④	[✓ ⑤]	⑥	⑦
	B:	①	②	③	④	⑤	⑥	⑦
5. Explains concepts clearly with appropriate use of examples.	A:	①	②	③	④	[✓ ⑤]	⑥	⑦
	B:	①	②	③	④	⑤	⑥	⑦
6. Communicates enthusiasm and interest in the course material.	A:	①	②	③	④	[✓ ⑤]	⑥	⑦
	B:	①	②	③	④	⑤	⑥	⑦
7. Attends to students' questions and answers them clearly and effectively.	A:	①	②	③	④	[✓ ⑤]	⑥	⑦
	B:	①	②	③	④	⑤	⑥	⑦
8. Is available for individual consultation, by appointment or stated office hours, to students with problems relating to the course.	A:	①	②	③	④	[✓ ⑤]	⑥	⑦
	B:	①	②	③	④	⑤	⑥	⑦
9. Ensures that student work is graded fairly, with helpful comments and feedback where appropriate.	A:	①	②	③	④	[✓ ⑤]	⑥	⑦
	B:	①	②	③	④	⑤	⑥	⑦
10. Ensures that student work is graded within a reasonable time.	A:	①	②	③	④	[✓ ⑤]	⑥	⑦
	B:	①	②	③	④	⑤	⑥	⑦
11. All things considered, performs effectively as a university teacher.	A:	①	②	③	④	[✓ ⑤]	⑥	⑦
	B:	①	②	③	④	⑤	⑥	⑦

PART I CONTINUES ON THE REVERSE SIDE

Statements about the course: Respond to the statements below, using the following 7-point scale.

SIDE 2

	very low	low	below average	average	above average	high	very high
12. Compared to other courses at the same level (100,200,300,400), the work load is	①	②	③	④	⑤	[✓ ⑥]	⑦
13. Compared to other courses at the same level, the level of difficulty of the material is	①	②	③	④	⑤	[✓ ⑥]	⑦
14. The value of the required reading is	①	②	③	④	⑤	[✓ ⑥]	⑦
15. (If applicable) The value of the tutorials is	①	②	③	④	⑤	[✓ ⑥]	⑦
16. (If applicable) The value of the laboratories is	①	②	③	④	⑤	[✓ ⑥]	⑦
17. (If applicable) The value of the seminars is	①	②	③	④	⑤	[✓ ⑥]	⑦
18. (If applicable) The value of the language conversation classes is	①	②	③	④	⑤	[✓ ⑥]	⑦
19. The value of the overall learning experience is	①	②	③	④	⑤	[✓ ⑥]	⑦

20. Considering your experience with this course, and disregarding your need for it to meet program or degree requirements, would you still have taken this course? [✓ Yes] ○ No

Statements about yourself:

21. Number of full course credits already earned (prior to this session):
    ① 0-4½ ② 5-9½ ③ 10-14½ ④ 15-19½ [✓ ⑤ ≥20]
22. Status of the course for you:
    ① Program Requirement ② Selected from a required list in a program ③ Breadth Requirement ④ Optional
23. Your level of enthusiasm to take this course at the time of initial registration:
    ① low ② medium [✓ ③ high]
24. Your expected grade in this course:
    ① <50 ② 50-59 ③ 60-69 ④ 70-79 [✓ ⑤ ≥80]

Additional statements or questions which may be supplied in class:

25. ① ② ③ ④ ⑤ ⑥ ⑦
26. ① ② ③ ④ ⑤ ⑥ ⑦
27. ① ② ③ ④ ⑤ ⑥ ⑦
28. ① ② ③ ④ ⑤ ⑥ ⑦
29. ① ② ③ ④ ⑤ ⑥ ⑦
30. ① ② ③ ④ ⑤ ⑥ ⑦
31. ① ② ③ ④ ⑤ ⑥ ⑦
32. ① ② ③ ④ ⑤ ⑥ ⑦
33. ① ② ③ ④ ⑤ ⑥ ⑦
34. ① ② ③ ④ ⑤ ⑥ ⑦
35. ① ② ③ ④ ⑤ ⑥ ⑦
36. ① ② ③ ④ ⑤ ⑥ ⑦

PART II: PLEASE ANSWER ONLY AFTER COMPLETING PART I. Please use the space below to provide supplementary comments on the instructor(s) or course. For example, you may wish to give the reasons for your numerical evaluations or provide specific suggestions for improving the instruction in the course.

END OF SURVEY. THANK YOU FOR YOUR COOPERATION AND PARTICIPATION.

STUDENT SURVEY FORM
UNIVERSITY OF TORONTO

FACULTY OF
ARTS & SCIENCE

Note that survey results will be available to the instructor(s) only after final course marks have been submitted.

PART I: INSTRUCTIONS. PLEASE READ FIRST.

Using an HB pencil or a blue or black ball-point pen (but not a felt marking pen), fill completely the numbered oval corresponding to your response for each statement. If using a pen, do not alter original response by making another selection.

Part II *(on the reverse side)* requires a written answer.

Course Identification: Please print course and section you are evaluating

COURSE M A T 4 0 1 H 1 S

SECTION L5 1 0 1

INSTRUCTOR(S):

A: D. Bar Natan

B:

1. If evaluating only one instructor, write the name in the upper (A) box. If evaluating two instructors, write their names, one in box A and the other in box B.

DO NOT EVALUATE TEACHING ASSISTANTS ON THIS FORM

Statements about the instructor(s):

Respond to the statements below for instructor A (and instructor B) bearing in mind that there are wide variations in class size and subject matter in Arts and Science.

		extremely poor	very poor	poor	adequate	good	very good	outstanding
2. Communicates goals and requirements of the course clearly and explicitly.	A:	1	2	3	4	●	6	7
	B:	1	2	3	4	5	6	7
3. Uses methods of evaluation (e.g. papers, assignments, tests) that appropriately reflect the subject matter and provide a fair evaluation of student learning.	A:	1	2	3	●	5	6	7
	B:	1	2	3	4	5	6	7
4. Presents material in an organized, well-planned manner.	A:	1	2	3	4	●	6	7
	B:	1	2	3	4	5	6	7
5. Explains concepts clearly with appropriate use of examples.	A:	1	2	3	●	5	6	7
	B:	1	2	3	4	5	6	7
6. Communicates enthusiasm and interest in the course material.	A:	1	2	3	4	5	●	7
	B:	1	2	3	4	5	6	7
7. Attends to students' questions and answers them clearly and effectively.	A:	1	2	3	●	5	6	7
	B:	1	2	3	4	5	6	7
8. Is available for individual consultation, by appointment or stated office hours, to students with problems relating to the course.	A:	1	2	3	4	●	6	7
	B:	1	2	3	4	5	6	7
9. Ensures that student work is graded fairly, with helpful comments and feedback where appropriate.	A:	1	2	3	4	●	6	7
	B:	1	2	3	4	5	6	7
10. Ensures that student work is graded within a reasonable time.	A:	1	2	3	4	5	●	7
	B:	1	2	3	4	5	6	7
11. All things considered, performs effectively as a university teacher.	A:	1	2	3	4	●	6	7
	B:	1	2	3	4	5	6	7

PART I CONTINUES ON THE REVERSE SIDE

Statements about the course: Respond to the statements below, using the following 7-point scale.

SIDE 2

	very low	low	below average	average	above average	high	very high
12. Compared to other courses at the same level (100,200,300,400), the work load is	①	②	③	④	●	⑥	⑦
13. Compared to other courses at the same level, the level of difficulty of the material is	①	②	③	●	⑤	⑥	⑦
14. The value of the required reading is	①	②	③	④	⑤	⑥	⑦
15. (If applicable) The value of the tutorials is	①	②	③	④	⑤	⑥	⑦
16. (If applicable) The value of the laboratories is	①	②	③	④	⑤	⑥	⑦
17. (If applicable) The value of the seminars is	①	②	③	④	⑤	⑥	⑦
18. (If applicable) The value of the language conversation classes is	①	②	③	●	⑤	⑥	⑦
19. The value of the overall learning experience is	①	②	③	●	⑤	⑥	⑦

20. Considering your experience with this course, and disregarding your need for it to meet program or degree requirements, would you still have taken this course? ○ Yes ● No

Statements about yourself:

21. Number of full course credits already earned (prior to this session):
① 0-4½ ② 5-9½ ● 10-14½ ④ 15-19½ ⑤ ≥20

22. Status of the course for you:
① Program Requirement ● Selected from a required list in a program ③ Breadth Requirement ④ Optional

23. Your level of enthusiasm to take this course at the time of initial registration:
① low ● medium ③ high

24. Your expected grade in this course:
① <50 ● 50-59 ③ 60-69 ④ 70-79 ⑤ ≥80

Additional statements or questions which may be supplied in class:

25. ① ② ③ ④ ⑤ ⑥ ⑦
26. ① ② ③ ④ ⑤ ⑥ ⑦
27. ① ② ③ ④ ⑤ ⑥ ⑦
28. ① ② ③ ④ ⑤ ⑥ ⑦
29. ① ② ③ ④ ⑤ ⑥ ⑦
30. ① ② ③ ④ ⑤ ⑥ ⑦
31. ① ② ③ ④ ⑤ ⑥ ⑦
32. ① ② ③ ④ ⑤ ⑥ ⑦
33. ① ② ③ ④ ⑤ ⑥ ⑦
34. ① ② ③ ④ ⑤ ⑥ ⑦
35. ① ② ③ ④ ⑤ ⑥ ⑦
36. ① ② ③ ④ ⑤ ⑥ ⑦

PART II: PLEASE ANSWER ONLY AFTER COMPLETING PART I. Please use the space below to provide supplementary comments on the instructor(s) or course. For example, you may wish to give the reasons for your numerical evaluations or provide specific suggestions for improving the instruction in the course.

END OF SURVEY. THANK YOU FOR YOUR COOPERATION AND PARTICIPATION.

STUDENT SURVEY FORM
UNIVERSITY OF TORONTO

FACULTY OF
ARTS & SCIENCE

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PART I: INSTRUCTIONS. PLEASE READ FIRST.

Using an HB pencil or a blue or black ball-point pen (but not a felt marking pen), fill completely the numbered oval corresponding to your response for each statement. If using a pen, do not alter original response by making another selection.

Part II *(on the reverse side)* requires a written answer.

Course Identification: Please print course and section you are evaluating

COURSE MAT401

SECTION

INSTRUCTOR(S):

A: Dror

B:

1. If evaluating only one instructor, write the name in the upper (A) box. If evaluating two instructors, write their names, one in box A and the other in box B.

DO NOT EVALUATE TEACHING ASSISTANTS ON THIS FORM

Statements about the instructor(s):

Respond to the statements below for instructor A (and instructor B) bearing in mind that there are wide variations in class size and subject matter in Arts and Science.

		extremely poor	very poor	poor	adequate	good	very good	outstanding
2. Communicates goals and requirements of the course clearly and explicitly.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
3. Uses methods of evaluation (e.g. papers, assignments, tests) that appropriately reflect the subject matter and provide a fair evaluation of student learning.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
4. Presents material in an organized, well-planned manner.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
5. Explains concepts clearly with appropriate use of examples.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
6. Communicates enthusiasm and interest in the course material.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
7. Attends to students' questions and answers them clearly and effectively.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
8. Is available for individual consultation, by appointment or stated office hours, to students with problems relating to the course.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
9. Ensures that student work is graded fairly, with helpful comments and feedback where appropriate.	A:	1	2	3	●	5	6	7
	B:	1	2	3	4	5	6	7
10. Ensures that student work is graded within a reasonable time.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
11. All things considered, performs effectively as a university teacher.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7

PART I CONTINUES ON THE REVERSE SIDE

Statements about the course: Respond to the statements below, using the following 7-point scale.

SIDE 2

	very low	low	below average	average	above average	high	very high
12. Compared to other courses at the same level (100,200,300,400), the work load is	①	②	③	●	⑤	⑥	⑦
13. Compared to other courses at the same level, the level of difficulty of the material is	①	②	③	●	⑤	⑥	⑦
14. The value of the required reading is	①	②	③	④	⑤	●	⑦
15. (If applicable) The value of the tutorials is	①	②	③	④	⑤	⑥	⑦
16. (If applicable) The value of the laboratories is	①	②	③	④	⑤	⑥	⑦
17. (If applicable) The value of the seminars is	①	②	③	④	⑤	⑥	⑦
18. (If applicable) The value of the language conversation classes is	①	②	③	④	⑤	⑥	⑦
19. The value of the overall learning experience is	①	②	③	④	⑤	●	⑦

20. Considering your experience with this course, and disregarding your need for it to meet program or degree requirements, would you still have taken this course? ● Yes ○ No

Statements about yourself:

21. Number of full course credits already earned (prior to this session):
① 0-4½ ② 5-9½ ③ 10-14½ ● 15-19½ ⑤ ≥20

22. Status of the course for you:
● Program Requirement ② Selected from a required list in a program ③ Breadth Requirement ④ Optional

23. Your level of enthusiasm to take this course at the time of initial registration:
① low ② medium ● high

24. Your expected grade in this course:
① <50 ② 50-59 ③ 60-69 ● 70-79 ⑤ ≥80

Additional statements or questions which may be supplied in class:

25. ① ② ③ ④ ⑤ ⑥ ⑦
26. ① ② ③ ④ ⑤ ⑥ ⑦
27. ① ② ③ ④ ⑤ ⑥ ⑦
28. ① ② ③ ④ ⑤ ⑥ ⑦
29. ① ② ③ ④ ⑤ ⑥ ⑦
30. ① ② ③ ④ ⑤ ⑥ ⑦
31. ① ② ③ ④ ⑤ ⑥ ⑦
32. ① ② ③ ④ ⑤ ⑥ ⑦
33. ① ② ③ ④ ⑤ ⑥ ⑦
34. ① ② ③ ④ ⑤ ⑥ ⑦
35. ① ② ③ ④ ⑤ ⑥ ⑦
36. ① ② ③ ④ ⑤ ⑥ ⑦

PART II: PLEASE ANSWER ONLY AFTER COMPLETING PART I. Please use the space below to provide supplementary comments on the instructor(s) or course. For example, you may wish to give the reasons for your numerical evaluations or provide specific suggestions for improving the instruction in the course.

END OF SURVEY. THANK YOU FOR YOUR COOPERATION AND PARTICIPATION.

STUDENT SURVEY FORM
UNIVERSITY OF TORONTO

FACULTY OF
ARTS & SCIENCE

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Part II *(on the reverse side)* requires a written answer.

Course Identification: Please print course and section you are evaluating

COURSE MAT401H1S

SECTION L5101

INSTRUCTOR(S):

A: Dror Bar-Natan

B:

1. If evaluating only one instructor, write the name in the upper (A) box. If evaluating two instructors, write their names, one in box A and the other in box B.

DO NOT EVALUATE TEACHING ASSISTANTS ON THIS FORM

Statements about the instructor(s):

Respond to the statements below for instructor A (and instructor B) bearing in mind that there are wide variations in class size and subject matter in Arts and Science.

		extremely poor	very poor	poor	adequate	good	very good	outstanding
2. Communicates goals and requirements of the course clearly and explicitly.	A:	1	2	3	4	●	6	7
	B:	1	2	3	4	5	6	7
3. Uses methods of evaluation (e.g. papers, assignments, tests) that appropriately reflect the subject matter and provide a fair evaluation of student learning.	A:	1	2	3	4	●	6	7
	B:	1	2	3	4	5	6	7
4. Presents material in an organized, well-planned manner.	A:	1	2	3	4	5	●	7
	B:	1	2	3	4	5	6	7
5. Explains concepts clearly with appropriate use of examples.	A:	1	2	3	4	●	6	7
	B:	1	2	3	4	5	6	7
6. Communicates enthusiasm and interest in the course material.	A:	1	2	3	4	●	6	7
	B:	1	2	3	4	5	6	7
7. Attends to students' questions and answers them clearly and effectively.	A:	1	2	3	4	●	6	7
	B:	1	2	3	4	5	6	7
8. Is available for individual consultation, by appointment or stated office hours, to students with problems relating to the course.	A:	1	2	3	4	●	6	7
	B:	1	2	3	4	5	6	7
9. Ensures that student work is graded fairly, with helpful comments and feedback where appropriate.	A:	1	2	3	4	●	6	7
	B:	1	2	3	4	5	6	7
10. Ensures that student work is graded within a reasonable time.	A:	1	2	3	4	●	6	7
	B:	1	2	3	4	5	6	7
11. All things considered, performs effectively as a university teacher.	A:	1	2	3	4	●	6	7
	B:	1	2	3	4	5	6	7

PART I CONTINUES ON THE REVERSE SIDE

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Statements about the course: Respond to the statements below, using the following 7-point scale.

SIDE 2

	very low	low	below average	average	above average	high	very high
12. Compared to other courses at the same level (100,200,300,400), the work load is	①	②	③	④	⑤	●	⑦
13. Compared to other courses at the same level, the level of difficulty of the material is	①	②	③	④	⑤	●	⑦
14. The value of the required reading is	①	②	③	④	●	⑥	⑦
15. (If applicable) The value of the tutorials is	①	②	③	●	⑤	⑥	⑦
16. (If applicable) The value of the laboratories is	①	②	③	●	⑤	⑥	⑦
17. (If applicable) The value of the seminars is	①	②	③	●	⑤	⑥	⑦
18. (If applicable) The value of the language conversation classes is	①	②	③	●	⑤	⑥	⑦
19. The value of the overall learning experience is	①	②	③	●	⑤	⑥	⑦

20. Considering your experience with this course, and disregarding your need for it to meet program or degree requirements, would you still have taken this course? ○ Yes ● No

Statements about yourself:

21. Number of full course credits already earned (prior to this session):
① 0-4½ ② 5-9½ ③ 10-14½ ④ 15-19½ ● ≥20

22. Status of the course for you:
● Program Requirement ② Selected from a required list in a program ③ Breadth Requirement ④ Optional

23. Your level of enthusiasm to take this course at the time of initial registration:
● low ② medium ③ high

24. Your expected grade in this course:
① <50 ② 50-59 ③ 60-69 ● 70-79 ⑤ ≥80

Additional statements or questions which may be supplied in class:

25. ① ② ③ ④ ⑤ ⑥ ⑦
26. ① ② ③ ④ ⑤ ⑥ ⑦
27. ① ② ③ ④ ⑤ ⑥ ⑦
28. ① ② ③ ④ ⑤ ⑥ ⑦
29. ① ② ③ ④ ⑤ ⑥ ⑦
30. ① ② ③ ④ ⑤ ⑥ ⑦
31. ① ② ③ ④ ⑤ ⑥ ⑦
32. ① ② ③ ④ ⑤ ⑥ ⑦
33. ① ② ③ ④ ⑤ ⑥ ⑦
34. ① ② ③ ④ ⑤ ⑥ ⑦
35. ① ② ③ ④ ⑤ ⑥ ⑦
36. ① ② ③ ④ ⑤ ⑥ ⑦

PART II: PLEASE ANSWER ONLY AFTER COMPLETING PART I. Please use the space below to provide supplementary comments on the instructor(s) or course. For example, you may wish to give the reasons for your numerical evaluations or provide specific suggestions for improving the instruction in the course.

END OF SURVEY. THANK YOU FOR YOUR COOPERATION AND PARTICIPATION.

STUDENT SURVEY FORM
UNIVERSITY OF TORONTO

FACULTY OF
ARTS & SCIENCE

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PART I: INSTRUCTIONS. PLEASE READ FIRST.

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Part II *(on the reverse side)* requires a written answer.

Course Identification: Please print course and section you are evaluating

COURSE MAT401H1S SECTION 5101

INSTRUCTOR(S):

A: Dior.

B:

1. **If evaluating only one instructor, write the name in the upper (A) box. If evaluating two instructors, write their names, one in box A and the other in box B.**

DO NOT EVALUATE TEACHING ASSISTANTS ON THIS FORM

Statements about the instructor(s):

Respond to the statements below for instructor A (and instructor B) bearing in mind that there are wide variations in class size and subject matter in Arts and Science.

		extremely poor	very poor	poor	adequate	good	very good	outstanding
2. Communicates goals and requirements of the course clearly and explicitly.	A:	1	2	3	[✓ 4]	5	6	7
	B:	1	2	3	4	5	6	7
3. Uses methods of evaluation (e.g. papers, assignments, tests) that appropriately reflect the subject matter and provide a fair evaluation of student learning.	A:	1	2	[✓ 3]	4	5	6	7
	B:	1	2	3	4	5	6	7
4. Presents material in an organized, well-planned manner.	A:	1	2	3	4	[✓ 5]	6	7
	B:	1	2	3	4	5	6	7
5. Explains concepts clearly with appropriate use of examples.	A:	1	2	[✓ 3]	4	5	6	7
	B:	1	2	3	4	5	6	7
6. Communicates enthusiasm and interest in the course material.	A:	1	2	[✓ 3]	4	5	6	7
	B:	1	2	3	4	5	6	7
7. Attends to students' questions and answers them clearly and effectively.	A:	1	2	3	[✓ 4]	5	6	7
	B:	1	2	3	4	5	6	7
8. Is available for individual consultation, by appointment or stated office hours, to students with problems relating to the course.	A:	1	2	3	[✓ 4]	5	6	7
	B:	1	2	3	4	5	6	7
9. Ensures that student work is graded fairly, with helpful comments and feedback where appropriate.	A:	1	2	3	[✓ 4]	5	6	7
	B:	1	2	3	4	5	6	7
10. Ensures that student work is graded within a reasonable time.	A:	1	2	3	[✓ 4]	5	6	7
	B:	1	2	3	4	5	6	7
11. All things considered, performs effectively as a university teacher.	A:	1	2	3	[✓ 4]	5	6	7
	B:	1	2	3	4	5	6	7

PART I CONTINUES ON THE REVERSE SIDE

Statements about the course: Respond to the statements below, using the following 7-point scale.

SIDE 2

	very low	low	below average	average	above average	high	very high
12. Compared to other courses at the same level (100,200,300,400), the work load is	1	2	3	4	5	[✓ 6]	7
13. Compared to other courses at the same level, the level of difficulty of the material is	1	2	3	4	5	[✓ 6]	7
14. The value of the required reading is	1	2	3	4	5	[✓ 6]	7
15. (If applicable) The value of the tutorials is	1	2	3	4	5	[✓ 6]	7
16. (If applicable) The value of the laboratories is	1	2	3	4	5	[✓ 6]	7
17. (If applicable) The value of the seminars is	1	2	3	4	5	[✓ 6]	7
18. (If applicable) The value of the language conversation classes is	1	2	3	4	5	[✓ 6]	7
19. The value of the overall learning experience is	1	2	3	4	5	[✓ 6]	7

20. Considering your experience with this course, and disregarding your need for it to meet program or degree requirements, would you still have taken this course? ○ Yes ○ No

Statements about yourself:

21. Number of full course credits already earned (prior to this session):
○ 0-4½ ○ 5-9½ ○ 10-14½ ○ 15-19½ ○ ≥20

22. Status of the course for you:
○ Program Requirement ○ Selected from a required list in a program ○ Breadth Requirement ○ Optional

23. Your level of enthusiasm to take this course at the time of initial registration:
○ low ○ medium ○ high

24. Your expected grade in this course:
○ <50 ○ 50-59 ○ 60-69 ○ 70-79 ○ ≥80

Additional statements or questions which may be supplied in class:

25. ① ② ③ ④ ⑤ ⑥ ⑦
26. ① ② ③ ④ ⑤ ⑥ ⑦
27. ① ② ③ ④ ⑤ ⑥ ⑦
28. ① ② ③ ④ ⑤ ⑥ ⑦
29. ① ② ③ ④ ⑤ ⑥ ⑦
30. ① ② ③ ④ ⑤ ⑥ ⑦
31. ① ② ③ ④ ⑤ ⑥ ⑦
32. ① ② ③ ④ ⑤ ⑥ ⑦
33. ① ② ③ ④ ⑤ ⑥ ⑦
34. ① ② ③ ④ ⑤ ⑥ ⑦
35. ① ② ③ ④ ⑤ ⑥ ⑦
36. ① ② ③ ④ ⑤ ⑥ ⑦

PART II: PLEASE ANSWER ONLY AFTER COMPLETING PART I. Please use the space below to provide supplementary comments on the instructor(s) or course. For example, you may wish to give the reasons for your numerical evaluations or provide specific suggestions for improving the instruction in the course.

END OF SURVEY. THANK YOU FOR YOUR COOPERATION AND PARTICIPATION.

STUDENT SURVEY FORM
UNIVERSITY OF TORONTO

FACULTY OF
ARTS & SCIENCE

Note that survey results will be available to the instructor(s) only after final course marks have been submitted.

PART I: INSTRUCTIONS. PLEASE READ FIRST.

Using an HB pencil or a blue or black ball-point pen (but not a felt marking pen), fill completely the numbered oval corresponding to your response for each statement. If using a pen, do not alter original response by making another selection.

Part II *(on the reverse side)* requires a written answer.

Course Identification: Please print course and section you are evaluating

COURSE MAT401H1S SECTION L5101

INSTRUCTOR(S):

A: Dror Bar Natan

B:

1. If evaluating only one instructor, write the name in the upper (A) box. If evaluating two instructors, write their names, one in box A and the other in box B.

DO NOT EVALUATE TEACHING ASSISTANTS ON THIS FORM

Statements about the instructor(s):

Respond to the statements below for instructor A (and instructor B) bearing in mind that there are wide variations in class size and subject matter in Arts and Science.

		extremely poor	very poor	poor	adequate	good	very good	outstanding
2. Communicates goals and requirements of the course clearly and explicitly.	A:	1	2	3	4	●	6	7
	B:	1	2	3	4	5	6	7
3. Uses methods of evaluation (e.g. papers, assignments, tests) that appropriately reflect the subject matter and provide a fair evaluation of student learning.	A:	1	2	3	●	5	6	7
	B:	1	2	3	4	5	6	7
4. Presents material in an organized, well-planned manner.	A:	1	2	3	●	5	6	7
	B:	1	2	3	4	5	6	7
5. Explains concepts clearly with appropriate use of examples.	A:	1	2	3	●	5	6	7
	B:	1	2	3	4	5	6	7
6. Communicates enthusiasm and interest in the course material.	A:	1	2	3	4	●	6	7
	B:	1	2	3	4	5	6	7
7. Attends to students' questions and answers them clearly and effectively.	A:	1	2	3	●	5	6	7
	B:	1	2	3	4	5	6	7
8. Is available for individual consultation, by appointment or stated office hours, to students with problems relating to the course.	A:	1	2	3	4	5	●	7
	B:	1	2	3	4	5	6	7
9. Ensures that student work is graded fairly, with helpful comments and feedback where appropriate.	A:	1	2	3	4	5	●	7
	B:	1	2	3	4	5	6	7
10. Ensures that student work is graded within a reasonable time.	A:	1	2	3	4	5	●	7
	B:	1	2	3	4	5	6	7
11. All things considered, performs effectively as a university teacher.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7

PART I CONTINUES ON THE REVERSE SIDE

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Statements about the course: Respond to the statements below, using the following 7-point scale.

SIDE 2

	very low	low	below average	average	above average	high	very high
12. Compared to other courses at the same level (100,200,300,400), the work load is	1	2	3	4	5	6	●
13. Compared to other courses at the same level, the level of difficulty of the material is	1	2	3	4	5	6	●
14. The value of the required reading is	1	2	●	4	5	6	7
15. (If applicable) The value of the tutorials is	1	2	3	4	5	6	7
16. (If applicable) The value of the laboratories is	1	2	3	4	5	6	7
17. (If applicable) The value of the seminars is	1	2	3	4	5	6	7
18. (If applicable) The value of the language conversation classes is	1	2	3	4	5	6	7
19. The value of the overall learning experience is	1	2	3	4	5	6	7

20. Considering your experience with this course, and disregarding your need for it to meet program or degree requirements, would you still have taken this course? ○ Yes ● No

Statements about yourself:

21. Number of full course credits already earned (prior to this session):
    1 0-4½ 2 5-9½ 3 10-14½ ● 15-19½ 5 ≥20

22. Status of the course for you:
    ● Program Requirement 2 Selected from a required list in a program 3 Breadth Requirement 4 Optional

23. Your level of enthusiasm to take this course at the time of initial registration:
    ● low 2 medium 3 high

24. Your expected grade in this course:
    1 <50 2 50-59 3 60-69 4 70-79 5 ≥80

Additional statements or questions which may be supplied in class:

25. 1 2 3 4 5 6 7
26. 1 2 3 4 5 6 7
27. 1 2 3 4 5 6 7
28. 1 2 3 4 5 6 7
29. 1 2 3 4 5 6 7
30. 1 2 3 4 5 6 7
31. 1 2 3 4 5 6 7
32. 1 2 3 4 5 6 7
33. 1 2 3 4 5 6 7
34. 1 2 3 4 5 6 7
35. 1 2 3 4 5 6 7
36. 1 2 3 4 5 6 7

PART II: PLEASE ANSWER ONLY AFTER COMPLETING PART I. Please use the space below to provide supplementary comments on the instructor(s) or course. For example, you may wish to give the reasons for your numerical evaluations or provide specific suggestions for improving the instruction in the course.

END OF SURVEY. THANK YOU FOR YOUR COOPERATION AND PARTICIPATION.

STUDENT SURVEY FORM
UNIVERSITY OF TORONTO

FACULTY OF
ARTS & SCIENCE

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PART I: INSTRUCTIONS. PLEASE READ FIRST.

Using an HB pencil or a blue or black ball-point pen (but not a felt marking pen), fill completely the numbered oval corresponding to your response for each statement. If using a pen, do not alter original response by making another selection.

Part II *(on the reverse side)* requires a written answer.

Course Identification: Please print course and section you are evaluating

COURSE: M A T 4 0 1 H 1 S

SECTION: L 5 1 0 1

INSTRUCTOR(S):

A: Dror Bar Natan

B:

1. If evaluating only one instructor, write the name in the upper (A) box. If evaluating two instructors, write their names, one in box A and the other in box B.

DO NOT EVALUATE TEACHING ASSISTANTS ON THIS FORM

Statements about the instructor(s):

Respond to the statements below for instructor A (and instructor B) bearing in mind that there are wide variations in class size and subject matter in Arts and Science.

		extremely poor	very poor	poor	adequate	good	very good	outstanding
2. Communicates goals and requirements of the course clearly and explicitly.	A:	①	②	③	④	●	⑥	⑦
	B:	①	②	③	④	⑤	⑥	⑦
3. Uses methods of evaluation (e.g. papers, assignments, tests) that appropriately reflect the subject matter and provide a fair evaluation of student learning.	A:	①	②	③	④	⑤	●	⑦
	B:	①	②	③	④	⑤	⑥	⑦
4. Presents material in an organized, well-planned manner.	A:	①	②	③	④	●	⑥	⑦
	B:	①	②	③	④	⑤	⑥	⑦
5. Explains concepts clearly with appropriate use of examples.	A:	①	②	③	④	⑤	●	⑦
	B:	①	②	③	④	⑤	⑥	⑦
6. Communicates enthusiasm and interest in the course material.	A:	①	②	③	④	⑤	●	⑦
	B:	①	②	③	④	⑤	⑥	⑦
7. Attends to students' questions and answers them clearly and effectively.	A:	①	②	③	④	⑤	●	⑦
	B:	①	②	③	④	⑤	⑥	⑦
8. Is available for individual consultation, by appointment or stated office hours, to students with problems relating to the course.	A:	①	②	③	④	●	⑥	⑦
	B:	①	②	③	④	⑤	⑥	⑦
9. Ensures that student work is graded fairly, with helpful comments and feedback where appropriate.	A:	①	②	③	●	⑤	⑥	⑦
	B:	①	②	③	④	⑤	⑥	⑦
10. Ensures that student work is graded within a reasonable time.	A:	①	②	③	④	●	⑥	⑦
	B:	①	②	③	④	⑤	⑥	⑦
11. All things considered, performs effectively as a university teacher.	A:	①	②	③	④	⑤	●	⑦
	B:	①	②	③	④	⑤	⑥	⑦

PART I CONTINUES ON THE REVERSE SIDE

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Statements about the course: Respond to the statements below, using the following 7-point scale.

SIDE 2

	very low	low	below average	average	above average	high	very high
12. Compared to other courses at the same level (100,200,300,400), the work load is	①	②	③	●	⑤	⑥	⑦
13. Compared to other courses at the same level, the level of difficulty of the material is	①	②	③	④	⑤	●	⑦
14. The value of the required reading is	①	②	③	●	⑤	⑥	⑦
15. (If applicable) The value of the tutorials is	①	②	③	④	⑤	⑥	⑦
16. (If applicable) The value of the laboratories is	①	②	③	④	⑤	⑥	⑦
17. (If applicable) The value of the seminars is	①	②	③	④	⑤	⑥	⑦
18. (If applicable) The value of the language conversation classes is	①	②	③	④	⑤	⑥	⑦
19. The value of the overall learning experience is	①	②	③	④	●	⑥	⑦

20. Considering your experience with this course, and disregarding your need for it to meet program or degree requirements, would you still have taken this course?
○ Yes ● No

Statements about yourself:

21. Number of full course credits already earned (prior to this session):
① 0-4½ ② 5-9½ ③ 10-14½ ● 15-19½ ⑤ ≥20

22. Status of the course for you:
● Program Requirement ② Selected from a required list in a program ③ Breadth Requirement ④ Optional

23. Your level of enthusiasm to take this course at the time of initial registration:
① low ● medium ③ high

24. Your expected grade in this course:
① <50 ② 50-59 ③ 60-69 ● 70-79 ⑤ ≥80

Additional statements or questions which may be supplied in class:

25. ① ② ③ ④ ⑤ ⑥ ⑦
26. ① ② ③ ④ ⑤ ⑥ ⑦
27. ① ② ③ ④ ⑤ ⑥ ⑦
28. ① ② ③ ④ ⑤ ⑥ ⑦
29. ① ② ③ ④ ⑤ ⑥ ⑦
30. ① ② ③ ④ ⑤ ⑥ ⑦
31. ① ② ③ ④ ⑤ ⑥ ⑦
32. ① ② ③ ④ ⑤ ⑥ ⑦
33. ① ② ③ ④ ⑤ ⑥ ⑦
34. ① ② ③ ④ ⑤ ⑥ ⑦
35. ① ② ③ ④ ⑤ ⑥ ⑦
36. ① ② ③ ④ ⑤ ⑥ ⑦

PART II: PLEASE ANSWER ONLY AFTER COMPLETING PART I. Please use the space below to provide supplementary comments on the instructor(s) or course. For example, you may wish to give the reasons for your numerical evaluations or provide specific suggestions for improving the instruction in the course.

END OF SURVEY. THANK YOU FOR YOUR COOPERATION AND PARTICIPATION.

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UNIVERSITY OF TORONTO

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PART I: INSTRUCTIONS. PLEASE READ FIRST.

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Part II *(on the reverse side)* requires a written answer.

Course Identification: Please print course and section you are evaluating

COURSE MAT401H1S

SECTION L5101

INSTRUCTOR(S):

A: Dror Bar Natn

B:

1. If evaluating only one instructor, write the name in the upper (A) box. If evaluating two instructors, write their names, one in box A and the other in box B.

DO NOT EVALUATE TEACHING ASSISTANTS ON THIS FORM

Statements about the instructor(s):

Respond to the statements below for instructor A (and instructor B) bearing in mind that there are wide variations in class size and subject matter in Arts and Science.

		extremely poor	very poor	poor	adequate	good	very good	outstanding
2. Communicates goals and requirements of the course clearly and explicitly.	A:	①	②	③	④	⑤	●	⑦
	B:	①	②	③	④	⑤	⑥	⑦
3. Uses methods of evaluation (e.g. papers, assignments, tests) that appropriately reflect the subject matter and provide a fair evaluation of student learning.	A:	①	②	③	④	⑤	●	⑦
	B:	①	②	③	④	⑤	⑥	⑦
4. Presents material in an organized, well-planned manner.	A:	①	②	③	④	⑤	●	⑦
	B:	①	②	③	④	⑤	⑥	⑦
5. Explains concepts clearly with appropriate use of examples.	A:	①	②	③	④	⑤	●	⑦
	B:	①	②	③	④	⑤	⑥	⑦
6. Communicates enthusiasm and interest in the course material.	A:	①	②	③	④	⑤	●	⑦
	B:	①	②	③	④	⑤	⑥	⑦
7. Attends to students' questions and answers them clearly and effectively.	A:	①	②	③	④	●	⑥	⑦
	B:	①	②	③	④	⑤	⑥	⑦
8. Is available for individual consultation, by appointment or stated office hours, to students with problems relating to the course.	A:	①	②	③	④	●	⑥	⑦
	B:	①	②	③	④	⑤	⑥	⑦
9. Ensures that student work is graded fairly, with helpful comments and feedback where appropriate.	A:	①	②	③	●	⑤	⑥	⑦
	B:	①	②	③	④	⑤	⑥	⑦
10. Ensures that student work is graded within a reasonable time.	A:	①	②	③	④	⑤	●	⑦
	B:	①	②	③	④	⑤	⑥	⑦
11. All things considered, performs effectively as a university teacher.	A:	①	②	③	④	⑤	●	⑦
	B:	①	②	③	④	⑤	⑥	⑦

PART I CONTINUES ON THE REVERSE SIDE

Statements about the course: Respond to the statements below, using the following 7-point scale.

SIDE 2

	very low	low	below average	average	above average	high	very high
12. Compared to other courses at the same level (100,200,300,400), the work load is	①	②	③	④	●	⑥	⑦
13. Compared to other courses at the same level, the level of difficulty of the material is	①	②	③	④	●	⑥	⑦
14. The value of the required reading is	①	②	③	④	●	⑥	⑦
15. (If applicable) The value of the tutorials is	①	②	③	④	⑤	⑥	⑦
16. (If applicable) The value of the laboratories is	①	②	③	④	⑤	⑥	⑦
17. (If applicable) The value of the seminars is	①	②	③	④	⑤	⑥	⑦
18. (If applicable) The value of the language conversation classes is	①	②	③	④	⑤	⑥	⑦
19. The value of the overall learning experience is	①	②	③	④	●	⑥	⑦

20. Considering your experience with this course, and disregarding your need for it to meet program or degree requirements, would you still have taken this course? ● Yes ○ No

Statements about yourself:

21. Number of full course credits already earned (prior to this session):
① 0-4½ ② 5-9½ ● 10-14½ ④ 15-19½ ⑤ ≥20

22. Status of the course for you:
● Program Requirement ② Selected from a required list in a program ③ Breadth Requirement ④ Optional

23. Your level of enthusiasm to take this course at the time of initial registration:
① low ● medium ③ high

24. Your expected grade in this course:
① <50 ② 50-59 ③ 60-69 ④ 70-79 ● ≥80

Additional statements or questions which may be supplied in class:

25. ① ② ③ ④ ⑤ ⑥ ⑦
26. ① ② ③ ④ ⑤ ⑥ ⑦
27. ① ② ③ ④ ⑤ ⑥ ⑦
28. ① ② ③ ④ ⑤ ⑥ ⑦
29. ① ② ③ ④ ⑤ ⑥ ⑦
30. ① ② ③ ④ ⑤ ⑥ ⑦
31. ① ② ③ ④ ⑤ ⑥ ⑦
32. ① ② ③ ④ ⑤ ⑥ ⑦
33. ① ② ③ ④ ⑤ ⑥ ⑦
34. ① ② ③ ④ ⑤ ⑥ ⑦
35. ① ② ③ ④ ⑤ ⑥ ⑦
36. ① ② ③ ④ ⑤ ⑥ ⑦

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STUDENT SURVEY FORM
UNIVERSITY OF TORONTO

FACULTY OF
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Part II *(on the reverse side)* requires a written answer.

Course Identification: Please print course and section you are evaluating

COURSE: MAT401H1S

SECTION: L5101

INSTRUCTOR(S):

A: DROR BAR NATAN

B:

1. If evaluating only one instructor, write the name in the upper (A) box. If evaluating two instructors, write their names, one in box A and the other in box B.

DO NOT EVALUATE TEACHING ASSISTANTS ON THIS FORM

Statements about the instructor(s):

Respond to the statements below for instructor A (and instructor B) bearing in mind that there are wide variations in class size and subject matter in Arts and Science.

		extremely poor	very poor	poor	adequate	good	very good	outstanding
2. Communicates goals and requirements of the course clearly and explicitly.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
3. Uses methods of evaluation (e.g. papers, assignments, tests) that appropriately reflect the subject matter and provide a fair evaluation of student learning.	A:	1	2	3	4	●	6	7
	B:	1	2	3	4	5	6	7
4. Presents material in an organized, well-planned manner.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
5. Explains concepts clearly with appropriate use of examples.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
6. Communicates enthusiasm and interest in the course material.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
7. Attends to students' questions and answers them clearly and effectively.	A:	1	2	3	4	5	6	●
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	B:	1	2	3	4	5	6	7
9. Ensures that student work is graded fairly, with helpful comments and feedback where appropriate.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
10. Ensures that student work is graded within a reasonable time.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
11. All things considered, performs effectively as a university teacher.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7

PART I CONTINUES ON THE REVERSE SIDE

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Statements about the course: Respond to the statements below, using the following 7-point scale. *SIDE 2*

	very low	low	below average	average	above average	high	very high
12. Compared to other courses at the same level (100,200,300,400), the work load is	①	②	③	④	●	⑥	⑦
13. Compared to other courses at the same level, the level of difficulty of the material is	①	②	③	④	⑤	●	⑦
14. The value of the required reading is	①	②	③	④	⑤	●	⑦
15. (If applicable) The value of the tutorials is	①	②	③	④	⑤	⑥	⑦
16. (If applicable) The value of the laboratories is	①	②	③	④	⑤	⑥	⑦
17. (If applicable) The value of the seminars is	①	②	③	④	⑤	⑥	⑦
18. (If applicable) The value of the language conversation classes is	①	②	③	④	⑤	⑥	⑦
19. The value of the overall learning experience is	①	②	③	④	⑤	●	⑦

20. Considering your experience with this course, and disregarding your need for it to meet program or degree requirements, would you still have taken this course? ○ Yes ● No

Statements about yourself:

21. Number of full course credits already earned (prior to this session):
① 0-4½ ② 5-9½ ● 10-14½ ④ 15-19½ ⑤ ≥20

22. Status of the course for you:
● Program Requirement ② Selected from a required list in a program ③ Breadth Requirement ④ Optional

23. Your level of enthusiasm to take this course at the time of initial registration:
① low ● medium ③ high

24. Your expected grade in this course:
① <50 ② 50-59 ③ 60-69 ● 70-79 ⑤ ≥80

Additional statements or questions which may be supplied in class:

25. ① ② ③ ④ ⑤ ⑥ ⑦
26. ① ② ③ ④ ⑤ ⑥ ⑦
27. ① ② ③ ④ ⑤ ⑥ ⑦
28. ① ② ③ ④ ⑤ ⑥ ⑦
29. ① ② ③ ④ ⑤ ⑥ ⑦
30. ① ② ③ ④ ⑤ ⑥ ⑦
31. ① ② ③ ④ ⑤ ⑥ ⑦
32. ① ② ③ ④ ⑤ ⑥ ⑦
33. ① ② ③ ④ ⑤ ⑥ ⑦
34. ① ② ③ ④ ⑤ ⑥ ⑦
35. ① ② ③ ④ ⑤ ⑥ ⑦
36. ① ② ③ ④ ⑤ ⑥ ⑦

PART II: PLEASE ANSWER ONLY AFTER COMPLETING PART I. Please use the space below to provide supplementary comments on the instructor(s) or course. For example, you may wish to give the reasons for your numerical evaluations or provide specific suggestions for improving the instruction in the course.

END OF SURVEY. THANK YOU FOR YOUR COOPERATION AND PARTICIPATION.

STUDENT SURVEY FORM
UNIVERSITY OF TORONTO

FACULTY OF
ARTS & SCIENCE

Note that survey results will be available to the instructor(s) only after final course marks have been submitted.

PART I: INSTRUCTIONS. PLEASE READ FIRST.

Using an HB pencil or a blue or black ball-point pen (but not a felt marking pen), fill completely the numbered oval corresponding to your response for each statement. If using a pen, do not alter original response by making another selection.

Part II *(on the reverse side)* requires a written answer.

Course Identification: Please print course and section you are evaluating

COURSE MAT401H1S

SECTION

INSTRUCTOR(S):

A: Bar-Natan

B:

1. If evaluating only one instructor, write the name in the upper (A) box. If evaluating two instructors, write their names, one in box A and the other in box B.

DO NOT EVALUATE TEACHING ASSISTANTS ON THIS FORM

Statements about the instructor(s):

Respond to the statements below for instructor A (and instructor B) bearing in mind that there are wide variations in class size and subject matter in Arts and Science.

		extremely poor	very poor	poor	adequate	good	very good	outstanding
2. Communicates goals and requirements of the course clearly and explicitly.	A:	①	②	③	④	⑤	●	⑦
	B:	①	②	③	④	⑤	⑥	⑦
3. Uses methods of evaluation (e.g. papers, assignments, tests) that appropriately reflect the subject matter and provide a fair evaluation of student learning.	A:	①	②	③	●	⑤	⑥	⑦
	B:	①	②	③	④	⑤	⑥	⑦
4. Presents material in an organized, well-planned manner.	A:	①	②	③	④	●	⑥	⑦
	B:	①	②	③	④	⑤	⑥	⑦
5. Explains concepts clearly with appropriate use of examples.	A:	①	②	③	④	●	⑥	⑦
	B:	①	②	③	④	⑤	⑥	⑦
6. Communicates enthusiasm and interest in the course material.	A:	①	②	③	④	⑤	●	⑦
	B:	①	②	③	④	⑤	⑥	⑦
7. Attends to students' questions and answers them clearly and effectively.	A:	①	②	③	④	⑤	●	⑦
	B:	①	②	③	④	⑤	⑥	⑦
8. Is available for individual consultation, by appointment or stated office hours, to students with problems relating to the course.	A:	①	②	③	④	⑤	●	⑦
	B:	①	②	③	④	⑤	⑥	⑦
9. Ensures that student work is graded fairly, with helpful comments and feedback where appropriate.	A:	①	②	③	④	⑤	●	⑦
	B:	①	②	③	④	⑤	⑥	⑦
10. Ensures that student work is graded within a reasonable time.	A:	①	②	③	④	⑤	⑥	●
	B:	①	②	③	④	⑤	⑥	⑦
11. All things considered, performs effectively as a university teacher.	A:	①	②	③	④	●	⑥	⑦
	B:	①	②	③	④	⑤	⑥	⑦

PART I CONTINUES ON THE REVERSE SIDE

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Statements about the course: Respond to the statements below, using the following 7-point scale. *SIDE 2*

	very low	low	below average	average	above average	high	very high
12. Compared to other courses at the same level (100,200,300,400), the work load is	①	②	③	④	⑤	⑥	●
13. Compared to other courses at the same level, the level of difficulty of the material is	①	②	③	④	⑤	⑥	●
14. The value of the required reading is	①	②	③	④	⑤	●	⑦
15. (If applicable) The value of the tutorials is	①	②	③	④	⑤	⑥	⑦
16. (If applicable) The value of the laboratories is	①	②	③	④	⑤	⑥	⑦
17. (If applicable) The value of the seminars is	①	②	③	④	⑤	⑥	⑦
18. (If applicable) The value of the language conversation classes is	①	②	③	④	⑤	⑥	⑦
19. The value of the overall learning experience is	①	②	③	④	●	⑥	⑦

20. Considering your experience with this course, and disregarding your need for it to meet program or degree requirements, would you still have taken this course? ○ Yes ● No

Statements about yourself:

21. Number of full course credits already earned (prior to this session):
① 0-4½ ② 5-9½ ③ 10-14½ ● 15-19½ ⑤ ≥20

22. Status of the course for you:
● Program Requirement ② Selected from a required list in a program ③ Breadth Requirement ④ Optional

23. Your level of enthusiasm to take this course at the time of initial registration:
① low ● medium ③ high

24. Your expected grade in this course:
① <50 ② 50-59 ● 60-69 ④ 70-79 ⑤ ≥80

Additional statements or questions which may be supplied in class:

25. ① ② ③ ④ ⑤ ⑥ ⑦
26. ① ② ③ ④ ⑤ ⑥ ⑦
27. ① ② ③ ④ ⑤ ⑥ ⑦
28. ① ② ③ ④ ⑤ ⑥ ⑦
29. ① ② ③ ④ ⑤ ⑥ ⑦
30. ① ② ③ ④ ⑤ ⑥ ⑦
31. ① ② ③ ④ ⑤ ⑥ ⑦
32. ① ② ③ ④ ⑤ ⑥ ⑦
33. ① ② ③ ④ ⑤ ⑥ ⑦
34. ① ② ③ ④ ⑤ ⑥ ⑦
35. ① ② ③ ④ ⑤ ⑥ ⑦
36. ① ② ③ ④ ⑤ ⑥ ⑦

PART II: PLEASE ANSWER ONLY AFTER COMPLETING PART I. Please use the space below to provide supplementary comments on the instructor(s) or course. For example, you may wish to give the reasons for your numerical evaluations or provide specific suggestions for improving the instruction in the course.

Weekly problem sets not a good indicator of students' mastering of content - their completion is affected by too many other variables to be a fair assessment.

END OF SURVEY. THANK YOU FOR YOUR COOPERATION AND PARTICIPATION.

STUDENT SURVEY FORM
UNIVERSITY OF TORONTO

FACULTY OF
ARTS & SCIENCE

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PART I: INSTRUCTIONS. PLEASE READ FIRST.

Using an HB pencil or a blue or black ball-point pen (but not a felt marking pen), fill completely the numbered oval corresponding to your response for each statement. If using a pen, do not alter original response by making another selection.

Part II *(on the reverse side)* requires a written answer.

Course Identification: Please print course and section you are evaluating

COURSE MAT401H1S

SECTION L5101

INSTRUCTOR(S):

A: D. BAR NATAN

B:

1. If evaluating only one instructor, write the name in the upper (A) box. If evaluating two instructors, write their names, one in box A and the other in box B.

DO NOT EVALUATE TEACHING ASSISTANTS ON THIS FORM

Statements about the instructor(s):

Respond to the statements below for instructor A (and instructor B) bearing in mind that there are wide variations in class size and subject matter in Arts and Science.

		extremely poor	very poor	poor	adequate	good	very good	outstanding
2. Communicates goals and requirements of the course clearly and explicitly.	A:	1	2	3	4	●	6	7
	B:	1	2	3	4	5	6	7
3. Uses methods of evaluation (e.g. papers, assignments, tests) that appropriately reflect the subject matter and provide a fair evaluation of student learning.	A:	1	2	●	4	5	6	7
	B:	1	2	3	4	5	6	7
4. Presents material in an organized, well-planned manner.	A:	1	2	3	●	5	6	7
	B:	1	2	3	4	5	6	7
5. Explains concepts clearly with appropriate use of examples.	A:	1	2	3	4	5	●	7
	B:	1	2	3	4	5	6	7
6. Communicates enthusiasm and interest in the course material.	A:	1	2	3	4	5	●	7
	B:	1	2	3	4	5	6	7
7. Attends to students' questions and answers them clearly and effectively.	A:	1	2	3	4	5	●	7
	B:	1	2	3	4	5	6	7
8. Is available for individual consultation, by appointment or stated office hours, to students with problems relating to the course.	A:	1	2	3	4	●	6	7
	B:	1	2	3	4	5	6	7
9. Ensures that student work is graded fairly, with helpful comments and feedback where appropriate.	A:	1	2	3	●	5	6	7
	B:	1	2	3	4	5	6	7
10. Ensures that student work is graded within a reasonable time.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
11. All things considered, performs effectively as a university teacher.	A:	1	2	3	4	●	6	7
	B:	1	2	3	4	5	6	7

PART I CONTINUES ON THE REVERSE SIDE

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Statements about the course: Respond to the statements below, using the following 7-point scale.

SIDE 2

	very low	low	below average	average	above average	high	very high
12. Compared to other courses at the same level (100,200,300,400), the work load is	①	②	③	●	⑤	⑥	⑦
13. Compared to other courses at the same level, the level of difficulty of the material is	①	②	③	④	⑤	●	⑦
14. The value of the required reading is	①	②	③	④	⑤	⑥	⑦
15. (If applicable) The value of the tutorials is	①	②	③	④	⑤	⑥	⑦
16. (If applicable) The value of the laboratories is	①	②	③	④	⑤	⑥	⑦
17. (If applicable) The value of the seminars is	①	②	③	④	⑤	⑥	⑦
18. (If applicable) The value of the language conversation classes is	①	②	③	④	⑤	⑥	⑦
19. The value of the overall learning experience is	①	②	③	④	●	⑥	⑦

20. Considering your experience with this course, and disregarding your need for it to meet program or degree requirements, would you still have taken this course? ● Yes ○ No

Statements about yourself:

21. Number of full course credits already earned (prior to this session):
① 0-4½ ② 5-9½ ③ 10-14½ ● 15-19½ ⑤ ≥20

22. Status of the course for you:
● Program Requirement ② Selected from a required list in a program ③ Breadth Requirement ④ Optional

23. Your level of enthusiasm to take this course at the time of initial registration:
① low ② medium ● high

24. Your expected grade in this course:
① <50 ② 50-59 ● 60-69 ④ 70-79 ⑤ ≥80

Additional statements or questions which may be supplied in class:

25. ① ② ③ ④ ⑤ ⑥ ⑦
26. ① ② ③ ④ ⑤ ⑥ ⑦
27. ① ② ③ ④ ⑤ ⑥ ⑦
28. ① ② ③ ④ ⑤ ⑥ ⑦
29. ① ② ③ ④ ⑤ ⑥ ⑦
30. ① ② ③ ④ ⑤ ⑥ ⑦
31. ① ② ③ ④ ⑤ ⑥ ⑦
32. ① ② ③ ④ ⑤ ⑥ ⑦
33. ① ② ③ ④ ⑤ ⑥ ⑦
34. ① ② ③ ④ ⑤ ⑥ ⑦
35. ① ② ③ ④ ⑤ ⑥ ⑦
36. ① ② ③ ④ ⑤ ⑥ ⑦

PART II: PLEASE ANSWER ONLY AFTER COMPLETING PART I. Please use the space below to provide supplementary comments on the instructor(s) or course. For example, you may wish to give the reasons for your numerical evaluations or provide specific suggestions for improving the instruction in the course.

- TA does not mark assignments fairly
- Professor loses his train of thought quite often
- He is very enthusiastic about the material in class
- Exams are extremely difficult compared to the weekly assignments
- Material is not useful in real life.

END OF SURVEY. THANK YOU FOR YOUR COOPERATION AND PARTICIPATION.

STUDENT SURVEY FORM
UNIVERSITY OF TORONTO

FACULTY OF
ARTS & SCIENCE

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Part II *(on the reverse side)* requires a written answer.

Course Identification: Please print course and section you are evaluating

COURSE MAT401H1S

SECTION L5101

INSTRUCTOR(S):

A: Dror Bar-Natan

B:

1. **If evaluating only one instructor, write the name in the upper (A) box. If evaluating two instructors, write their names, one in box A and the other in box B.**

DO NOT EVALUATE TEACHING ASSISTANTS ON THIS FORM

Statements about the instructor(s):

Respond to the statements below for instructor A (and instructor B) bearing in mind that there are wide variations in class size and subject matter in Arts and Science.

		extremely poor	very poor	poor	adequate	good	very good	outstanding
2. Communicates goals and requirements of the course clearly and explicitly.	A:	①	②	③	④	⑤	●	⑦
	B:	①	②	③	④	⑤	⑥	⑦
3. Uses methods of evaluation (e.g. papers, assignments, tests) that appropriately reflect the subject matter and provide a fair evaluation of student learning.	A:	①	②	③	④	⑤	●	⑦
	B:	①	②	③	④	⑤	⑥	⑦
4. Presents material in an organized, well-planned manner.	A:	①	②	③	④	●	⑥	⑦
	B:	①	②	③	④	⑤	⑥	⑦
5. Explains concepts clearly with appropriate use of examples.	A:	①	②	③	④	●	⑥	⑦
	B:	①	②	③	④	⑤	⑥	⑦
6. Communicates enthusiasm and interest in the course material.	A:	①	②	③	④	⑤	⑥	●
	B:	①	②	③	④	⑤	⑥	⑦
7. Attends to students' questions and answers them clearly and effectively.	A:	①	②	③	④	⑤	●	⑦
	B:	①	②	③	④	⑤	⑥	⑦
8. Is available for individual consultation, by appointment or stated office hours, to students with problems relating to the course.	A:	①	②	③	④	⑤	⑥	●
	B:	①	②	③	④	⑤	⑥	⑦
9. Ensures that student work is graded fairly, with helpful comments and feedback where appropriate.	A:	①	②	③	④	⑤	●	⑦
	B:	①	②	③	④	⑤	⑥	⑦
10. Ensures that student work is graded within a reasonable time.	A:	①	②	③	④	⑤	⑥	●
	B:	①	②	③	④	⑤	⑥	⑦
11. All things considered, performs effectively as a university teacher.	A:	①	②	③	④	⑤	⑥	●
	B:	①	②	③	④	⑤	⑥	⑦

PART I CONTINUES ON THE REVERSE SIDE

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Statements about the course: Respond to the statements below, using the following 7-point scale.

SIDE 2

	very low	low	below average	average	above average	high	very high
12. Compared to other courses at the same level (100,200,300,400), the work load is	①	②	③	●	⑤	⑥	⑦
13. Compared to other courses at the same level, the level of difficulty of the material is	①	②	③	④	⑤	●	⑦
14. The value of the required reading is	①	②	③	●	⑤	⑥	⑦
15. (If applicable) The value of the tutorials is	①	②	③	④	⑤	⑥	⑦
16. (If applicable) The value of the laboratories is	①	②	③	④	⑤	⑥	⑦
17. (If applicable) The value of the seminars is	①	②	③	④	⑤	⑥	⑦
18. (If applicable) The value of the language conversation classes is	①	②	③	④	●	⑥	⑦
19. The value of the overall learning experience is	①	②	③	④	●	⑥	⑦

20. Considering your experience with this course, and disregarding your need for it to meet program or degree requirements, would you still have taken this course? ● Yes ○ No

Statements about yourself:

21. Number of full course credits already earned (prior to this session):
① 0-4½ ② 5-9½ ③ 10-14½ ● 15-19½ ⑤ ≥20
22. Status of the course for you:
● Program Requirement ② Selected from a required list in a program ③ Breadth Requirement ④ Optional
23. Your level of enthusiasm to take this course at the time of initial registration:
① low ② medium ③ high
24. Your expected grade in this course:
① <50 ② 50-59 ③ 60-69 ● 70-79 ⑤ ≥80

Additional statements or questions which may be supplied in class:

25. ① ② ③ ④ ⑤ ⑥ ⑦
26. ① ② ③ ④ ⑤ ⑥ ⑦
27. ① ② ③ ④ ⑤ ⑥ ⑦
28. ① ② ③ ④ ⑤ ⑥ ⑦
29. ① ② ③ ④ ⑤ ⑥ ⑦
30. ① ② ③ ④ ⑤ ⑥ ⑦
31. ① ② ③ ④ ⑤ ⑥ ⑦
32. ① ② ③ ④ ⑤ ⑥ ⑦
33. ① ② ③ ④ ⑤ ⑥ ⑦
34. ① ② ③ ④ ⑤ ⑥ ⑦
35. ① ② ③ ④ ⑤ ⑥ ⑦
36. ① ② ③ ④ ⑤ ⑥ ⑦

PART II: PLEASE ANSWER ONLY AFTER COMPLETING PART I. Please use the space below to provide supplementary comments on the instructor(s) or course. For example, you may wish to give the reasons for your numerical evaluations or provide specific suggestions for improving the instruction in the course.

Prof presents the material in the best manner, his enthusiasm and humor makes learning ~~about~~ the topics motivating and makes even some of the difficult material good.

That being said, the last month of material was extremely difficult, though I'm not sure what could have been done to fix this.

Great Course & Great Professor. ☺

END OF SURVEY. THANK YOU FOR YOUR COOPERATION AND PARTICIPATION.

STUDENT SURVEY FORM
UNIVERSITY OF TORONTO

FACULTY OF
ARTS & SCIENCE

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PART I: INSTRUCTIONS. PLEASE READ FIRST.

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Part II *(on the reverse side)* requires a written answer.

Course Identification: Please print course and section you are evaluating

COURSE MAT401H1S   SECTION L5101

INSTRUCTOR(S):

A: Dror Bar-Natan

B:

1. If evaluating only one instructor, write the name in the upper (A) box. If evaluating two instructors, write their names, one in box A and the other in box B.

DO NOT EVALUATE TEACHING ASSISTANTS ON THIS FORM

Statements about the instructor(s):

Respond to the statements below for instructor A (and instructor B) bearing in mind that there are wide variations in class size and subject matter in Arts and Science.

		extremely poor	very poor	poor	adequate	good	very good	outstanding
2. Communicates goals and requirements of the course clearly and explicitly.	A:	1	2	3	4	●	6	7
	B:	1	2	3	4	5	6	7
3. Uses methods of evaluation (e.g. papers, assignments, tests) that appropriately reflect the subject matter and provide a fair evaluation of student learning.	A:	1	2	●	4	5	6	7
	B:	1	2	3	4	5	6	7
4. Presents material in an organized, well-planned manner.	A:	1	2	3	4	●	6	7
	B:	1	2	3	4	5	6	7
5. Explains concepts clearly with appropriate use of examples.	A:	1	2	3	4	●	6	7
	B:	1	2	3	4	5	6	7
6. Communicates enthusiasm and interest in the course material.	A:	1	2	3	4	5	●	7
	B:	1	2	3	4	5	6	7
7. Attends to students' questions and answers them clearly and effectively.	A:	1	2	3	4	●	6	7
	B:	1	2	3	4	5	6	7
8. Is available for individual consultation, by appointment or stated office hours, to students with problems relating to the course.	A:	1	2	3	4	●	6	7
	B:	1	2	3	4	5	6	7
9. Ensures that student work is graded fairly, with helpful comments and feedback where appropriate.	A:	1	2	●	4	5	6	7
	B:	1	2	3	4	5	6	7
10. Ensures that student work is graded within a reasonable time.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
11. All things considered, performs effectively as a university teacher.	A:	1	2	3	4	5	●	7
	B:	1	2	3	4	5	6	7

PART I CONTINUES ON THE REVERSE SIDE

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Statements about the course: Respond to the statements below, using the following 7-point scale.

SIDE 2

	very low	low	below average	average	above average	high	very high
12. Compared to other courses at the same level (100,200,300,400), the work load is	①	②	③	④	⑤	●	⑦
13. Compared to other courses at the same level, the level of difficulty of the material is	①	②	③	④	⑤	●	⑦
14. The value of the required reading is	①	②	③	●	⑤	⑥	⑦
15. (If applicable) The value of the tutorials is	①	②	③	④	⑤	⑥	⑦
16. (If applicable) The value of the laboratories is	①	②	③	④	⑤	⑥	⑦
17. (If applicable) The value of the seminars is	①	②	③	④	⑤	⑥	⑦
18. (If applicable) The value of the language conversation classes is	①	②	③	④	⑤	⑥	⑦
19. The value of the overall learning experience is	①	②	③	●	⑤	⑥	⑦

20. Considering your experience with this course, and disregarding your need for it to meet program or degree requirements, would you still have taken this course? ● Yes ○ No

Statements about yourself:

21. Number of full course credits already earned (prior to this session):
① 0-4½ ② 5-9½ ③ 10-14½ ● 15-19½ ⑤ ≥20

22. Status of the course for you:
① Program Requirement ② Selected from a required list in a program ③ Breadth Requirement ④ Optional

23. Your level of enthusiasm to take this course at the time of initial registration:
① low ② medium ● high

24. Your expected grade in this course:
① <50 ② 50-59 ③ 60-69 ● 70-79 ⑤ ≥80

Additional statements or questions which may be supplied in class:

25. ① ② ③ ④ ⑤ ⑥ ⑦
26. ① ② ③ ④ ⑤ ⑥ ⑦
27. ① ② ③ ④ ⑤ ⑥ ⑦
28. ① ② ③ ④ ⑤ ⑥ ⑦
29. ① ② ③ ④ ⑤ ⑥ ⑦
30. ① ② ③ ④ ⑤ ⑥ ⑦
31. ① ② ③ ④ ⑤ ⑥ ⑦
32. ① ② ③ ④ ⑤ ⑥ ⑦
33. ① ② ③ ④ ⑤ ⑥ ⑦
34. ① ② ③ ④ ⑤ ⑥ ⑦
35. ① ② ③ ④ ⑤ ⑥ ⑦
36. ① ② ③ ④ ⑤ ⑥ ⑦

PART II: PLEASE ANSWER ONLY AFTER COMPLETING PART I. Please use the space below to provide supplementary comments on the instructor(s) or course. For example, you may wish to give the reasons for your numerical evaluations or provide specific suggestions for improving the instruction in the course.

- assignments every week keep you on track
- marking is too rough

END OF SURVEY. THANK YOU FOR YOUR COOPERATION AND PARTICIPATION.

STUDENT SURVEY FORM
UNIVERSITY OF TORONTO

FACULTY OF
ARTS & SCIENCE

Note that survey results will be available to the instructor(s) only after final course marks have been submitted.

PART I: INSTRUCTIONS. PLEASE READ FIRST.

Using an HB pencil or a blue or black ball-point pen (but not a felt marking pen), fill completely the numbered oval corresponding to your response for each statement. If using a pen, do not alter original response by making another selection.

Part II *(on the reverse side)* requires a written answer.

Course Identification: Please print course and section you are evaluating

COURSE MAT401H1S

SECTION L5101

INSTRUCTOR(S):

A: Dror Bar-Natan

B:

1. If evaluating only one instructor, write the name in the upper (A) box. If evaluating two instructors, write their names, one in box A and the other in box B.

DO NOT EVALUATE TEACHING ASSISTANTS ON THIS FORM

Statements about the instructor(s):

Respond to the statements below for instructor A (and instructor B) bearing in mind that there are wide variations in class size and subject matter in Arts and Science.

		extremely poor	very poor	poor	adequate	good	very good	outstanding
2. Communicates goals and requirements of the course clearly and explicitly.	A:	①	②	③	④	●	⑥	⑦
	B:	①	②	③	④	⑤	⑥	⑦
3. Uses methods of evaluation (e.g. papers, assignments, tests) that appropriately reflect the subject matter and provide a fair evaluation of student learning.	A:	①	②	③	●	⑤	⑥	⑦
	B:	①	②	③	④	⑤	⑥	⑦
4. Presents material in an organized, well-planned manner.	A:	①	②	③	④	●	⑥	⑦
	B:	①	②	③	④	⑤	⑥	⑦
5. Explains concepts clearly with appropriate use of examples.	A:	①	②	③	●	⑤	⑥	⑦
	B:	①	②	③	④	⑤	⑥	⑦
6. Communicates enthusiasm and interest in the course material.	A:	①	②	③	④	⑤	●	⑦
	B:	①	②	③	④	⑤	⑥	⑦
7. Attends to students' questions and answers them clearly and effectively.	A:	①	②	③	④	●	⑥	⑦
	B:	①	②	③	④	⑤	⑥	⑦
8. Is available for individual consultation, by appointment or stated office hours, to students with problems relating to the course.	A:	①	②	③	④	⑤	⑥	⑦
	B:	①	②	③	④	⑤	⑥	⑦
9. Ensures that student work is graded fairly, with helpful comments and feedback where appropriate.	A:	①	②	③	④	●	⑥	⑦
	B:	①	②	③	④	⑤	⑥	⑦
10. Ensures that student work is graded within a reasonable time.	A:	①	②	③	④	⑤	●	⑦
	B:	①	②	③	④	⑤	⑥	⑦
11. All things considered, performs effectively as a university teacher.	A:	①	②	③	④	●	⑥	⑦
	B:	①	②	③	④	⑤	⑥	⑦

PART I CONTINUES ON THE REVERSE SIDE

PRINTED BY NCS CANADA 1-800-665-8774 DesignExpert™ by Pearson NCS Mark Reflex® forms MM201053-5 15 ED06

Statements about the course: Respond to the statements below, using the following 7-point scale. *SIDE 2*

	very low	low	below average	average	above average	high	very high
12. Compared to other courses at the same level (100,200,300,400), the work load is	○ 1	○ 2	○ 3	○ 4	●	○ 6	○ 7
13. Compared to other courses at the same level, the level of difficulty of the material is	○ 1	○ 2	○ 3	○ 4	●	○ 6	○ 7
14. The value of the required reading is	○ 1	○ 2	○ 3	○ 4	●	○ 6	○ 7
15. (If applicable) The value of the tutorials is	○ 1	○ 2	○ 3	○ 4	○ 5	○ 6	○ 7
16. (If applicable) The value of the laboratories is	○ 1	○ 2	○ 3	○ 4	○ 5	○ 6	○ 7
17. (If applicable) The value of the seminars is	○ 1	○ 2	○ 3	○ 4	○ 5	○ 6	○ 7
18. (If applicable) The value of the language conversation classes is	○ 1	○ 2	○ 3	○ 4	○ 5	○ 6	○ 7
19. The value of the overall learning experience is	○ 1	○ 2	○ 3	●	○ 5	○ 6	○ 7

20. Considering your experience with this course, and disregarding your need for it to meet program or degree requirements, would you still have taken this course? ● Yes ○ No

Statements about yourself:

21. Number of full course credits already earned (prior to this session):
○ 1 0-4½ ○ 2 5-9½ ● 10-14½ ○ 4 15-19½ ○ 5 ≥20

22. Status of the course for you:
○ 1 Program Requirement ● Selected from a required list in a program ○ 3 Breadth Requirement ○ 4 Optional

23. Your level of enthusiasm to take this course at the time of initial registration:
○ 1 low ● medium ○ 3 high

24. Your expected grade in this course:
○ 1 <50 ○ 2 50-59 ● 60-69 ○ 4 70-79 ○ 5 ≥80

Additional statements or questions which may be supplied in class:

25. ○1 ○2 ○3 ○4 ○5 ○6 ○7
26. ○1 ○2 ○3 ○4 ○5 ○6 ○7
27. ○1 ○2 ○3 ○4 ○5 ○6 ○7
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31. ○1 ○2 ○3 ○4 ○5 ○6 ○7
32. ○1 ○2 ○3 ○4 ○5 ○6 ○7
33. ○1 ○2 ○3 ○4 ○5 ○6 ○7
34. ○1 ○2 ○3 ○4 ○5 ○6 ○7
35. ○1 ○2 ○3 ○4 ○5 ○6 ○7
36. ○1 ○2 ○3 ○4 ○5 ○6 ○7

PART II: PLEASE ANSWER ONLY AFTER COMPLETING PART I. Please use the space below to provide supplementary comments on the instructor(s) or course. For example, you may wish to give the reasons for your numerical evaluations or provide specific suggestions for improving the instruction in the course.

Good prof. Very enthusiastic and friendly.
MAT 301 should be prereq for class while
MAT 235/7 not useful as prereq.
Course material got very hard towards end!!

END OF SURVEY. THANK YOU FOR YOUR COOPERATION AND PARTICIPATION.

STUDENT SURVEY FORM
UNIVERSITY OF TORONTO

FACULTY OF
ARTS & SCIENCE

Note that survey results will be available to the instructor(s) only after final course marks have been submitted.

PART I: INSTRUCTIONS. PLEASE READ FIRST.

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Part II *(on the reverse side)* requires a written answer.

Course Identification: Please print course and section you are evaluating

COURSE M A T 4 0 1 H 1 S

SECTION L 5 1 0 1

INSTRUCTOR(S):

A: Dror Bar-Natan

B:

1. If evaluating only one instructor, write the name in the upper (A) box. If evaluating two instructors, write their names, one in box A and the other in box B.

DO NOT EVALUATE TEACHING ASSISTANTS ON THIS FORM

Statements about the instructor(s):

Respond to the statements below for instructor A (and instructor B) bearing in mind that there are wide variations in class size and subject matter in Arts and Science.

		extremely poor	very poor	poor	adequate	good	very good	outstanding
2. Communicates goals and requirements of the course clearly and explicitly.	A:	1	2	3	4	●	6	7
	B:	1	2	3	4	5	6	7
3. Uses methods of evaluation (e.g. papers, assignments, tests) that appropriately reflect the subject matter and provide a fair evaluation of student learning.	A:	1	2	3	4	●	6	7
	B:	1	2	3	4	5	6	7
4. Presents material in an organized, well-planned manner.	A:	1	2	3	●	5	6	7
	B:	1	2	3	4	5	6	7
5. Explains concepts clearly with appropriate use of examples.	A:	1	2	3	●	5	6	7
	B:	1	2	3	4	5	6	7
6. Communicates enthusiasm and interest in the course material.	A:	1	2	3	4	●	6	7
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7. Attends to students' questions and answers them clearly and effectively.	A:	1	2	3	●	5	6	7
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8. Is available for individual consultation, by appointment or stated office hours, to students with problems relating to the course.	A:	1	2	3	●	5	6	7
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9. Ensures that student work is graded fairly, with helpful comments and feedback where appropriate.	A:	1	2	3	●	5	6	7
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10. Ensures that student work is graded within a reasonable time.	A:	1	2	3	4	●	6	7
	B:	1	2	3	4	5	6	7
11. All things considered, performs effectively as a university teacher.	A:	1	2	3	●	5	6	7
	B:	1	2	3	4	5	6	7

PART I CONTINUES ON THE REVERSE SIDE

Statements about the course: Respond to the statements below, using the following 7-point scale.

SIDE 2

	very low	low	below average	average	above average	high	very high
12. Compared to other courses at the same level (100,200,300,400), the work load is	①	②	③	●	⑤	⑥	⑦
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19. The value of the overall learning experience is	①	②	●	④	⑤	⑥	⑦

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29. ① ② ③ ④ ⑤ ⑥ ⑦
30. ① ② ③ ④ ⑤ ⑥ ⑦
31. ① ② ③ ④ ⑤ ⑥ ⑦
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35. ① ② ③ ④ ⑤ ⑥ ⑦
36. ① ② ③ ④ ⑤ ⑥ ⑦

PART II: PLEASE ANSWER ONLY AFTER COMPLETING PART I. Please use the space below to provide supplementary comments on the instructor(s) or course. For example, you may wish to give the reasons for your numerical evaluations or provide specific suggestions for improving the instruction in the course.

- material is extremely difficult
- prof is extremely knowledgeable
- assignment questions good measure of tests

END OF SURVEY. THANK YOU FOR YOUR COOPERATION AND PARTICIPATION.

Return HW9.

0. Two
   1. we'll finish early
   2. ✓ fix second.

Math 401 Pols Eqns Fields, April 9 2008, Week 13 (last)

1. on The final.
2. Fix blunder.
3. state Lemmas.
4. Proof of $\psi \circ \Phi = I$
5. Proof of Lemmas.

on board: for a splitting ext. E

$$\begin{tikzcd}
E \arrow[d, dash, "{[E:K]}"] \arrow[r, leftrightarrow] & \{e\} = Gal(E/E) \arrow[d, dash, "|H|"] \\
K \arrow[d, dash, "{[K:F]}"] \arrow[r, leftrightarrow] & H = Gal(E/K) \arrow[d, dash, "{[G:H]}"] \\
F \arrow[r, leftrightarrow] & G = Gal(E/F)
\end{tikzcd}$$

If K is splitting $H \lhd G$ & $Gal(K/F) = G/H = Gal(E/F)/Gal(E/K)$

Blunder:

$$\begin{tikzcd}
F(a_1,\dots a_n) \subset E_n \arrow[r, dash, "="] \arrow[d, dash] & K_n = S_{E_n}(F) \arrow[d, dash] \\
\vdots & \\
F(a_1, a_2) \subset E_2 \arrow[r, dash] \arrow[d, dash] & K_2 = S_{E_2}(F) \arrow[d, dash] \\
F(a_0) \subset E_1 \arrow[r, dash] \arrow[d, dash] & K_1 = S_{E_1}(F) \arrow[d, dash] \\
F = E_0 \arrow[r, dash] & K_0 = S_{E_0}(F)
\end{tikzcd}$$

$\Phi : K \to Gal(E/K)$

$\psi : H \mapsto E_H$

Claim $\psi \circ \Phi = I$, or $E_{Gal(E/K)} = K$

Lemma 2 Uniqueness of splitting fields

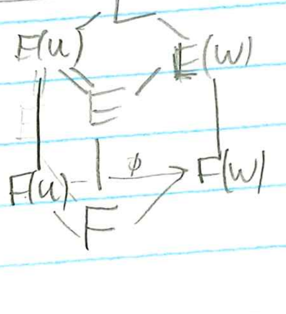
Lemma 4 splitting fields are good at splitting

Proof

Proof of lemma 2

$$\begin{tikzcd}
E_1 \arrow[r, dashed, "\exists \Phi"] & E_2
\end{tikzcd}$$

Proof of lemma 4.



u a root of p
w another

u₁ a root of p₁.

$$\begin{tikzcd}
F_1(u_1) \arrow[r, "\Phi"] \arrow[d, dash] & F_2(u_2) \arrow[d, dash] \\
F_1 \arrow[r, "\phi"] \arrow[d, dash, "in"'] & F_2 \arrow[d, dash, "in"] \\
p_1 \mid f_1 \arrow[r] & \phi(f_1) = f_2 \\
& p_2
\end{tikzcd}$$

$p_2 = \phi(p_1)$

08-401/The Final Exam

From Drorbn

08-401/Navigation Panel [Show]

As announced (http://www.artsci.utoronto.ca/current/undergraduate/exams/aprmay08) by the powers above, our final exam will take place on the *evening* of Monday April 28 between 7PM and 10PM, at BN2S (Large Gymnasium, South End, Benson Building, 320 Huron Street (south of Harbord Street), Second Floor).

The exam will be similar in style to the Term Test (also see On the Term Test), and perhaps even more similar in style to last years' final (PDF). The material is **everything** covered in class. There will be two types of questions (or maybe sometimes the two types will be mixed within a single question):

- You may be asked to prove a theorem proven in class. The reason we prove theorems in class is that these proofs are **valuable**. Therefore I expect you to know them.
- You may be asked to solve exercises from the relevant chapters of the book, or minor variations thereof. These may be questions that were assigned as homework, but also, these may be questions that were not assigned before.

Office Hours. I (Dror) will hold extended office hours before the final, on Friday April 25 1-4PM and on the exam date, Monday April 28 10AM-12PM, at or near my office, Bahen 6178.

Preparing for the Test. Read, reread and rereread everything and solve lots of exercises from the book.

My (Dror's) system when I was an undergrad was to prepare a 4-6 page 100-200 item list of points covered in class. I'd only summarize each point with one sentence, without giving any details and without trying to be precise, much like the list that I prepared for the class of February 7 (see On the Term Test). I would then go over my list again and again and again, crossing out every item for which I was sure I could complete all the details and supply all the proofs. I would only stop when there was nothing left to cross out.

Good Luck!

Retrieved from "http://katlas.math.toronto.edu/drorbn/index.php?title=08-401/The_Final_Exam"

- This page was last modified 14:54, 9 April 2008.

UNIVERSITY OF TORONTO
Faculty of Arts and Sciences
APRIL-MAY EXAMINATIONS 2007
Math 401H1S Polynomial Equations and Fields

Instructor: Dror Bar-Natan
Date: April 24, 2007

Duration. You have 3 hours to write this exam.

Allowed Material. Basic calculators, not capable of displaying text or sounding speech.

Solve 6 of the following 7 questions. The questions carry equal weight though different parts of the same question may be weighted differently.

Neatness counts! Language counts! The *ideal* written solution to a problem looks like a proof from the textbook; neat and clean and made of complete and grammatical sentences. Definitely phrases like "there exists" or "for every" cannot be skipped. Lectures are mostly made of spoken words, and so the blackboard part of proofs given during lectures often omits or shortens key phrases. The ideal written solution to a problem does not do that.

Good Luck!

Solve 6 of the following 7 problems. Neatness counts! Language counts!

Problem 1. Let R be a commutative ring with unity and let A be an ideal of R. Prove that R/A is a field if and only if A is maximal.

Problem 2. Let $\mathbb{Z}[\sqrt{2}] = \{a + b\sqrt{2} | a, b \in \mathbb{Z}\}$ and let $H = \left\{ \begin{pmatrix} a & 2b \\ b & a \end{pmatrix} \middle| a, b \in \mathbb{Z} \right\}$. Show that $\mathbb{Z}[\sqrt{2}]$ and H are isomorphic as rings.
(Remember that in math-talk the word "show" is equivalent to the word "prove".)

Problem 3. Let $f(x) = x^3 + 6 \in \mathbb{Z}_7[x]$. Write $f(x)$ as a product of irreducible polynomials over $\mathbb{Z}_7$.
(Remember to explain why each of the factors you end up writing really is irreducible!)

Problem 4. Let E/F be a field extension and let $a \in E$ satisfy $p(a) = 0$ where $p \in F[x]$ is a polynomial with coefficients in F. Define the field $F(a)$ and prove that it is isomorphic to the field $F[x]/\langle p(x) \rangle$.

Problem 5. Let E/F be a field extension and let a and b be elements of the bigger field E.

1. Show that $[F(a) : F]$ is finite if and only if a is algebraic over F.
2. Prove that if both a and b are algebraic over F, then so is $a + b$.

Problem 6. Show that the Galois group of a polynomial of degree n (i.e., of the splitting field of a polynomial of degree n over some base field) has order dividing $n!$.

Problem 7. Let F be the field $\mathbb{Q}(i)$ and let E be the field $\mathbb{Q}(\sqrt[4]{2}, i)$.

1. Compute $G := \mathrm{Gal}(E/F)$.
2. Find all the subgroups H of G.
3. For exactly one non-trivial subgroup of G (that is, a subgroup that is neither $\{e\}$ nor G), describe the fixed field E_H.
(The word "describe" here means "find $a \in E$ so that $E_H = F(a)$".)

Good Luck!

Error in ref card

TA @ Hi
Thu 5PM math lounge

HW8 returned
HW9 due
HW10 on web by midnight tomorrow.

Math 401 Pols, Eqns, Fields, April 2 2008, Week 12

on board

If E/F a splitting ext,

$\{K : E/K/F\} \leftrightarrow \{H : H < \mathrm{Gal}(E/F)\}$

$K \longmapsto \mathrm{Gal}(E/K)$

$E_H \longleftarrow\!\mapsto H$

{field ext} $\Leftrightarrow$ {groups}

{ext by radicals} $\to$ {solvable? groups}

$S_{\mathbb{Q}}(3x^5-15x+5) \to S_5$ (not solvable)

1. inclusion reversing

2&3

$$\begin{tikzcd}
E \arrow[r, leftrightarrow] \arrow[d, dash, "{[E:K]}"'] & \mathrm{id} \Rightarrow \mathrm{Gal}(E/E) \arrow[d, dash, "|H|"] \\
K \arrow[r, leftrightarrow] \arrow[d, dash, "{[K:F]}"'] & H = \mathrm{Gal}(E/K) \arrow[d, dash, "{[G:H]}"] \\
F \arrow[r, leftrightarrow] & G = \mathrm{Gal}(E/F)
\end{tikzcd}$$

If K is splitting, H is normal and

$\mathrm{Gal}(K/F) = G/H = \mathrm{Gal}(E/F)/\mathrm{Gal}(E/K)$

Thm $\mathrm{Gal}(S_F(x^n-a)/F)$ is solvable.

"If E is the splitting field of x^n-a over a field of char 0 ($a \in F$), then $\mathrm{Gal}(E/F)$ is solvable"

Proof Def w "a primitive root of unity of order n"

Case I F contains a primitive root of unity w.

$\Rightarrow$ The roots of x^n-a are $b, wb, \ldots$

$\sigma : b \mapsto w^i b \quad \tau : b \mapsto w^k b \quad \Rightarrow$ they commute.

Case II otherwise, find an even bigger ext. E' that contains w. (debt)

a. then $F(w) \subset E$

b. $\mathrm{Gal}(F(w)/F)$ is Abelian

c. $\mathrm{Gal}(E/F(w))$ is Abelian

$$\begin{tikzcd}
E \arrow[d] \\
F(w) \arrow[d, dash] \\
F
\end{tikzcd}$$

does the job?

Thm If $f \in F[x]$ splits in $F(a_1 \ldots a_n)$ where $a_i^{n_i} \in F(a_1 \ldots a_{i-1})$, then $\mathrm{Gal}(S_F(f)/F)$ is solvable.

Math 401 Pols Eqns, Fields, week 12 cont.

Thm $\mathrm{Gal}(\underbrace{S_{\mathbb{Q}}(\underbrace{3x^5-15x+5}_{p})}_{E}/\mathbb{Q})_{=G} = S_5$

0. $G \subset S_5$.

Proof 1. p is irred $\Rightarrow [\mathbb{Q}(\alpha):\mathbb{Q}]=5 \Rightarrow 5 \mid [E:\mathbb{Q}]$

$\Rightarrow 5 \mid |G| \Rightarrow G$ has an element of order 5.

2. By drawing, p has exactly 3 real roots $\Rightarrow$ 2 complex

$\Rightarrow z \mapsto \bar{z}$ is not the id $\Rightarrow G$ has an element of order 2.

~~3. $G \subset S_5$~~

Thm Any subgroup of S_5 containing an element α of order 5 & an element β of order 2 is S_5.

* * *

Thm An irreducible poly over a field of char 0 has no multiple roots (in any extension)

Def f'; linearity, Leibnitz.

Prop f has a multiple root (in some extension) iff f & f' have a common factor of deg > 0. in $F[x]$

Prop ⇒ Thm obvious

Pf of Prop $\Rightarrow$ $(x-a)^2 \mid f \Rightarrow (x-a) \mid f' \Rightarrow x-a$ is a common factor.

Consider $\langle f, f' \rangle = \langle p \rangle$ $\Rightarrow 1 = af+bg$
assume no common factor
$\Rightarrow x-a \mid 1$ in E

$\Leftarrow$ if $f(a)=f'(a)=0 \Rightarrow a$ is a ~~common~~ multiple root.

08-401/The Fundamental Theorem

From Drorbn

The statement appearing here, which is a weak version of the full **fundamental theorem of Galois theory**, is taken from Gallian's book and is meant to match our discussion in class. The proof is taken from Hungerford's book, except modified to fit our notations and conventions and simplified as per our weakened requirements.

Here and everywhere below our base field F will be a field of characteristic 0.

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Statement

Theorem. Let E be a splitting field over F. Then there is a bijective correspondence between the set $\{K : E/K/F\}$ of intermediate field extensions K lying between F and E and the set $\{H : H < \mathrm{Gal}(E/F)\}$ of subgroups H of the Galois group $\mathrm{Gal}(E/F)$ of the original extension E/F:

$$\{K : E/K/F\} \quad \leftrightarrow \quad \{H : H < \mathrm{Gal}(E/F)\}.$$

The bijection is given by mapping every intermediate extension K to the subgroup $\mathrm{Gal}(E/K)$ of elements in $\mathrm{Gal}(E/F)$ that preserve K,

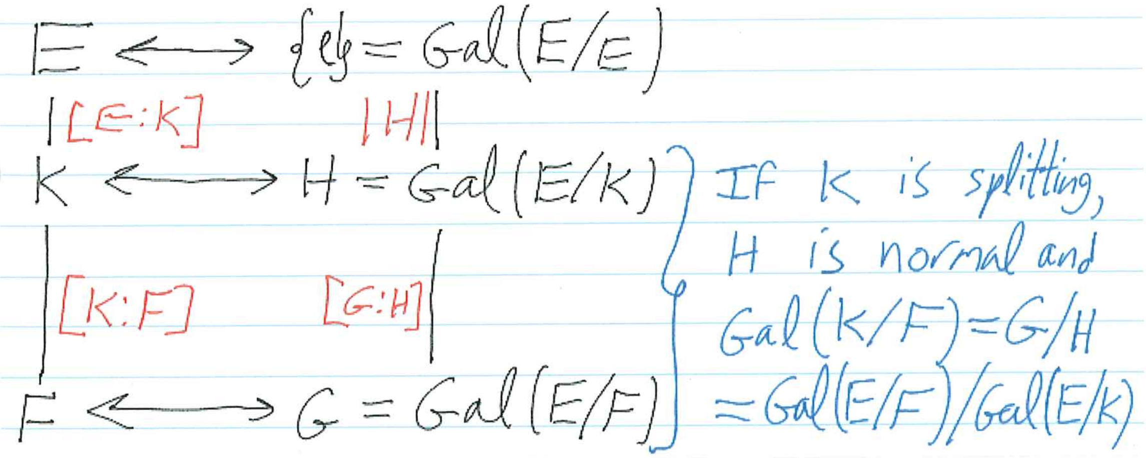
$$\Phi: \quad K \mapsto \mathrm{Gal}(E/K) := \{\phi : E \to E : \phi|_K = I\},$$

and reversely, by mapping every subgroup H of $\mathrm{Gal}(E/F)$ to its fixed field E_H:

$$\Psi: \quad H \mapsto E_H := \{x \in E : \forall h \in H,\ hx = x\}.$$

This correspondence has the following further properties:

1. It is inclusion-reversing: if $H_1 \subset H_2$ then $E_{H_1} \supset E_{H_2}$ and if $K_1 \subset K_2$ then $\mathrm{Gal}(E/K_1) > \mathrm{Gal}(E/K_2)$.
2. It is degree/index respecting: $[E : K] = |\mathrm{Gal}(E/K)|$ and $[K : F] = [\mathrm{Gal}(E/F) : \mathrm{Gal}(E/K)]$.
3. Splitting fields correspond to normal subgroups: If K in $E/K/F$ is the splitting field of a polynomial in $F[x]$ then $\mathrm{Gal}(E/K)$ is normal in $\mathrm{Gal}(E/F)$ and $\mathrm{Gal}(K/F) \cong \mathrm{Gal}(E/F)/\mathrm{Gal}(E/K)$.



The Fundamental Theorem of Galois Theory, all in one.

Lemmas

The four lemmas below belong to earlier chapters but we skipped them in class (the last one was also skipped by Gallian).

Zeros of Irreducible Polynomials

Lemma 1. An irreducible polynomial over a field of characteristic 0 has no multiple roots.

Proof. See the proof of Theorem 20.6 on page 362 of Gallian's book. □

Uniqueness of Splitting Fields

Lemma 2. Let $\phi : F_1 \to F_2$ be an isomorphism of fields, let $f_1 \in F_1[x]$ be a polynomial and let $f_2 = \phi(f_1)$, and let E_1 and E_2 be splitting fields for f_1 and f_2 over F_1 and F_2, respectively. Then there is an isomorphism $\bar{\phi} : E_1 \to E_2$ (generally not unique) that extends ϕ .

Proof. See the proof of Theorem 20.4 on page 360 of Gallian's book. □

The Primitive Element Theorem

The celebrated "Primitive Element Theorem" is just a lemma for us:

Lemma 3. Let a and b be algebraic elements of some extension E of F. Then there exists a single element c of E so that $F(a,b) = F(c)$. (And so by induction, every finite extension of E is "simple", meaning, is generated by a single element, called "a primitive element" for that extension).

Proof. See the proof of Theorem 21.6 on page 375 of Gallian's book. □

Splitting Fields are Good at Splitting

Lemma 4. (Compare with Hungerford's Theorem 10.15 on page 355). If E is a splitting field of some polynomial f over F and some irreducible polynomial $p \in F[x]$ has a root v in E, then p splits in E.

Proof. Let L be a splitting field of p over E. We need to show that if w is a root of p in L, then $w \in E$ (so all the roots of p are in E and hence p splits in E). Consider the two extensions

$E = E(v)/F(v)$ and $E(w)/F(w)$.

The "smaller fields" $F(v)$ and $F(w)$ in these two extensions are isomorphic as they both arise by adding a root of the same irreducible polynomial (p) to the base field F. The "larger fields" $E = E(v)$ and $E(w)$ in these two extensions are both the splitting fields of the same polynomial (f) over the respective "small fields", as E/F is a splitting extension for f and we can use the sub-lemma below. Thus by the uniqueness of splitting extensions (lemma 2), the isomorphism between $F(v)$ and $F(w)$ extends to an isomorphism between $E = E(v)$ and $E(w)$, and in particular these two fields are isomorphic and so $[E : F] = [E(v) : F] = [E(w) : F]$. Since all the degrees involved are finite it follows from the last equality and from $[E(w) : F] = [E(w) : E][E : F]$ that $[E(w) : E] = 1$ and therefore $E(w) = E$. Therefore $w \in E$. □

Sub-lemma. If E/F is a splitting extension of some polynomial $f \in F[x]$ and z is an element of some larger extension L of E, then $E(z)/F(z)$ is also a splitting extension of f.

Proof. Let $u_1, \ldots, u_n$ be all the roots of f in E. Then they remain roots of f in $E(z)$, and since f completely splits already in E, these are *all* the roots of f in $E(z)$. So

$$E(z) = F(u_1, \ldots, u_n)(z) = F(z)(u_1, \ldots, u_n),$$

and $E(z)$ is obtained by adding all the roots of f to $F(z)$. □

Proof of The Fundamental Theorem

The Bijection

Proof of $\Psi \circ \Phi = I$. More precisely, we need to show that if K is an intermediate field between E and F, then $E_{\mathrm{Gal}(E/K)} = K$. The inclusion $E_{\mathrm{Gal}(E/K)} \supset K$ is easy, so we turn to prove the other inclusion. Let $v \in E - K$ be an element of E which is not in K. We need to show that there is some automorphism $\phi \in \mathrm{Gal}(E/K)$ for which $\phi(v) \neq v$; if such a ϕ exists it follows that $v \notin E_{\mathrm{Gal}(E/K)}$ and this implies the other inclusion. So let p be the minimal polynomial of v over K. It is not of degree 1; if it was, we'd have that $v \in K$ contradicting the choice of v. By lemma 4 and using the fact that E is a splitting extension, we know that p splits in E, so E contains all the roots of p. Over a field of characteristic 0 irreducible polynomials cannot have multiple roots (lemma 1) and hence p must have at least one other root; call it w. Since v and w have the same minimal polynomial over K, we know that $K(v)$ and $K(w)$ are isomorphic; furthermore, there is an isomorphism $\phi_0 : K(v) \to K(w)$ so that $\phi_0|_K = I$ yet $\phi_0(v) = w$. But E is a splitting field of some polynomial f over F and hence also over $K(v)$ and over $K(w)$. By the uniqueness of splitting fields (lemma 2), the isomorphism ϕ_0 can be extended to an isomorphism $\phi : E \to E$; i.e., to an automorphism of E. but then $\phi|_K = \phi_0|_K = I$ so $\phi \in \mathrm{Gal}(E/K)$, yet $\phi(v) = w \neq v$, as required. □

Proof of $\Phi \circ \Psi = I$. More precisely we need to show that if $H < \mathrm{Gal}(E/F)$ is a subgroup of the Galois group of E over F, then $H = \mathrm{Gal}(E/E_H)$. The inclusion $H < \mathrm{Gal}(E/E_H)$ is easy. Note that H is finite since we've proven previously that Galois groups of finite extensions are finite and hence $\mathrm{Gal}(E/F)$ is finite. We will prove the following sequence of inequalities:

$$|H| \leq |\mathrm{Gal}(E/E_H)| \leq [E : E_H] \leq |H|$$

This sequence and the finiteness of $|H|$ imply that these quantities are all equal and since $H < \mathrm{Gal}(E/E_H)$ it follows that $H = \mathrm{Gal}(E/E_H)$ as required.

The first inequality above follows immediately from the inclusion $H < \mathrm{Gal}(E/E_H)$.

By the Primitive Element Theorem (Lemma 3) we know that there is some element $u \in E$ so that $E = E_H(u)$. Let p be the minimal polynomial of u over E_H. Distinct elements of $\mathrm{Gal}(E/E_H)$ map u to distinct roots of p, but p has exactly $\deg p$ roots. Hence $|\mathrm{Gal}(E/E_H)| \leq \deg p = [E : E_H]$, proving the second inequality above.

Let $\sigma_1, \ldots, \sigma_n$ be an enumeration of all the elements of H, let $u_i := \sigma_i u$ (with u as above), and let f be the polynomial

$$f = \prod_{i=1}^{n}(x - u_i).$$

Clearly, $f \in E[x]$. Furthermore, if $\tau \in H$, then left multiplication by τ permutes the σ_i's (this is always true in groups), and hence the sequence $(\tau u_i = \tau\sigma u_i)_{i=1}^{n}$ is a permutation of the sequence $(u_i)_{i=1}^{n}$, hence

$$\tau f = \prod_{i=1}^{n}(x - \tau u_i) = \prod_{i=1}^{n}(x - u_i) = f,$$

and hence $f \in E_H[x]$. Clearly $f(u) = 0$, so $p|f$, so $[E : E_H] = \deg p \leq \deg f = n = |H|$, proving the third inequality above. ☐

The Properties

Property 1. If $H_1 \subset H_2$ then $E_{H_1} \supset E_{H_2}$ and if $K_1 \subset K_2$ then $\mathrm{Gal}(E/K_1) > \mathrm{Gal}(E/K_1)$.

Proof of Property 1. Easy. ☐

Property 2. $[E : K] = |\mathrm{Gal}(E/K)|$ and $[K : F] = [\mathrm{Gal}(E/F) : \mathrm{Gal}(E/K)]$.

Proof of Property 2. If $K = E_H$, then $|\mathrm{Gal}(E/K)| = |\mathrm{Gal}(E/E_H)| = [E : E_H] = [E : K]$ as was shown within the proof of $\Phi \circ \Psi = I$. But every K is E_H for some H, so $|\mathrm{Gal}(E/K)| = [E : K]$ for every K between E and F. The second equality follows from the first and from the multiplicativity of the degree/order/index in towers of extensions and in towers of groups:

$$[K : F] = \frac{[E : F]}{[E : K]} = \frac{|\mathrm{Gal}(E/F)|}{|\mathrm{Gal}(E/K)|} = [\mathrm{Gal}(E/F) : \mathrm{Gal}(E/K)]. \quad \square$$

Property 3. If K in $E/K/F$ is the splitting field of a polynomial in $F[x]$ then $\mathrm{Gal}(E/K)$ is normal in $\mathrm{Gal}(E/F)$ and $\mathrm{Gal}(K/F) \cong \mathrm{Gal}(E/F)/\mathrm{Gal}(E/K)$.

Proof of Property 3. We will define a surjective (onto) group homomorphism $\rho : \mathrm{Gal}(E/F) \to \mathrm{Gal}(K/F)$ whose kernel is $\mathrm{Gal}(E/K)$. This shows that $\mathrm{Gal}(E/K)$ is normal in $\mathrm{Gal}(E/F)$ (kernels of homomorphisms are always normal) and then by the first isomorphism theorem for groups, we'll have that $\mathrm{Gal}(K/F) \cong \mathrm{Gal}(E/F)/\mathrm{Gal}(E/K)$.

Let σ be in $\mathrm{Gal}(E/F)$ and let u be an element of K. Let p be the minimal polynomial of u in $F[x]$. Since K is a splitting field, lemma 4 implies that p splits in $K[x]$, and hence all the other roots of p are also in K. As $\sigma(u)$ is a root of p, it follows that $\sigma(u) \in K$ and hence $\sigma(K) \subset K$. But since σ is an isomorphism, $[\sigma(K) : F] = [K : F]$ and hence $\sigma(K) = K$. Hence the restriction $\sigma|_K$ of σ to K is an automorphism of K, so we can define $\rho(\sigma) = \sigma|_K$.

Clearly, ρ is a group homomorphism. The kernel of ρ is those automorphisms of E whose restriction to K is the identity. That is, it is $\mathrm{Gal}(E/K)$. Finally, as E/F is a splitting extension, so is E/K. So every automorphism of K extends to an automorphism of E by the uniqueness statement for splitting extensions (lemma 2). But this means that ρ is onto. ☐

course evaluations?   HW 8 due   HW returned
HW 9 on web soon.

Math 401 Pols, Eqn's, Fields, March 26 2008, Week 11

on board:

$\{\text{Field exts}\} \xleftrightarrow[\text{F.T.}]{\text{The}} \{\text{groups}\}$

3. $\boxed{\{\text{ext. by radicals}\} \longleftrightarrow \boxed{\text{solvable groups}}}$

$S_{\mathbb{Q}}(3x^5-15x+5) \longrightarrow \boxed{S_5 \text{ (non-solv)}}$ 2

Plan: 1. solvable groups
  2. S_5 isn't solvable
  3. Ext. by radical → solvable groups

Later: 4. compute $Gal(S_{\mathbb{Q}}(3x^5_{\mathbb{Q}}-15x+5)/\mathbb{Q})$
  5. Prove the F.T. (?).

Isomorphism Thms 1. If $\phi: G_1 \to G_2$ then $\ker\phi \triangleleft G_1$ and $G_1/\ker\phi \cong \operatorname{im}\phi$

2. If $N \triangleleft H \triangleleft G$ and $N \triangleleft G$ then $G/H \cong {}^{G/N}\!/_{H/N}$

3. $H < G$, $N \triangleleft G$ $H/_{N\cap H} \cong \dfrac{H\cdot N}{N}$

$[h] \longrightarrow [h]$ $h_1 = h_2 n_2 n n_1^{-1}$

$[h] \longleftarrow [h\cdot n]$ □.

$h_1 n_1 = h_2 n_2 n \Rightarrow$

Def solvable G: $\exists_{\{e\}} H_0 \ldots H_n \overset{\shortparallel}{G}$

$H_{k-1} \triangleleft H_k$ H_k/H_{k-1} Abelian.

Example: 1. Abelian
  2. S_3

Theorem If $N \triangleleft G$, N & G/N solvable $\Leftarrow$ G is solvable.

Pf $\Rightarrow$ Given $N \overline{\triangleright} H_n \triangleright H_{n-1}$ . . $\triangleright H_0 = \{e\}$ H_i/H_{i-1} abelian

$G/N \overline{\triangleright} K_m \triangleright$ . . $\triangleright K_0 = \{[e]\} = \{N\}$ . . K_i/K_{i-1} abelian

consider

$$G \overset{=}{\triangleright} \underbrace{\pi^{-1}(K_m) > \pi^{-1}(K_{m-1}) \quad \ldots \quad > \pi^{-1}(K_0)}_{} = \underbrace{N > H_n \ldots > H_0}_{\text{obvious}}$$

Claim $\pi^{-1}(K_j) \triangleright \pi^{-1}(K_{j-1})$ &

$\pi^{-1}(K_j)/\pi^{-1}(K_{j-1}) \cong K_j/K_j$ (hence abelian)

Pf $a \in \pi^{-1}(K_{j-1})$ $b \in \pi^{-1}(K_j)$ $a = [a']$, $b = [b']$ $b^{-1}ab = [b'^{-1}a'b'] \in \pi^{-1}(K_j)$

$\pi(\pi^{-1}(K_j)) = K_j$ so $K_j \cong \pi^{-1}(K_j)/N$ . . . . . . /cont.

Math 401 March 26 2008, Cont.

Example S_4 is solvable

Pf $\phi : S_4 \twoheadrightarrow S_3$ so $S_4 \triangleright \overset{\text{an 8 element group}}{\ker\phi} \triangleright \{e\}$

Quo is S_3.

Theorem 2 S_5 is not solvable.

Consider $A = \{$the subgroup generated by 3-cycles$\}$.

Lemma 1 $[A,A] = A$

Lemma 2 If $H' \triangleleft H''$, H''/H' is Abelian and $H'' > A$,

then $H' > [A,A]$

The two lemmas prove the theorem.

Pf of lemma 2 easy

Pf of lemma 1 Consider $[(ijk),(kmn)]$

Back to theorem 1 $\Leftarrow$ Assume G is solvable.

$G = H_n \triangleright H_{n-1} \ldots \quad H_0 = \{e\}$ (Abel. Quo)

take H_kN/N within G/N

$$\frac{H_kN/N}{H_{k-1}N/N} \simeq \frac{H_kN}{H_{k-1}N} \xleftarrow[\text{to}]{\text{onto}} H_k/H_{k-1}.$$

If time:
* Define ext. by radicals
* state "solvable in ~~ext by radicals~~ an ext. by radicals" $\Rightarrow$ solvable group.
* Do the case $x^n - a = 0$, WEF.

MAT401H1 WK13 11/4/07

- Fields $\xleftrightarrow{\text{The fundamental Theorem}}$ Groups.

Spliting f of $3x^5 - 15x + 5 \longrightarrow S_5$ is not solvable.

Extensions describable by radicals $\longrightarrow$ solvable groups.

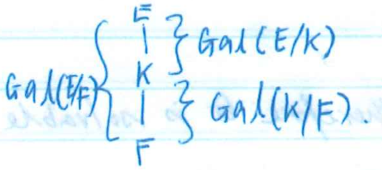
G is solvable: can find tower
$G = H_n \triangleright H_{n-1} \triangleright \dots \triangleright H_1 \triangleright H_0 = \{e\}$.
So that $\forall k$ H_k / H_{k-1} is abelian.

THEOREM ⓪: S_5 is not solvable.

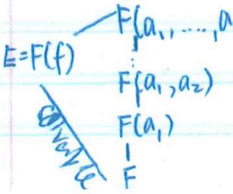
THEOREM ①: $H \triangleleft G \Rightarrow G$ is solvable iff H and G/H are solvable.

The fundamental Theorem
Given E a splitting extension of F, char$F = 0$, then
$\{K : E/K/F\} \longleftrightarrow \{H : H < Gal(E/F)\}$
$K \mapsto Gal(E/K)$ $Gal(E/K) \subset Gal(E/F)$.
$E_H \longleftarrow H$

① Inclusion reversing.
② Degree/order/index respecting.
$[E:K] = |Gal(E/K)|$ etc.
③ If K is a splitting extension, then $Gal(E/K) \triangleleft Gal(E/F)$ and
$Gal(K/F) \cong Gal(E/F)/Gal(E/K)$.



- DEFINITION: $f \in F[x]$ is solvable in terms of radicals if there exists a tower of fiel
extension such that ① for each i, there is a natural number k_i such that
$a_i^{k_i} \in F(a_1, \dots, a_{i-1})$, $a_1^{k_1} \in F$
② f splits in $F(a_1, \dots, a_n)$



$$\begin{tikzcd}
 & & E_n = E_{n-1}(x^{k_n} - b_n) \arrow[d, dash, dotted] \\
 & F(a_1, \dots, a_n) \arrow[ur, dash] \arrow[d, dash, dotted] & \\
E(f) \arrow[uurr, dash, bend left] \arrow[ddr, dash] & F(a_1, a_2) \arrow[d, dash] & E_2 = E_1(x^{k_2} - b_2) \arrow[d, dash] \\
 & F(a_1) \arrow[d, dash] & E_1 = E_0(x^{k} - b_1). \\
 & F \arrow[ur, dash] &
\end{tikzcd}$$

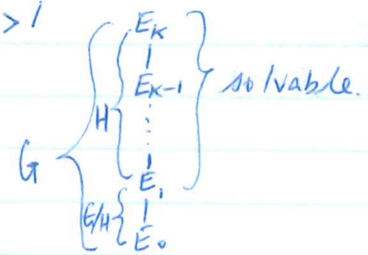
- LEMMA: $Gal(E_k/E_{k-1})$ is solvable for $1 \leq k \leq n$. Precisely if F is of char 0 and $E = F(x^k - b)$ then $Gal(E/F)$ is solvable

- CLAIM: $Gal(E_k/F)$ is solvable for all K. In fact in any k-step tower of extensions, in which the Gal group is solvable, the whole tower is solvable.

PROOF

If $k = 1$, this is the lemma.

$k > 1$



By inductive hypothesis, $Gal(E_k/E_1)$ is solvable and $Gal(E_1/E_0)$ is solvable. E_1 is indeed a splitting extension.

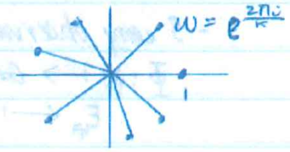
Let $G = Gal(E_k/E_0)$, $H = Gal(E_k/E_1)$

By property ③, $G/H = Gal(E_1/E_0)$.

H is solvable, G/H is solvable and therefore G is solvable.

By ③, $Gal(E(f)/F)$ is a quotient of $Gal(E_n/F)$, so it is solvable.

- $E = F(x^k - b)$



$\omega = e^{\frac{2\pi i}{k}}$

- CASE ① $b=1$

$F(x^k-1) = F(\omega)$

$\tau, \sigma \in Gal(F(\omega)/F)$

$\sigma(\omega) = \omega^j$

$\tau(\omega) = \omega^i$

- CLAIM: If $\sigma, \tau \in Gal(F(\omega)/F)$, then $\sigma\tau = \tau\sigma$.

PROOF

Indeed, $(\sigma\tau)(\omega) = \sigma(\tau(\omega)) = \sigma(\omega^i) = (\sigma(\omega))^i = (\omega^j)^i = \omega^{ji}$

$(\tau\sigma)(\omega) = \tau(\sigma(\omega)) = \tau(\omega^j) = (\tau(\omega))^j = (\omega^i)^j = \overset{\shortparallel}{\omega^{ij}}$

- CASE②. $b \neq 1$

$E = F(x^k-b)$.

$$\begin{tikzcd}
 & E = F(x^k-b) \ni \beta \arrow[dl, dash] \arrow[dd, dash] \\
F(\omega) \arrow[dr, dash, "\text{abelian}"'] & \\
 & F
\end{tikzcd}$$

take $\beta \in E$ st $\beta^k = b$.

Consider $\omega\beta \in F(\beta, \omega)$.

$$\begin{tikzcd}
F(\omega, \beta) \arrow[d, dash] \\
F(\omega) \arrow[d, dash] \\
F
\end{tikzcd}$$

$(\omega\beta)^k = \omega^k \beta^k = 1 \cdot b = b$.

So $\omega\beta$ is a root of $x^k - b$.

So $\omega\beta \in E$

$\Rightarrow \omega = \frac{\omega\beta}{\beta} \in E$

- CLAIM: $Gal(F(x^k-b))/F(\omega))$ is abelian.

PROOF

The roots of x^k-b are $\beta, \omega\beta, \omega^2\beta, \dots, \omega^{k-1}\beta$

$F(x^k-b) = F(\omega)(\beta)$

if $\sigma, \tau \in Gal(F(\omega)(\beta)/F(\omega))$, then $\sigma\tau = \tau\sigma$

Indeed $\sigma(\beta) = \omega^j\beta$, $\tau(\beta) = \omega^i\beta$

$(\sigma\tau)(\beta) = \sigma(\tau(\beta)) = \sigma(\omega^i\beta) = \sigma(\omega^i)\sigma(\beta) = \omega^i\omega^j\beta = \omega^{i+j}\beta$

$(\tau\sigma)(\beta) = \dots \quad \dots = \omega^{j+i}\beta$ //

So $\sigma\tau = \tau\sigma$.

- 3 easy observations.

$\Phi : K \longmapsto Gal(E/K)$

$E_H \longleftarrow\!\shortmid\ H : \psi$

$\psi \circ \Phi = I : K \xmapsto{\Phi} Gal(E/K) \xmapsto{\psi} E_{Gal(E/K)} \overset{?}{=} K$

$\boxed{\text{Easy} \Rightarrow E_{Gal(E/K)} \supset K}$ Hard $\Rightarrow E_{Gal(E/K)} \subset K$

Assume $a \in K$ and $\sigma \in Gal(E/K)$.

$\sigma|_K = I \Rightarrow \sigma(a) = I(a) = a$.

$a \in E_{Gal(E/K)}$.

$\Rightarrow K \subset E_{Gal(E/K)}$.

$\Phi \circ \psi = I : H \xmapsto{\psi} E_H \xrightarrow{\Phi} Gal(E/E_H) \overset{?}{=} H$

$\boxed{\text{Easy} \Rightarrow H < Gal(E/E_H)}$ Hard $\Rightarrow H > Gal(E/E_H)$

Let $h \in H$

$h|_{E_H} = I$

X

MAT401H1 WK12 4/4/07.

- Galois theory.

Extension fields $\xleftrightarrow{\text{"the fundamental theorem"}}$ Groups

Extensions described by roots. $\longrightarrow$ solvable groups.

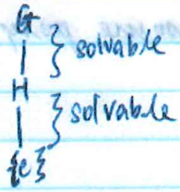
Splitting field of $3x^5-15x+5$ over $\mathbb{Q}$ $\longrightarrow$ S_5 is not solvable.

DONE

Solvable G: $\exists$ a tower $\{e\}=H_0 \triangleleft H_1 \triangleleft \cdots \triangleleft H_n = G$ s.t. $\forall k$ H_k/H_{k-1} is abelian.

THEOREM: If $H \triangleleft G$ then G is solvable $\Leftrightarrow$ H and G/H are solvable.

THEOREM: S_5 is not solvable.



$\Rightarrow G$ is solvable.

- DEFINITION: Let E/F be an extension. $\mathrm{Gal}(E/F)$ = "the galois group of E/F"

$= \left\{ \sigma : E \to E \;\middle|\; \begin{array}{l} ① \ \sigma \text{ is an automophism} \\ ② \ \sigma|_F = I \Leftrightarrow \forall \alpha \in F \ \sigma\alpha = \alpha \end{array} \right\}$

- CLAIM: $\mathrm{Gal}(E/F)$ is a group under composition:

$\sigma, \tau \in \mathrm{Gal}(E/F)$ $\quad \sigma \cdot \tau := \sigma \circ \tau$

PROOF

① $\sigma \circ \tau$ is in $\mathrm{Gal}(E/F)$

If $\alpha \in F$ $\quad \sigma(\tau\alpha) = \sigma(\alpha) = \alpha$.

② associativity is obvious

③ $I : E \to E$ is 1 of $\mathrm{Gal}(E/F)$

④ If $\sigma \in \mathrm{Gal}(E/F)$, then $\sigma^{-1} \in \mathrm{Gal}(E/F)$.

- CLAIM: $E = F(a_1 \ldots a_n)/F$ then "σ is determined by $\sigma(a_1) \ldots \sigma(a_n)$"

(ie σ_1 & $\sigma_2 \in \mathrm{Gal}(E/F)$ and $\sigma_1(a_1) = \sigma_2(a_1) = \cdots = \sigma_1(a_n) = \sigma_2(a_n)$.

then $\sigma_1 = \sigma_2$)

PROOF

Consider the set $\{\alpha \in E : \sigma_1(\alpha) = \sigma_2(\alpha)\} = K$

then ① $K \supset F$

② $K \supset \{a_1 \dots . a_n\}$

③ K is a field.

$\alpha, \beta \in K$

$$\begin{array}{r} \sigma_1(\alpha) = \sigma_2(\alpha) \\ + \quad \sigma_1(\beta) = \sigma_2(\beta) \\ \hline \end{array}$$

$\sigma_1(\alpha+\beta) = \sigma_1(\alpha) + \sigma_1(\beta) = \sigma_2(\alpha) + \sigma_2(\beta) = \sigma_2(\alpha+\beta)$.

$\Rightarrow \alpha+\beta \in K$, $\alpha \cdot \beta \in K$

$\Rightarrow K \supset E$

$\Rightarrow$ Since $K \subset E$, then $K = E$.

$\Rightarrow \sigma_1 = \sigma_2$ everywhere.

- CLAIM: If $\sigma \in Gal(E/K)$ and $a \in E$ is a root of some poly $f \in F[x]$ then $\sigma(a)$ is also a root of f.

COROLLARY

If E/F is finitely galois, then $|Gal(E/F| < \infty$

PROOF

$a \in E$, $f(a) = 0$, $f \in F[x]$

$f(\sigma a) = \sum a_i \sigma k(a^i) = \sum \sigma(a_i)\sigma(a^i)$

where $f = \sum a_i x^i$ and $a_i \in F$ $= \sigma(\sum a_i a^i) = \sigma(f(a)) = 0/0 = 0$.

*EXAMPLE

$E = \mathbb{Q}(\sqrt{3}, \sqrt{5})/\mathbb{Q}$.

root of $x^2 - 3$

root of $x^2 - 5 = 0$ $\pm\sqrt{5}$ are all roots.

$\pm\sqrt{3}$ are all roots

$\sigma \in Gal(E/F)$ is determined by $\sigma(\sqrt{3}) \in \{\pm\sqrt{3}\}$. and $\sigma(\sqrt{5}) \in \{\pm\sqrt{5}\}$

4 choices

$\sqrt{3} \rightarrow$	$\sqrt{3}$	$-\sqrt{3}$	$\sqrt{3}$	$-\sqrt{3}$
$\sqrt{5} \rightarrow$	$\sqrt{5}$	$\sqrt{5}$	$-\sqrt{5}$	$\sqrt{5}$
	I	α	β	$\alpha\beta$

$\alpha\beta : \sqrt{3} \xrightarrow{\beta} \sqrt{3} \longrightarrow -\sqrt{3}$ $\qquad \alpha^2 = I \quad \beta^2 = I$

$\sqrt{5} \xrightarrow{\beta} -\sqrt{5} \longrightarrow -5$

Is α really an automorphism?

$E = \{a_1 + a_2\sqrt{3} + a_3\sqrt{5} + a_4\sqrt{3}\sqrt{5} \; : a_i \in \mathbb{Q}\}$

$\downarrow$

$a_1 - a_2\sqrt{3} + a_3\sqrt{5} - a_4\sqrt{3}\sqrt{5}$

We have an explicit formula for α.
Can check explicitly that α (and β) respect + and ×, hence they are really automorphism.

$\Rightarrow Gal\{E/F\} = \{1, \alpha, \beta, \alpha\cdot\beta\}$

$\Rightarrow Gal(E/F) = \mathbb{Z}_2 \times \mathbb{Z}_2$.

- DEFINITION: If $H < Gal(E/F)$, set E_H = "the fixed field of H" $= \{a \in E : \forall h \in H, h(a) = a\}$

- CLAIM: E_H is a field containing F

-

$$\begin{tikzcd}
 & Z_2 \times Z_2 \arrow[dl, dash, "2"'] \arrow[d, dash, "2"] \arrow[dr, dash, "2"] & \\
\langle\alpha\rangle \arrow[dr, dash, "2"'] & \langle\beta\rangle \arrow[d, dash, "2"] & \langle\alpha\beta\rangle \arrow[dl, dash, "2"] \\
 & \{e\} &
\end{tikzcd}$$

$$\Downarrow$$

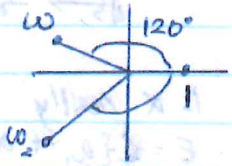
$$\begin{tikzcd}
 & E_{\langle\alpha,\beta\rangle} = \mathbb{Q} \arrow[dl, dash, "2"'] \arrow[d, dash] \arrow[dr, dash, "2"] & \\
E_{\langle\alpha\rangle} = \mathbb{Q}(\sqrt{5}) \arrow[dr, dash, "2"'] & \mathbb{Q}(\sqrt{3}) \arrow[d, dash, "2"] & E_{\langle\alpha\beta\rangle} = \mathbb{Q}(\sqrt{3}\sqrt{5}) \arrow[dl, dash, "2"] \\
 & E_{\{e\}} = E = \mathbb{Q}(\sqrt{3}, \sqrt{5}) &
\end{tikzcd}$$

* EXAMPLE

$E = \mathbb{Q}(\sqrt[3]{2}, \omega)$

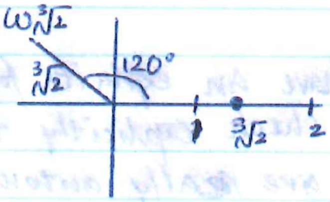
root of x^3-2 $\qquad$ $e^{2\pi i/3} = -\frac{1}{2} + \frac{\sqrt{3}}{2} i$

$\omega^3 - 1 = 0$



$\omega^3 - 1 = 0.$

$(\omega^2+\omega+1)(\omega-1) = 0.$



$\omega^2 + \omega + 1 = 0.$

roots $= \{\omega, \omega^2\}.$

root of $x^3 - 2$

↓

roots $= \{\sqrt[3]{2}, \omega\sqrt[3]{2}, \omega^2\sqrt[3]{2}\}.$

$\sqrt[3]{2} \to$	$\sqrt[3]{2}$	$\omega\sqrt[3]{2}$	$\omega^2\sqrt[3]{2}$	$\sqrt[3]{2}$	$\omega\sqrt[3]{2}$	$\omega^2\sqrt[3]{2}$
$\omega \to$	ω	ω	ω	ω^2	ω^2	ω^2
	I	β		α	$\beta\alpha$	$\alpha\beta$

EXERCISE: All of these really define automorphism.

$\alpha\beta: \omega \xmapsto{\beta} \omega \xmapsto{\alpha} \omega^2$

$\qquad \sqrt[3]{2} \xmapsto{\beta} \omega\sqrt[3]{2} \xrightarrow{\alpha} \omega^2\sqrt[3]{2}$

$\beta\alpha: \omega \xmapsto{\alpha} \omega^2 \xmapsto{\beta} \omega^2$

$\qquad \sqrt[3]{2} \xmapsto{\alpha} \sqrt[3]{2} \xmapsto{\beta} \omega\sqrt[3]{2}$

$\alpha\beta \neq \beta\alpha$

So $Gal(E/F) = G$ is not commutative.

$|G| = 6 \Rightarrow G = S_3.$

\- The Fundamental theorem of Galois Theory

" If char $F = 0$ and E/F is a splitting field, then there is a bijective (1-1 and onto) correspondence between $\left\{ \begin{array}{l} \text{Intermediate extensions} \\ K: E/K/F \end{array} \right\}$ and

$\left\{ \begin{array}{l} \text{subgroups} \\ H < Gal(E/F) \end{array} \right\}. \; \underset{**}{\overset{*}{\rightleftarrows}}$

$* \quad K \mapsto Gal(E/K) < Gal(E/F).$

$** \quad H \mapsto E_H$ of course $E/E_H/F$

5

Ⓐ So that

0. * and ** are inverses of each other.

1. inclusion-reversing

$H_1 < H_2 \Rightarrow E_{H_1} \supset E_{H_2}$

$K_1 < K_2 \Rightarrow Gal(E/K_1) > Gal(E/K_2)$

2. degrees / indexes / orders are respected.

$[E:K] = |Gal(E/K)|$

$[K:F] = [Gal(E/F) : Gal(E/K)] = \frac{|Gal(E/F)|}{|Gal(E/K)|}$

3. If K/F is a splitting extension, then $Gal(E/K) \trianglelefteq Gal(E/F)$ (normal subgroup).

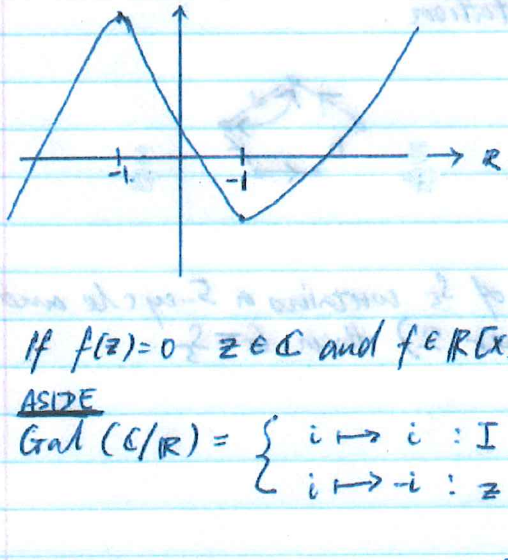
Furthermore, $Gal(K/F) = \frac{Gal(E/F)}{Gal(E/K)}$

Ⓑ - Extension describable by roots $\longrightarrow$ solvable groups.

Ⓒ splitting field of $3x^5 - 15x + 5$ $\longrightarrow$ S_5

- PROOF OF Ⓒ

Consider $f = 3x^5 - 15x + 5$. (f is irreducible by Eisenstein's Criterion)



$f' = 15x^4 - 15$

$f' = 0 \Rightarrow x^4 = 1 \Rightarrow x = \pm 1$

$f(1) = -7 \quad f(-1) = 17$

$\underline{f \text{ has } 5 \text{ roots in all.}}$

$\Rightarrow f$ has 3 real roots.

$\Rightarrow f$ has 2 further complex roots, z and $\bar{z}$.

If $f(z) = 0$ $z \in \mathbb{C}$ and $f \in \mathbb{R}[x]$ then $f(\bar{z}) = 0$.

ASIDE

$$Gal(\mathbb{C}/\mathbb{R}) = \left\{ \begin{array}{l} i \mapsto i : I \\ i \mapsto -i : z \mapsto \bar{z} \end{array} \right\} = \mathbb{Z}/2$$

Let E be the splitting field of f. $G = Gal(E/\mathbb{Q}) < \underset{\text{of } \{\alpha_1, \dots, \alpha_5\}}{\overset{\text{grp of}}{\text{permutations}}} = S_5$

Let $\alpha_1 \dots \alpha_5$ be the roots of F.

Let $\sigma \in G$. $\sigma : \begin{array}{l} \alpha_1 \longrightarrow \alpha_{i_1} \\ \alpha_2 \longrightarrow \alpha_{i_2} \\ \vdots \\ \alpha_5 \longrightarrow \alpha_{i_5} \end{array}$ σ being invertible must permute the roots.

$E = Q(\alpha_1 \dots \alpha_5)$

① $z \longmapsto \overline{z}$

$$\begin{tikzcd} \mathbb{C} \arrow[r] & \mathbb{C} \\ E \arrow[u, dash] \arrow[r] & E \arrow[u, dash] \end{tikzcd}$$

restricts to an element of $Gal(E/Q) = G$ that keeps three roots in place and switches the other two. (2 cycle).

② Let $[Q(\alpha_1) : Q] = \deg f = 5$

the minimal polynomial of α_1 is $\frac{1}{3}f$.

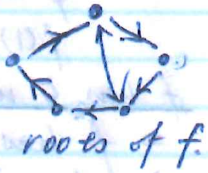
$[Q(\alpha) : Q]$ = degree of min poly α

$$\left.\begin{tikzcd} E \arrow[d, dash] \\ Q(\alpha_1) \arrow[d, dash, "5"] \\ Q \end{tikzcd}\right\} \text{multiple of 5.}$$

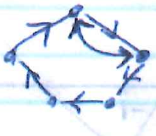
$|Gal(E/Q)| = [E:Q]$ = divisible by 5, not divisible by 25

By Sylow, $Gal(E/Q)$ has a subgroup of order 5.

$\Rightarrow$ Contains a cyclic permutation.



roots of f.



- PROPOSITION: If a subgroup G of S_5 contains a 5-cycle and a transposition (keeps 3 and switches other 2) then $G = S_5$

PROOF

Rubik's style exercise

MAT401H1 WK11 28/3/07.

- GOAL: Some polynomials cannot be "solved" using $+, -, \times, \div, \sqrt[n]{\ }$

- Galois theory

FIELDS $\xleftrightarrow{\text{(rough)}}$ GROUPS
EXTENSIONS The Fundamental Theorem of Galois

extensions that are describable using "radicals" $\longrightarrow$ "Solvable groups"

Splitting field $\longrightarrow$ "hard"
$3x^5 - 15x + 5$ S_5 $|S_5| = 120$.
$\mathbb{Q}(\alpha_1 \ldots \alpha_5)$

- REMINDERS
Group: $\times$; associative but not necessarily commutative, e, inverses.
$H < G$: $H \subset G$. closed under $\times$ and it is a group.

- $gl_2(\mathbb{R}) = \{A \in M_{2\times 2}(\mathbb{R}) : \det A \neq 0\} = \{\begin{pmatrix} * & * \\ * & * \end{pmatrix} : \begin{pmatrix} * & * \\ * & * \end{pmatrix}^{-1} \text{ exists}\}$.
$gl_2(\mathbb{Z}) < gl_2(\mathbb{R})$ where $gl_2(\mathbb{Z}) = \{A \in M_{2\times 2}(\mathbb{Z}) : \det A \neq 0\}$.

- Rubik's cube group $< S_8 \times S_{12}$
$G_1 \times G_2 = \{g_1, g_2)\}$.
$(g_1, g_2) \cdot (g_1', g_2') = (g_1 g_1', g_2 g_2')$.

- $[g] = \{gh : h \in H\}$.
$g_1 \overset{H}{\sim} g_2$ iff $\exists h \in H$ such that $g_1 = g_2 h$. $\Leftrightarrow g_2^{-1} g_1 \in H$
$G/H = \{[g] : g \in G\}$
$|G/H| = |G|/|H| = [G:H] =$ the index of H in G.

- DEFINITION: $(N \triangleleft G)$ N is normal in G if $\forall g \in G$ $\underset{\{g^{-1}ng \,:\, n \in N\}}{\underset{\|}{g^{-1}Ng}} = N \Leftrightarrow Ng = gN$.

* EXAMPLE

G = rigid motions of the plane.

$\{f : \mathbb{C} \to \mathbb{C} : f(z) = az + b \quad |a| = 1, b \in \mathbb{C}\}$

$f_2 \circ f_1(z) = f_2(a_1 z + b_1) = a_2(a_1 z + b_1) + b_2$

$= \underbrace{a_1 a_2 z + b_1 a_2}_{a} + \underbrace{b_1 a_2 + b_2}_{b.}$

$N = \{f(z) = 1 \cdot z + b\}$

$(z + b_1) + b_2 = z + (b_1 + b_2).$

$(z \mapsto az + b)^{-1} = (y \mapsto a^{-1}y - a^{-1}b).$

$y = az + b$

$a^{-1}y = z + a^{-1}b$

$a^{-1}y - a^{-1}b = z.$

$(az + b) \circ (1z + b_1) \circ (a^{-1}z \; a^{-1}b)$

$= a(a^{-1}z - a^{-1}b + b_1) + b$

$= z - b + ab_1 + b = \underline{z + ab_1 \in N}$

- THEOREM: If N is normal in G, G/N is a group under $[g_1][g_2] = [g_1 g_2]$

- $f_1 = a_1 z + b_1 \quad f_2 = a_2 z + b_2 \quad N = \{f(z) = 1 \cdot z + b\}$

$f_1 \sim f_2 \Leftrightarrow f_1 = f_2(z \mapsto z + b)$

$= (z \longmapsto a_2(z + b) + b_2).$

$(z \mapsto a_1 z + b_1) = (z \longmapsto a_2 z + a_2 b + b_2)$

$\Leftrightarrow a_1 = a_2$

$G/N = \{a : |a| = 1\}.$

- ISOMORPHISM THEOREMS.

① $\phi : G_1 \to G_2$ $\ker\phi \triangleleft G_1$ and $G_1/\ker\phi \cong \operatorname{im} G_2$

② $N \triangleleft H \triangleleft G$ and $N \triangleleft G$ then $G/H \cong G/H / H/N$.

③ $H < G$, $N \triangleleft G$

$H/N \cap H = H \cdot N/N$

$$\begin{tikzcd} & G \arrow[dl, phantom, "\supset"] \arrow[dr, phantom, "\triangleright"] & \\ H & & N \end{tikzcd}$$

- DEFINITION: G is called "solvable" if you can find a finite chain of subgroups $\{e\} = H_0 \triangleleft H_1 \triangleleft H_2 \triangleleft \ldots \triangleleft H_n = G$ such that H_k/H_{k-1} is abelian (commutative).

* EXAMPLE

$G = \{ az + b \}$

Take $\{e\} = H_0 \lhd H_1 = G$ "shortest chain"

Normal since $g^{-1} e g = e$.

But $G/\{e\} = G$ not abelian.

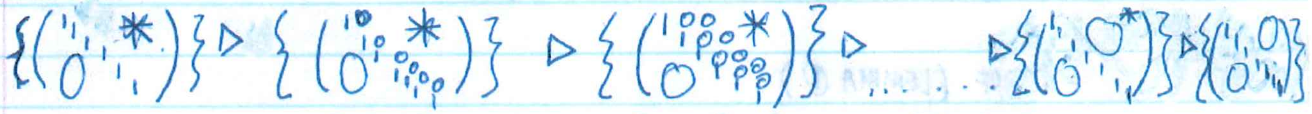
$\{e\} = H_0 \lhd H_1 = N \lhd H_2 = G$

$\{z+b\}$

$H_1/H_0 = N \cong (\mathbb{C}, +)$ abelian.

$H_2/H_1 = \{ a : |a| = 1 \} \subset (\mathbb{C}, \times)$ abelian.

* EXAMPLE



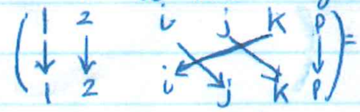
- THEOREM: S_p is not solvable if $p \geq 5$.

$S_p = \{ \sigma : \{1 \dots p\} \to \{1 \dots p\} : \sigma^{-1} \text{ exists} \}$

PROOF

Suppose a chain $\{e\} = H_0 \lhd H_1 \lhd H_2 \lhd \dots \lhd H_{n-1} \lhd H_n = S_p$ exist such that H_k / H_{k-1} is abelian is given.

Let A be the subgroup of S_p generated by 3 cycles.



$= (ijk)$.

The smallest subgroup of S_p containing all (ijk)'s.

- DEFINITION: Given $H < G$, $[H, H]$ = the "commutator subgroup of H"
= the smallest subgroup of G containing all things of the form $\{ h_1 h_2 h_1^{-1} h_2^{-1} : h_1, h_2 \in H \}$.

If H is abelian, $[H, H] = \{e\}$.

- LEMMA ①: $[A, A] = A$ (in our case)

- LEMMA ②: If $H' \lhd H''$ and H''/H' is abelian and $H'' > A$ then $H'' > [A, A]$

- (Continuation of proof)

$H_n = S_p > A$.

By lemma ②, $H_{n-1} > [A, A] = A$

$H_{n-2} > [A, A] = A$.

$\vdots$

$\underset{\{e\}}{\overset{\shortparallel}{H_0}} > \underset{\{e\}}{\overset{\nparallel}{A}}$ $(\Rightarrow\Leftarrow)$.

- PROOF (LEMMA ②).

Let $a, b \in A$. Need to show that $aba^{-1}b^{-1} \in H'$ (Work in H''/H').
Need to show in H''/H', that $[a][b][a^{-1}][b^{-1}] = [e]$.
H''/H' is abelian so that is obvious.

- PROOF (LEMMA ①)

$a = (ijk)$ $\quad a^{-1} = (kji)$ $\quad b = (kmn)$ $\quad b^{-1} = (nmk)$.

$ijk \underset{a^{-1}}{\overset{a}{\rightleftarrows}} kij$ $\qquad i, j, k, m, n$ are different.

$$aba^{-1}b^{-1} = (ijk)\cdot(kmn)\cdot(kji)\cdot(nmk)$$

$$= \left(\begin{matrix} 1 \\ \downarrow \\ 1 \end{matrix}\ \begin{matrix} 2 \\ \downarrow \\ 2 \end{matrix}\ \boxed{\begin{matrix} i \\ \downarrow \\ m \end{matrix}}\ \begin{matrix} j \\ \downarrow \\ j \end{matrix}\ \boxed{\begin{matrix} k \\ \downarrow \\ i \end{matrix}}\ \boxed{\begin{matrix} m \\ \downarrow \\ k \end{matrix}}\ \begin{matrix} n \\ \downarrow \\ n \end{matrix}\ \dots\right)$$

$$m \longleftarrow k \longleftarrow i \longleftarrow i$$

$$j \longleftarrow j \longleftarrow j \longleftarrow j$$

$$= (imk)$$

$\forall i, m, k$, $(imk) \in [A, A]$

All three cycles are in $[A, A]$ so $[A, A] \supset A$; $\underbrace{A \supset [A, A]}_{\text{obvious}}$.

$\Rightarrow A = [A, A]$.

- LEMMA ①: Suppose $N \lhd G$, N is solvable and G/N is solvable. Then G is solvable.

PROOF

$N \rhd H_n \rhd H_{n-1} \rhd \dots \rhd H_0 = \{e\}$ $\quad H_k/H_{k-1}$ is abelian.

$G/N \rhd K_m \rhd K_{m-1} \rhd \dots \rhd K_0 = \{[e]\} = \{N\}$. such that K_k/K_{k-1} is abelian.

ASIDE

$\pi : G \to G/N$

$\pi^{-1}(k) \check{<} G \leftarrow K < G/N.$

$G > \pi^{-1}(k_m) > \pi^{-1}(k_{m-1}) > \dots > \pi^{-1}(k_0) = N > H_n > H_{n-1} > \dots > H_0$

Verify that all the inclusions are normal

Furthermore $\pi^{-1}(K_k)/\pi^{-1}(K_{k-1}) \cong K_k/K_{k-1}$ abelian.

- LEMMA ⓪: If $N \lhd G$ and G is solvable then G/N is solvable.

PROOF

$\{e\} = H_0 \lhd H_1 \lhd H_2 \lhd \dots \lhd H_n = G \qquad H_k/H_{k-1}$ is abelian

$$\begin{array}{cccccc} N\{e\}/N & H_0N/N & < H_1N/N & < H_2N/N < & \dots & < H_nN/N \\ \| & \| & & & & \| \\ N/N & = N/N & & & & G/N \end{array}$$

- CLAIM: If $H_{k-1} \lhd H_k$ and $N \lhd G$ then $H_{k-1}N/N \lhd H_kN/N$

PROOF

$[h] = [h \cdot n] \quad h \in H_k \quad n \in N.$

Take $[h'] \in H_{k-1}N/N$ and $[h] \in H_kN/N$

$$[h]^{-1}[h'][h] = [\underbrace{h^{-1}\underset{\underset{H_{k-1}}{\cap}}{h'}\underset{\underset{H}{\cap}}{h}}_{\in H_{k-1}}] \in H_{k-1}N/N.$$

- CLAIM: Abelian is $\dfrac{H_kN/N}{H_{k-1}N/N} = \dfrac{H_kN}{H_{k-1}N} \xleftarrow[\substack{\phi \\ \text{onto}}]{\text{homomorphism}} \dfrac{H_k}{H_{k-1}}$

By first isomorphism theorem, $\dfrac{H_kN}{H_{k-1}N} = \operatorname{im}\phi = \dfrac{H_k/H_{k-1}}{\ker\phi} = \dfrac{\text{Abelian}}{\text{something}} = \text{Abelian}.$

$h \in H_k. \quad \phi([h]_{H_{k-1}}). = [h]_{H_{k-1}N}.$

Verify that ① ϕ is well-defined ② ϕ is onto.

HW6 returned
HW7 due
HW8 on web by noon tomorrow

Math 401 Pols, Eqns, Fields, March 19 2008, week 10.

on board: First two sections of "Quick Reference"

Discussion of front page of quick ref.

Def Group: A set G with a "multiplication" $(g_1, g_2) \mapsto g_1 g_2 \in G$
s.t. 1. Associative
2. $\exists$ identity (unique, 1 or e)
3. $\exists$ inverses.

$H < G$, G/H, $|G/H| = [G:H] = |G|/|H|$ "the index of H in G"

$N \triangleleft G$, $N \triangleleft G \Rightarrow G/N$ is a group ... there are various iso theorems to be stated later.

Def $E/F \Rightarrow Gal(E/F)$
(claim: it is a group)

Def $H < Gal(E/F) \Rightarrow E_H$ "the fixed ~~set~~ field of H in E"   claim E_H is a field.

The fundamental thm of Galois Theory: If char $F = 0$ and E is a splitting field over F (these exist and are unique), then there are a bijections

$$\begin{tikzcd} K : E/K/F \arrow[r, bend left, "\Phi : K \mapsto Gal(E/K)"] & \text{Subgroups of } Gal(E/F) \arrow[l, bend left, "E_H \leftarrow\!\shortmid H : \Psi"] \end{tikzcd}$$

Furthermore, Φ & Ψ are inverses of each other
1. inclusion reversing.
2. degree/index respecting /order $[E:K] = |Gal(E/K)|$
$[K:F] = [Gal(E/F) : Gal(E/K)]$

Example 1 $E = \mathbb{Q}(\sqrt{3}, \sqrt{5}) / \mathbb{Q} = F$

Claim given E/F if $\alpha \in E$ is a root of $f \in F[x]$, ~~then~~ and $\phi \in Gal(E/F)$, then $\phi(\alpha)$ is also a root of f.

$$\begin{array}{llcccc}
\text{so} & \sqrt{3} \longrightarrow & \sqrt{3} & -\sqrt{3} & \sqrt{3} & -\sqrt{3} \\
 & \sqrt{5} \longrightarrow & \sqrt{5} & \sqrt{5} & -\sqrt{5} & -\sqrt{5} \\
 & & \epsilon & \alpha & \beta & \alpha\beta
\end{array}$$

furthermore, every aut ϕ is determined by $\phi(\sqrt{3})$ & $\phi(\sqrt{5})$

So $G = Gal(E/F)$ is $\mathbb{Z}_2 \oplus \mathbb{Z}_2$; lets figure out the fixed fields . . .

Example 2 The splitting field of $x^3 - 2$ over $\mathbb{Q}$

$E = \mathbb{Q}(\sqrt[3]{2}, w) / \mathbb{Q} = F$, with $w^3 = 1$ or $w^2 + w + 1 = 0$

continue as in book.

Def solvable group, ~~Extension~~ solvable by radicals.

Example 5 is a bit more complicated than our previous examples. In particular, the automorphism group is non-Abelian.

EXAMPLE 5 Direct calculations show that $\omega = -1/2 + i\sqrt{3}/2$ satisfies the equations $\omega^3 = 1$ and $\omega^2 + \omega + 1 = 0$. Now, consider the extension $Q(\omega, \sqrt[3]{2})$ of Q. We may describe the automorphisms of $Q(\omega, \sqrt[3]{2})$ by specifying how they act on ω and $\sqrt[3]{2}$. There are six in all:

ε	α	β	β^2	$\alpha\beta$	$\alpha\beta^2$
$\omega \to \omega$	$\omega \to \omega^2$	$\omega \to \omega$	$\omega \to \omega$	$\omega \to \omega^2$	$\omega \to \omega^2$
$\sqrt[3]{2} \to \sqrt[3]{2}$	$\sqrt[3]{2} \to \sqrt[3]{2}$	$\sqrt[3]{2} \to \omega\sqrt[3]{2}$	$\sqrt[3]{2} \to \omega^2\sqrt[3]{2}$	$\sqrt[3]{2} \to \omega^2\sqrt[3]{2}$	$\sqrt[3]{2} \to \omega\sqrt[3]{2}$

Since $\alpha\beta \neq \beta\alpha$, we know that $\mathrm{Gal}(Q(\omega, \sqrt[3]{2})/Q)$ is isomorphic to S_3. (See Theorem 7.2.) The lattices of subgroups and subfields are shown in Figure 32.5.

The lattices in Figure 32.5 have been arranged so that the field occupying the same position as some group is the fixed field of that group. For instance, $Q(\omega\sqrt[3]{2})$ is the fixed field of $\{\varepsilon, \alpha\beta\}$.

◆

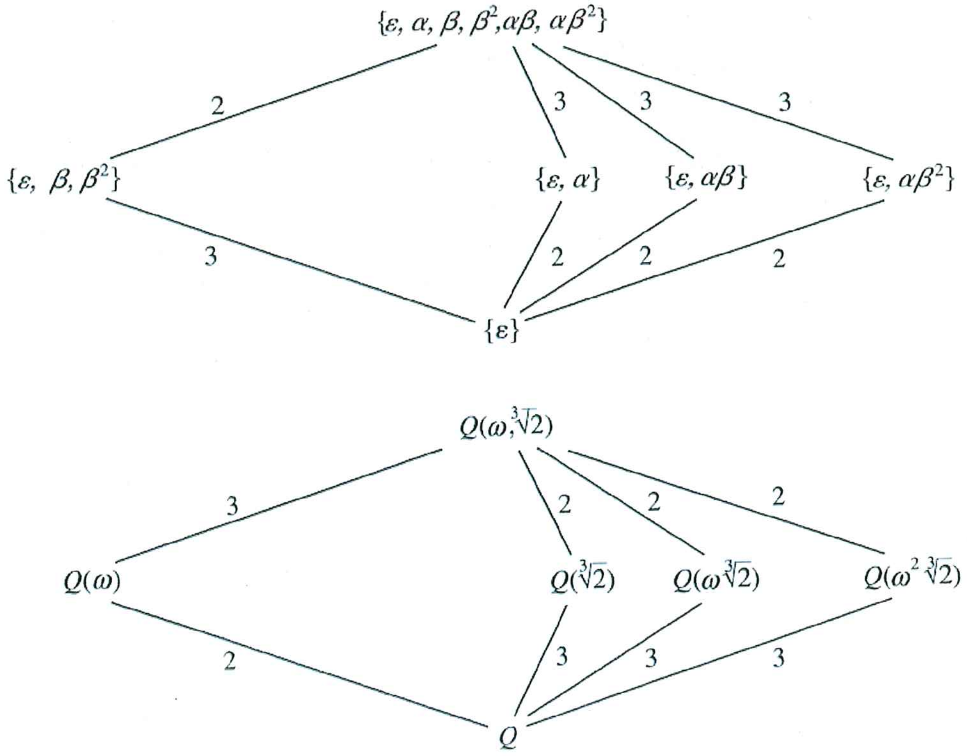


Figure 32.5 Lattice of subgroups of $\mathrm{Gal}(Q(\omega, \sqrt[3]{2})/Q)$ and lattice of subfields of $Q(\omega, \sqrt[3]{2})$, where $\omega = -1/2 + i\sqrt{3}/2$.

$r_1 = \sqrt[3]{2}$ $r_2 = w\sqrt[3]{2}$ $r_3 = w^2\sqrt[3]{2}$

More on split(x^3-2): $E = \mathbb{Q}(\sqrt[3]{2}, w) = \mathbb{Q}(r_1\ r_2\ r_3)$

$\alpha_1 : r_2 \leftrightarrow r_3$

$\alpha_2 : r_1 \leftrightarrow r_3$

$\beta : r_1 \to r_2$
$r_2 \to r_3$
$r_3 \to r_1$

deg 6 extension,
So all of S_3
occurs.

Subgroups $\langle \alpha_1 \rangle = \{e, \alpha_1\}$ $\langle \alpha_2 \rangle = \{e, \alpha_2\}$

07-401/Class Notes for April 11

From Drorbn

07-401/Navigation Panel [Show]

Contents

Today's Agenda

- Today's the deadline for the prize problem from 07-401/Homework Assignment 7!
- Reminder of the Fundamental Theorem of Galois Theory.
- Proof of the insolubility of the quintic assuming the Fundamental Theorem.
- Proofs of the easy parts of the Fundamental Theorem.
- A short discussion of the final and and the time leading to it.
- **Course Evaluation Forms** and a **Post-Mortem** discussion in the spirit of 0506-1300/Post Mortem (http://katlas.math.toronto.edu/0506-Topology/index.php?title=Post_Mortem) and of 06-240/Classnotes For Thursday December 7.
- With luck, **early dismissal!**

The Final Exam

As announced (http://www.artsci.utoronto.ca/current/undergraduate/exams/april-may-2007-exam-schedule) by the powers above, our final exam will take place on the *evening* of Tuesday April 24 between 7PM and 10PM, at New College Residence (NR) room 25.

The exam will be similar in style to the Term Test (also see On the Term Test). The material is **everything** covered in class. Everything in the test will be taken from our text book, and there will be two types of questions (or maybe sometimes the two types will be mixed within a single question):

- You may be asked to prove a theorem proven in class. The reason we prove theorems in class is that these proofs are **valuable**. Therefore I expect you to know them.
- You may be asked to solve exercises from the relevant chapters of the book, or minor variations thereof. These may be questions that were assigned as homework, but also, these may be questions that were not assigned before.

Office Hours. I (Dror) will hold extended office hours before the final, on Monday April 23 11AM-1PM and on the exam date, Wednesday April 24 10AM-12PM. You will be able to pick up all your graded assignments then and also on my last "normal" office hour, on Wednesday April 18 10:30AM-11:30AM. All office hours will be held at or near my office, Bahen 6178

Preparing for the Test. Read, reread and rereread everything and solve lots of exercises from the book.

My (Dror's) system when I was an undergrad was to prepare a 4-6 page 100-200 item list of points covered in class. I'd only summarize each point with one sentence, without giving any details and without trying to be precise, much like the list that I prepared for the class of February 7 (see On the Term Test). I would then go over my list again and again and again, crossing out every item for which I was sure I could complete all the details and supply all the proofs. I would only stop when there was nothing left to cross out.

Good Luck!

The Fundamental Theorem of Galois Theory

It seems we will not have time to prove the Fundamental Theorem of Galois Theory in full. Thus this note is about what we will be missing. The statement appearing here, which is a weak version of the full theorem, is taken from Gallian's book and is meant to match our discussion in class. The proof is taken from Hungerford's book, except modified to fit our notations and conventions and simplified as per our weakened requirements.

Here and everywhere below our base field F will be a field of characteristic 0.

Statement

Theorem. Let E be a splitting field over F. Then there is a bijective correspondence between the set $\{K : E/K/F\}$ of intermediate field extensions K lying between F and E and the set $\{H : H < \mathrm{Gal}(E/F)\}$ of subgroups H of the Galois group $\mathrm{Gal}(E/F)$ of the original extension E/F:

$$\{K : E/K/F\} \quad \leftrightarrow \quad \{H : H < \mathrm{Gal}(E/F)\}\,.$$

The bijection is given by mapping every intermediate extension K to the subgroup $\mathrm{Gal}(E/K)$ of elements in $\mathrm{Gal}(E/F)$ that preserve K,

$$\Phi : \quad K \mapsto \mathrm{Gal}(E/K),$$

and reversely, by mapping every subgroup H of $\mathrm{Gal}(E/F)$ to its fixed field E_H:

$$\Psi : \quad H \mapsto E_H\,.$$

This correspondence has the following further properties:

1. It is inclusion-reversing: if $H_1 \subset H_2$ then $E_{H_1} \supset E_{H_2}$ and if $K_1 \subset K_2$ then $\mathrm{Gal}(E/K_1) > \mathrm{Gal}(E/K_1)$.
2. It is degree/index respecting: $[E : K] = |\mathrm{Gal}(E/K)|$ and $[K : F] = [\mathrm{Gal}(E/F) : \mathrm{Gal}(E/K)]$.
3. Splitting fields correspond to normal subgroups: If K in $E/K/F$ is the splitting field of a polynomial in $F[x]$ then $\mathrm{Gal}(E/K)$ is normal in $\mathrm{Gal}(E/F)$ and $\mathrm{Gal}(K/F) \cong \mathrm{Gal}(E/F)/\mathrm{Gal}(E/K)$.

Lemmas

The two lemmas below belong to earlier chapters but we skipped them in class.

The Primitive Element Theorem

The celebrated "Primitive Element Theorem" is just a lemma for us:

Lemma 1. Let a and b be algebraic elements of some extension E of F. Then there exists a single element c of E so that $F(a, b) = F(c)$. (And so by induction, every finite extension of E is "simple", meaning, is generated by a single element, called "a primitive element" for that extension).

Proof. See the proof of Theorem 21.6 on page 375 of Gallian's book. ☐

Splitting Fields are Good at Splitting

Lemma 2. (Compare with Hungerford's Theorem 10.15 on page 355). If E is a splitting field of some polynomial f over F and some irreducible polynomial $p \in F[x]$ has a root v in E, then p splits in E.

Proof. Let L be a splitting field of p over E. We need to show that if w is a root of p in L, then $w \in E$ (so all the roots of p are in E and hence p splits in E). Consider the two extensions

$E = E(v)/F(v)$ and $E(w)/F(w)$.

The "smaller fields" $F(v)$ and $F(w)$ in these two extensions are isomorphic as they both arise by adding a root of the same irreducible polynomial (p) to the base field F. The "larger fields" $E = E(v)$ and $E(w)$ in these two extensions are both the splitting fields of the same polynomial (f) over the respective "small fields", as E/F is a splitting extension for f and we can use the sub-lemma below. Thus by the uniqueness of splitting extensions, the isomorphism between $F(v)$ and $F(w)$ extends to an isomorphism between $E = E(v)$ and $E(w)$, and in particular these two fields are isomorphic and so $[E : F] = [E(v) : F] = [E(w) : F]$. Since all the degrees involved are finite it follows from the last equality and from $[E(w) : F] = [E(w) : E][E : F]$ that $[E(w) : E] = 1$ and therefore $E(w) = E$. Therefore $w \in E$. $\square$

Sub-lemma. If E/F is a splitting extension of some polynomial $f \in F[x]$ and z is an element of some larger extension L of E, then $E(z)/F(z)$ is also a splitting extension of f.

Proof. Let $u_1, \dots, u_n$ be all the roots of f in E. Then they remain roots of f in $E(z)$, and since f completely splits already in E, these are *all* the roots of f in $E(z)$. So

$$E(z) = F(u_1, \dots, u_n)(z) = F(z)(u_1, \dots, u_n),$$

and $E(z)$ is obtained by adding all the roots of f to $F(z)$. $\square$

Proof of The Fundamental Theorem

The Bijection

Proof of $\Psi \circ \Phi = I$. More precisely, we need to show that if K is an intermediate field between E and F, then $E_{\mathrm{Gal}(E/K)} = K$. The inclusion $E_{\mathrm{Gal}(E/K)} \supset K$ is easy, so we turn to prove the other inclusion. Let $v \in E - K$ be an element of E which is not in K. We need to show that there is some automorphism $\phi \in \mathrm{Gal}(E/K)$ for which $\phi(v) \neq v$; if such a ϕ exists it follows that $v \notin E_{\mathrm{Gal}(E/K)}$ and this implies the other inclusion. So let p be the minimal polynomial of v over K. It is not of degree 1; if it was, we'd have that $v \in K$ contradicting the choice of v. By lemma 2 and using the fact that E is a splitting extension, we know that p splits in E, so E contains all the roots of p. Over a field of characteristic 0 irreducible polynomials cannot have multiple roots and hence p must have at least one other root; call it w. Since v and w have the same minimal polynomial over K, we know that $K(v)$ and $K(w)$ are isomorphic; furthermore, there is an isomorphism $\phi_0 : K(v) \to K(w)$ so that $\phi_0|_K = I$ yet $\phi_0(v) = w$. But E is a splitting field of some polynomial f over F and hence also over $K(v)$ and over $K(w)$. By the uniqueness of splitting fields, the isomorphism ϕ_0 can be extended to an isomorphism $\phi : E \to E$; i.e., to an automorphism of E. but then $\phi|_K = \phi_0|_K = I$ so $\phi \in \mathrm{Gal}(E/K)$, yet $\phi(v) = w \neq v$, as required. $\square$

Proof of $\Phi \circ \Psi = I$. More precisely we need to show that if $H < \mathrm{Gal}(E/F)$ is a subgroup of the Galois group of E over F, then $H = \mathrm{Gal}(E/E_H)$. The inclusion $H < \mathrm{Gal}(E/E_H)$ is easy. Note that H is finite since we've proven previously that Galois groups of finite extensions are finite and hence $\mathrm{Gal}(E/F)$ is finite. We will prove the following sequence of inequalities:

$$|H| \leq |\mathrm{Gal}(E/E_H)| \leq [E : E_H] \leq |H|$$

This sequence and the finiteness of $|H|$ imply that these quantities are all equal and since $H < \mathrm{Gal}(E/E_H)$ it follows that $H = \mathrm{Gal}(E/E_H)$ as required.

The first inequality above follows immediately from the inclusion $H < \mathrm{Gal}(E/E_H)$.

By the Primitive Element Theorem (Lemma 1) we know that there is some element $u \in E$ so that $E = E_H(u)$. Let p be the minimal polynomial of u over E_H. Distinct elements of $\mathrm{Gal}(E/E_H)$ map u to distinct roots of p, but p has exactly $\deg p$ roots. Hence $|\mathrm{Gal}(E/E_H)| \leq \deg p = [E : E_H]$, proving the second inequality above.

Let $\sigma_1, \ldots, \sigma_n$ be an enumeration of all the elements of H, let $u_i := \sigma_i u$ (with u as above), and let f be the polynomial

$$f = \prod_{i=1}^{n} (x - u_i).$$

Clearly, $f \in E[x]$. Furthermore, if $\tau \in H$, then left multiplication by τ permutes the σ_i's (this is always true in groups), and hence the sequence $(\tau u_i = \tau\sigma u_i)_{i=1}^{n}$ is a permutation of the sequence $(u_i)_{i=1}^{n}$, hence

$$\tau f = \prod_{i=1}^{n} (x - \tau u_i) = \prod_{i=1}^{n} (x - u_i) = f,$$

and hence $f \in E_H[x]$. Clearly $f(u) = 0$, so $p|f$, so $[E : E_H] = \deg p \leq \deg f = n = |H|$, proving the third inequality above. □

The Properties

Property 1. If $H_1 \subset H_2$ then $E_{H_1} \supset E_{H_2}$ and if $K_1 \subset K_2$ then $\mathrm{Gal}(E/K_1) > \mathrm{Gal}(E/K_1)$.

Proof of Property 1. Easy. □

Property 2. $[E : K] = |\mathrm{Gal}(E/K)|$ and $[K : F] = [\mathrm{Gal}(E/F) : \mathrm{Gal}(E/K)]$.

Proof of Property 2. If $K = E_H$, then $|\mathrm{Gal}(E/K)| = |\mathrm{Gal}(E/E_H)| = [E : E_H] = [E : K]$ as was shown within the proof of $\Phi \circ \Psi = I$. But every K is E_H for some H, so $|\mathrm{Gal}(E/K)| = [E : K]$ for every K between E and F. The second equality follows from the first and from the multiplicativity of the degree/order/index in towers of extensions and in towers of groups:

$$[K : F] = \frac{[E : F]}{[E : K]} = \frac{|\mathrm{Gal}(E/F)|}{|\mathrm{Gal}(E/K)|} = [\mathrm{Gal}(E/F) : \mathrm{Gal}(E/K)]. \quad \square$$

Property 3. If K in $E/K/F$ is the splitting field of a polynomial in $F[x]$ then $\mathrm{Gal}(E/K)$ is normal in $\mathrm{Gal}(E/F)$ and $\mathrm{Gal}(K/F) \cong \mathrm{Gal}(E/F)/\mathrm{Gal}(E/K)$.

Proof of Property 3. We will define a surjective (onto) group homomorphism $\rho : \mathrm{Gal}(E/F) \to \mathrm{Gal}(K/F)$ whose kernel is $\mathrm{Gal}(E/K)$. This shows that $\mathrm{Gal}(E/K)$ is normal in $\mathrm{Gal}(E/F)$ (kernels of homomorphisms are always normal) and then by the first isomorphism theorem for groups, we'll have that $\mathrm{Gal}(K/F) \cong \mathrm{Gal}(E/F)/\mathrm{Gal}(E/K)$.

Let σ be in $\mathrm{Gal}(E/F)$ and let u be an element of K. Let p be the minimal polynomial of u in $F[x]$. Since K is a

Math 401 Pols, Eqns, Fields, April 11 2007, Week 13.

On board:

General Scheme	Solvable fields	Fund theorem.

Cont. as on April 4

Discussion of The Final.

Post Mortem / Evaluations

Dror's regrets:	Good as is	Bad or need improvement.
* First half too slow * Fund theorem not completed * Huge drop in Attendance * Time block. Aspects of class lectures, HW, TT, Web, book, ...		

HW10 (last) on web by noon tomorrow.

Math 401 Pols, Eqns, Fields, April 4 2007, Week 12.

on board.

Galois Theory

$\text{Fields} \xleftrightarrow{\text{fund Thm}} \text{Groups}$

extensions describable by roots $\longrightarrow$ solvable groups

splitting field of $3x^5-15x+5$ $\longrightarrow$ S_5 not solvable

Done:

solvable G:

$\{e\}=H_0 \lhd H_1 \lhd H_2 \lhd \dots \lhd H_n = G$

s.t. H_k/H_{k-1} Abelian $\forall\ 1 \le k \le n$

* If $H \lhd G$ then G solvable iff H & G/H are.

* S_5 is not solvable

Def $E/F \Rightarrow Gal(E/F)$

Claim it is a group

Claim If $\sigma \in Gal(E/F)$ and a is a root of $f \in F[x]$, then $\sigma(a)$ is a root too,

Claim $\sigma \in Gal(F(a_1 \dots a_n)/F)$ is determined by $\sigma(a_1)$ $\sigma(a_n)$

Example 1 $E = \mathbb{Q}(\sqrt{3},\sqrt{5})/\mathbb{Q} = F$

$\sqrt{3} \longrightarrow$	$\sqrt{3}$	$-\sqrt{3}$	$\sqrt{3}$	$-\sqrt{3}$
$\sqrt{5} \longrightarrow$	$\sqrt{5}$	$\sqrt{5}$	$-\sqrt{5}$	$-\sqrt{5}$
	I	α	β	$\alpha\beta$

So $Gal(E/F) = \mathbb{Z}_2 \times \mathbb{Z}_2$

Def The Fixed field E_H of $H < Gal(E/F)$

Claim It is a field.

Continue example 1 $E_{\langle\alpha\rangle} = \mathbb{Q}(\sqrt{5})$ $E_{\langle\beta\rangle} = \mathbb{Q}(\sqrt{3})$

$E_{\langle\alpha,\beta\rangle} = \mathbb{Q}$ $E_{\langle I\rangle} = \mathbb{Q}(\sqrt{3},\sqrt{5})$

$E_{\langle\alpha\beta\rangle} = \mathbb{Q}(\sqrt{15})$

Thus Figure 32.4

Make page 551 handout!

Example 2. $E = \mathbb{Q}(\omega = e^{2\pi i/3} = -\frac{1}{2}+\frac{\sqrt{3}}{2}, \sqrt[3]{2})$ as in text. /cont

* State the fundamental Theorem of Galois Theory as on April 11

Proposition Let E ~~be~~ the splitting field of $3x^5 - 15x + 5$ over $\mathbb{Q}$. Then

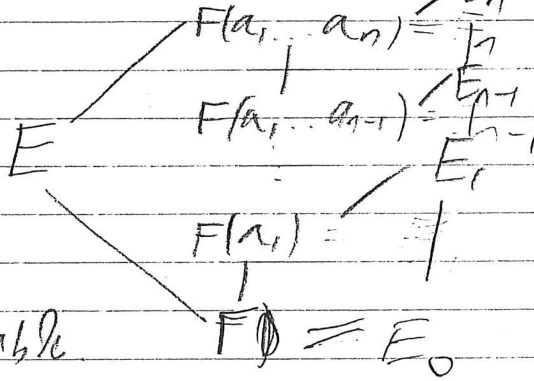
$$Gal(E/\mathbb{Q}) = S_5$$

Done to here Proof - - -

Def f solvable by radicals: f splits in some extension $F(a_1 \ldots a_n)$ s.t. $\exists k_1 \ldots k_n$ ~~st~~ with $a_1^{k_1} \in F$ and $a_i^{k_i} \in F(a_1 \ldots a_{i-1})$ $2 \le i \le n$.

Thm
~~Claim~~ The Galois group of each step, $Gal(E_k/E_{k-1})$ is ~~abelian~~ solvable.

$\Rightarrow Gal(E/F)$ is solvable



Pf of thm need to show that if E/F is the splitting extention of $x^n - b$ ~~$\in \mathbb{Q}$~~, the $Gal(E/F)$ is solvable.

Case I $b = 1$

Case II b general, $w_n \in F$

Case III general case.

If time - The Primitive Element Theorem

* HW9 on web by noon tomorrow.

* Some discussion of HW7 JFF.

Math 401 Pols, Eqns, Fields, March 28 2007, Week 11

Galois Theory

Fields $\xleftrightarrow{\text{Fund Thm}}$ Groups

extensions ~~Not~~ describable by roots $\longleftrightarrow$ solvable groups

splitting field of $3x^5-15x+5$ $\longleftrightarrow$ The 120-element group S_5 <u>not</u> <u>solvable</u>

Goal: Some polynomials cannot be "solved" using $+ - \times \div \sqrt{\ }$

To do list. 0. Quick reminders about group

1. Precise statement of the Fundamental Thm
2. Examples
3. On Solvable groups:
   1. definition
   2. basic properties
   3. S_5 isn't solvable

   } just group theory.
4. * "Extensions by radicals" → solvable groups.
5. \# splitting field of $3x^5-15x+5$ and
6. proof of the fundamental theorem,

<u>Reminders</u> G: (assoiciative, identity, inverses) $H<G$ G/H $|G/H|=[G:H]=|G|/|H|$

$N \triangleleft G \Rightarrow G/N$ is a group. Iso2Thm $HN/N \cong H/_{H\cap N}$ Iso3Thm $N \triangleleft G \Rightarrow$ $G/N/_{H/N} \cong$

<u>Def</u> $E/F \Rightarrow Gal(E/F)$ <u>Claim</u> It is a group.

<u>Def</u> $H < Gal(E/F)$ $E_H =$ The fixed set of H <u>Claim</u> it is a field.

<u>The fundamental theorem of Galois theory</u> If $\operatorname{char} F = 0$ and E is a splitting field over F then there are a bijections

$$\begin{tikzcd} \overset{\text{Int}}{\{K : E/K/F\}} \arrow[r, bend left, "K \mapsto Gal(E/K)"] & \text{Subgroups of } Gal(E/F) \arrow[l, bend left, "E_H \leftarrow H"] \end{tikzcd}$$

<u>furthermore</u> 0. Inverses of each other.

1. inclusion reversing: $H_1 < H_2 \Rightarrow E_{H_1} \supset E_{H_2}$
   $K_1 \subset K_2 \Rightarrow Gal(E/K_1) > Gal(E/$

2. degree index respecting: $[E:K] = |Gal(E/K)|$; $[K:F] = [Gal(E/F) : Gal\ E/K$

3. splitting ⇒ normal: If K in $E/K/F$ is a splitting field, the $Gal(E/K)$ is normal in $Gal(E/F)$ and

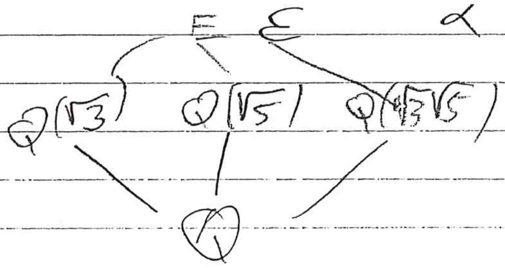
$$Gal(K/F) \cong Gal(E/F)/Gal(E/K)$$

.../over.

Example 1 $E = \mathbb{Q}(\sqrt{3}, \sqrt{5}) / \mathbb{Q} = F$

1. Every aut. ϕ is det by $\phi(\sqrt{3})$ & $\phi(\sqrt{5})$

$$\begin{array}{ccccc} \sqrt{3} \to & \sqrt{3} & -\sqrt{3} & \sqrt{3} & -\sqrt{3} \\ \sqrt{5} \to & \sqrt{5} & \sqrt{5} & -\sqrt{5} & -\sqrt{5} \\ & \varepsilon & \alpha & \beta & \alpha\beta \end{array}$$



Example 2 $E = \mathbb{Q}(w, \sqrt[3]{2})$ where $w = e^{\frac{2\pi i}{3}} = \sqrt[3]{1}$, the other

$= \cos 120° + i \sin 120°$

$= \frac{1}{2} + i\frac{\sqrt{3}}{2}$

$\alpha: \begin{array}{l} w \mapsto w^2 \\ \sqrt[3]{2} \mapsto \sqrt[3]{2} \end{array}$ $\beta: \begin{array}{l} w \mapsto w \\ \sqrt[3]{2} \mapsto w\sqrt[3]{2} \end{array}$

continue as in book

Def A solvable group

Thm If G is solvable and $N \lhd G$, then G/N is solvable.

Thm If N & G/N are solvable, then so is G.

Thm S_n is not solvable for $n \geq 5$

Thm

HW7 on web by noon tommorow.

Math 401 Pols, Eqns, Fields, March 14 2007, Week 9

Today's agenda:
1. Review of last time
2. A word about derivatives
3. Algebraic numbers: "Alg. numbers form an alg. closed field" (rough statement)

Last week's key theme: "Extending a field".

(Passive) 1. E/F, $a_1 \ldots a_n \in E$, $F(a_1, a_2 \ldots) = F(a_2, a_1)(a_3, \ldots)$

Existence & uniqueness are obvious.

Active/creative

2. Adding a single root: p irreducible in $F[x]$

Existence $F[x]/\langle p \rangle$

Uniqueness Always $F[x]/\langle p \rangle \cong F(a)$ if $p(a)=0$

strong uniqueness: later $\phi : F \to F'$ iso, $\phi(p) = p'$ $p(a)=0$, $p(a')=0$ $\Rightarrow F'(a') \cong F(a)$

3. Splitting a polynomial $f \in F[x]$

$f = a(x-a_1) \ldots (x-a_n)$ in E & E minimal w/ this property

$E = F(a_1, \ldots a_n)$

Existence: In steps

Strong uniqueness: In steps

Uniqueness: Follows.

Theorem $f \in F[x]$ has a multiple zero in some extension iff f & f' have a common factor of $\deg > 0$, iff f & f' have a common root in some extension.

Def f'

Lemma 1. $(cf)' = cf'$

2. $(f+g)' = f' + g'$

3 Leibnitz law

4 $c' = 0$

5. $f' = 0 \Rightarrow f = c$ if char $F = 0$!

Pf of thm $\Leftarrow$: common factor $\Rightarrow$ common root $\Rightarrow$ multiple root. $f = (x-a)g \Rightarrow g(a) = 0 \Rightarrow g = (x-a)h$.

$\Rightarrow$ multiple root $\Rightarrow$ common root $\Rightarrow$ common factor in $E[x]$

Lemma If f, g are rel. prime in $F[x]$, they ~~are~~ remain relatively prime in $E[x]$ for any extension E/F.

Corollary: In char 0, ~~[illegible]~~ irreducible polynomials have no multiple roots. ..[illegible]

Goal: Given E/F, say that $a \in E$ is "algebraic" over F if it is the root of a polynomial in F.

Theorem 1. If a, b are algebraic, then so are
$a+b$, $a-b$, ab, a/b,
(and so the algebraic numbers form a field)

2. If $a_0 \ldots a_n$ are algebraic over F &
b is a root of $f(x) = \sum a_i x^i$, then
b is algebraic

(So, - - -)

Def E/F algebraic.

Definition E/F finite, $[E:F]$ | E/F algebraic

Theorem E/F finite $\Rightarrow$ E/F algebraic.

Theorem a is algebraic over F $\Leftrightarrow$ $F(a)/F$ is finite.

Theorem If $K/E/F$, then $[K:F] = [K:E][E:F]$

Proof of goal

Discussion of the just for fun exercise.

$$\sqrt[3]{7} - \sqrt[4]{\sqrt{5} / \sqrt[3]{\sqrt{2} + \sqrt[5]{7}}}$$

HW7 on webb [illegible] by noon tomorrow

Math 401 Pols Eqns Fields, March 12 2008, week 9.

Key from last $F(a)=F[x]/<p>$

Agenda 1. Alg. numbers.
2. Geometric constructions.
3. Odds & ends from chap 21

Goal 1 a. If a & b alre alg., so are $a+b$, $a-b$, ab, a/b.
b. a_i alg, b solves $\sum a_i x^i = 0$ $\rightarrow$ b is alg.

Def Alg. element.

Def $[E:F]$ the dimension of E as a v.s. over F.
say that E/F is finite if $[E:F]$ is finite.

① Thm a is alg over F $\Leftrightarrow$ $F(a)/F$ is finite.

Thm If $E/E/F$ then $[K:F]=[K:E][E:F]$.

Proof of goal 1

Example
$\{\mathbb{Q}[\sqrt[4]{3}, \sqrt[3]{4}\} : \mathbb{Q}] = 12$.

Geometric constructions using a ruler & a compass. (Given a unit interval).

Odds & ends as on March 21, 2007

cannot costruct $\cos 20°$.

TT Apps Deadline today?
HW8 on web by noon
tomorrow

Math 401 Pols, Eqns, Fields, March 21 2007, Week 10

Today's agenda
1. Finish "alg numbers are alg closed"
2. Geometric constructions
3. Odd and ends from chapter 21
4. Start of Galois theory?

Last week: Def Given E/F, $a \in E$ is "algebraic" if a is a root of $f \in F[x]$.

Goal: 1. If $a, b \in E$ are alg., so are $a+b, a-b, ab, a/b$ (alg numbers form a field)

2. a_i alg, b solve $\sum a_i x^i = 0 \Rightarrow$ b is alg. ("alg nums are alg closed")

$[E:F]$:= dim of E as a v.s over F.

Thm a alg over F $\Leftrightarrow$ $F(a)/F$ is finite.

Thm If K/E/F, then $[K:F] = [K:E][E:F]$

Proof of goal.

Geom const. using a ruler & a compass.

Given a unit interval, cannot construct $\sqrt[3]{2}$

given an angle α, cannot construct $\alpha/3$

π is transcendental (not algebraic) $\Rightarrow$ cannot "square a circle".

---

odds and ends: E/F, $a \in E$ F(a) is a "simple" extension.

Thm a alg $\Rightarrow$ $F(a) \cong F[x]/\langle p \rangle$
where p is the (unique) minimal polynomial of a

a trans $\Rightarrow$ $F(a) \cong F(x) = \left\{ \frac{f(x)}{g(x)} \right\}$ = The ~~quotient~~ field of quotients field of $F[x]$.

Def The minimal poly

Claim It exists and is unique

Claim If p is minpoly(a), and $f(a)=0$, then $p | f$.

Example $[\mathbb{Q}(\sqrt[3]{2}, \sqrt{3}) : \mathbb{Q}] = 12$

Example Find the min poly of $\sqrt{3}+\sqrt{5}$ over $\mathbb{Q}$.

/over

Thm If char $F=0$ & $\alpha,\beta \in E/F$, Then $\exists \gamma \in E$ st. $F(\alpha,\beta)=F(\gamma)$. γ is "primitive element of E/F"

(so by induction, finite dg extensions are simple)

---

start of Galois:

$\text{Gal}(E/F)$ is a group

$H<G \Rightarrow E_H$ is $E/E_H/F$

$\shortparallel$

"The fixed field of H".

Math 401 Pols Eqns Fields, March 5 2008, week 8

* Discussion of TT   * HW6 on web by noon tomorrow.

* Some philosophy:

Def A field extension E/F * A symmetry of E/F "an automorphism of E/F"

Will talk about two ways of extending fields:

later → 1. By adding $\sqrt[2]{a}$, $\sqrt[3]{a}$, … etc.   less symmetric

today → 2. By throwing in all roots of a given poly.   highly symmetric

"splitting fields exists and are roughly unique".

Thm (Kronecker) Let $f \in F[x]$ be non-constant. Then $\exists\ E/F$ in which f has a root

Examples $x^2+1/\mathbb{R}$, $x^5+2x^2+2x+2 = (x^2+1)(x^3+2x+2)/\mathbb{Z}/3$.

Def $f \in F[x]$ splits in E/F; A splitting field E of f over F

Example x^2-2 over $\mathbb{Q}$ & over $\mathbb{R}$.

Def E/F, $a_1 \ldots a_n \in E$, $F(a_1 \ldots a_n)$

= "the smallest subfield of E that contains F & $\{a_i\}$" = "The intersection" =

= "Everything in E that can be reached from $F \cup \{a_1 \ldots a_n\}$ by alg operations"

claim If $f \in F[x]$ splits in E, and its roots in E are $a_1 \ldots a_n$, then $F(a_1 \ldots a_n)$ is a splitting field of f over F.

Examples $x^4-x^2-2 = (x^2-2)(x^2+1)$ over $\mathbb{Q}$

x^2+x+2 over $\mathbb{Z}/3$.

08-401 week 8, continued.

Thm Let $f \in F[x]$. A splitting field for f over F exists.

Thm Let $p \in F[x]$ be irred. if $a \in E/F$ & $p(a)=0$

then $F(a) \cong F[x]/\langle p \rangle$ via $\phi : F[x]/\langle p \rangle \to F(a)$
s.t. $\phi([x]) = a$. Furthermore, if $\deg p = n$,

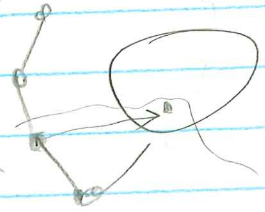
then every element of $F(a)$ can be written as a combination

$$c_0 + c_1 a + c_2 a^2 + \dots + c_{n-1} a^{n-1}$$

with all $c_i \in F$, in a unique way.

Corollary If $p \in F[x]$ is irred, $a \in E/F$ and $b \in E'/F$ are roots of p, then $F(a) \cong F(b)$.

Thm Any two splitting fields of $f \in F[x]$ over F are isomorphic



Generalization If $\phi : F \to F'$ is an iso of fields, E a splitting field of $f \in F[x]$ over F, E' a splitting field of $f' := \phi(f) \in F'[x]$ over F', then $\exists$ iso $\psi : E \to E'$ s.t. $\psi|_F = \phi$.

Lemma $p \in F[x]$ irred, $\phi : F \to F'$ iso, $p' = \phi(p) \in F'[x]$, a ~~ab~~ root of f is some ext of F, a' a root of f' in some ext. of F', then $\exists \psi : F(a) \to F'(a')$ iso, s.t. $\psi|_F = \phi$.

Template:08-401/Results of the Term Test

From Drorbn

The Results

[print]

A total of 27 students took the exam; before appeals the average grade is 64.66 and the standard deviation is 18.79.

The full list of grades is: (31 43 43 46 47 49 50 52 53 54 55 60 60 61 61 62 65 73 75 75 76 82 83 92 99 99 100).

The results are quite similar to what I expected them to be, perhaps a bit on the low side.

Individual grades are on CCNet (http://ccnet.utoronto.ca/20081/mat401h1s/) .

How should you read your grade?

- If you got 100 you should pat yourself on your shoulder and feel good.
- If you got something like 95, you're doing great. You made a few relatively minor mistakes; find out what they are and try to avoid them next time.
- If you got something like 75 you're doing fine but you did miss something significant, probably more than just a minor thing. Figure out what it was and make a plan to fix the problem for next time.
- If you got something like 55 you should be concerned. You are still in position to improve greatly and get an excellent grade at the end, but what you missed is quite significant and you are at the risk of finding yourself far behind. You must analyze what happened - perhaps it was a minor mishap, but more likely you misunderstood something major or something major is missing in your background. Find out what it is and try to come up with a realistic strategy to overcome the difficulty!
- If you got something like 35, most likely you are not gaining much from this class and you should consider dropping it, unless you are convinced that you fully understand the cause of your difficulty (you were very sick, you really couldn't study at all for the two weeks before the exam because of some unusual circumstances, something like that) and you feel confident you have a fix for next time. The deadline for dropping a class this semester is soon: Sunday March 9.

Appeals.

Remember! Grading is a difficult process and mistakes **always** happen - solutions get misread, parts are forgotten, grades are not added up correctly. You **must** read your exam and make sure that you understand how it was graded. If you disagree with anything, don't hesitate to complain! Dror graded everything, so appeals should go directly to him.

The deadline to start the appeal process is Wednesday March 19 at class time.

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- This page was last modified 14:08, 2 March 2008.

HW 6 on Web by noon tomorrow.

07-401/Class Notes for March 7

From Drorbn

07-401/Navigation Panel [Hide]

#	Week of...	Links
1	Jan 10	About, Notes, HW1
2	Jan 17	HW2, Notes
3	Jan 24	HW3, Photo, Notes
4	Jan 31	HW4, Notes
5	Feb 7	HW5, Notes
6	Feb 14	On TT, Notes
R	Feb 21	Reading week
7	Feb 28	Term Test
8	Mar 7	HW6, **Notes**
9	Mar 14	HW7
10	Mar 21	HW8
11	Mar 28	HW9
12	Apr 4	HW10
13	Apr 11	
S	Apr 16-20	Study Period
F	Apr 24	Final

Add your name / see who's in!

Register of Good Deeds

Contents

Class Plan

Some discussion of the term test and HW6.

Some discussion of our general plan.

Extension Fields

Definition. An extension field E of F.

Theorem. For every non-constant polynomial f in $F[x]$ there is an extension E of F in which f has a zero.

Example x^2+1 over $\mathbb{R}$.

Example $x^5+2x^2+2x+2=(x^2+1)(x^3+2x+2)$ over $\mathbb{Z}/3$.

Definition. $F(a_1,\dots,a_n)$.

Theorem. If a is a root of an irreducible polynomial $p\in F[x]$, within some extension field E of F, then $F(a)\cong F[x]/\langle p\rangle$, and $\{1,a,a^2,\dots,a^{n-1}\}$ (here $n=\deg p$) is a basis for $F(a)$ over F.

Corollary. In this case, $F(a)$ depends only on p.

Splitting Fields

Definition. $f\in F[x]$ splits in E/F, a splitting field for f over F.

Theorem. A splitting field always exists.

Example. $x^4-x^2-2=(x^2-2)(x^2+1)$ over $\mathbb{Q}$. [illegible]

Example. Factor $x^2+x+2 \in \mathbb{Z}_3[x]$ within its splitting field $\mathbb{Z}_3[x]/\langle x^2+x+2\rangle$. $(x-\beta)(x+\beta+1)$

Theorem. Any two splitting fields for $f \in F[x]$ over F are isomorphic.

should be stated more specifically.

Lemma 1. If $p \in F[x]$ irreducible over F, $\phi : F \to F'$ an isomorphism, a a root of p (in some E/F), a' a root of $\phi(p)$ in some E'/F', then $F(a) \cong F'(a')$.

Lemma 2. Isomorphisms can be extended to splitting fields.

Lemma 2 strengthen

$$\begin{tikzcd} E \arrow[r, "\Phi"] \arrow[d, dash] & E' \arrow[d, dash] \\ F \arrow[r, "\phi"] & F' \end{tikzcd}$$

$f \in F[x] \to \phi(f) \in F'[x]$

Pf Induct on deg f; deg $f=1$, [illegible]

$$\begin{tikzcd} E \arrow[r, "\Phi"] \arrow[d, dash] & E' \arrow[d, dash] \\ F(a) \arrow[r, "\bar{\Phi}"] \arrow[d, dash] & F(a') \arrow[d, dash] \\ F \arrow[r, "\phi"] & F' \end{tikzcd}$$

$f \in F[x]$ $\phi(f) \in F'[x]$

choose a root of f a' root of $\phi(f)$

Zeros of Irreducible Polynomials

Definition. The derivative of a polynomial.

Claim. The derivative operation is linear and satisfies Leibnitz's law.

Theorem. $f \in F[x]$ has a multiple zero in some extension field of F iff f and f' have a common factor of positive degree. Pf $\Leftarrow$ $f=(x-\alpha)g$

Lemma. The property of "being relatively prime" is preserved under extensions.

Theorem. Let $f \in F[x]$ be irreducible. If $\mathrm{char} F = 0$, then f has no multiple zeros in any extension of F. If $\mathrm{char} F = p > 0$, then f has multiple zeros (in some extension) iff it is of the form $g(x^p)$ for some $g \in F[x]$.

Definition. A perfect field. (char 0 or $F^p = F$.)

Theorem. A finite field is perfect.

Theorem. An irreducible polynomial over a perfect field has no multiple zeros (in any extension).

Theorem. Let $f \in F[x]$ be irreducible and let E be the splitting field of f over F. Then in E all zeros of f have the same multiplicity.

Corollary. f as above must have the form $a(x-a_1)^n \cdots (x-a_k)^n$ for some $a \in F$ and $a_1, \ldots, a_k \in E$.

Example. $x^2 - t \in \mathbb{Z}_2(t)[x]$ is irreducible and has a single zero of multiplicity 2 within its splitting field over $\mathbb{Z}_2(t)[x]$.

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- This page was last modified 17:41, 7 March 2007.

08-401/On the Term Test

From Drorbn

Our Term Test will take place on February 27 at 6:20PM at Galbraith (GB) 304 on 35 St. George Street, across from the Bahen Centre for Information Technology. It will be two hours long.

08-401/Navigation Panel [Hide]

#	Week of...	Links
1	Jan 9	About, Notes, HW1
2	Jan 16	HW2, Notes
3	Jan 23	HW3, Photo, Notes
4	Jan 30	HW4, Notes
5	Feb 6	HW5, Notes
6	Feb 13	**On TT**, Notes
R	Feb 20	Reading week
7	Feb 27	Term Test
8	Mar 5	HW6, Notes
9	Mar 12	HW7, Notes
10	Mar 19	HW8, Notes
11	Mar 26	HW9, Notes
12	Apr 2	HW10, Notes
13	Apr 9	Notes
S	Apr 14-18	Study Period
F		Final

Add your name / see who's in!

Register of Good Deeds

The material is **everything** covered in class until and including the class of February 13, 2007. Everything in the test be taken from our text book, and there will be two types of questions (or maybe sometimes the two types will be mixed within a single question):

- You may be asked to prove a theorem proven in class. The reason we prove theorems in class is that these proofs are **important**. Therefore I expect you to know them.
- You may be asked to solve exercises from the relevant chapters of the book, or minor variations thereof. These may be questions that were assigned as homework, but also, these may be questions that were not assigned before.

Office Hours. Dror will hold extended office hours on the week of the Term Test, on Monday from 2PM to 4PM, on Tuesday at the usual time (12:30-1:30) and on Wednesday from 1:30PM to 3PM, all at or near Bahen 6178. Yichao Zhang will hold his usual office hours, on Tuesday 1:00-3:00 at the Math Aid Centre, Sidney Smith 1071.

Preparing for the Test. Read, reread and rereread everything and solve lots of exercises from the book.

My (Dror's) system when I was an undergrad was to prepare a 2-3 page 50-100 item list of points covered in class. I'd only summarize each point with one sentence, without giving any details and without trying to be precise, much like the list below that I prepared for the class of February 6. I would then go over my list again and again and again, crossing out every item for which I was sure I could complete all the details and supply all the proofs. I would only stop when there was nothing left to cross out.

Good Luck!

Summary of the class of February 7:

- A long division computation.
- $f(a)$ is the remainder of the division of $f(x)$ by $x-a$.
- a is a zero of $f(x)$ iff $x-a$ is a factor of $f(x)$.
- The multiplicity of a zero.
- A polynomial of degree n has at most n roots, counting multiplicities.
- The roots of x^2+3x+2 in $\mathbb{Z}/6$.
- The roots of x^n-1.
- **Definition.** A Principle Ideal Domain (PID).
- $F[x]$ is a PID and a criterion for $I=\langle g(x)\rangle$.
- **Example.** the complex numbers and $\mathbb{R}[x]/\langle x^2+1\rangle$.
- **Definition.** Units in a ring.
- **Definition.** Reducible and irreducible polynomials.
- Reducibility in degrees 2 and 3.
- Primitive polynomials.
- The product of primitive polynomials is primitive.
- The content of a polynomial.
- The content of a product is the product of the contents.
- If $f\in\mathbb{Z}[x]$ is reducible in $\mathbb{Q}[x]$, it is reducible already in $\mathbb{Z}[x]$.

Retrieved from "http://katlas.math.toronto.edu/drorbn/index.php?title=08-401/On_the_Term_Test"

- This page was last modified 18:14, 13 February 2008.

No credit for "trying".

Solve 5 of the following 6 problems. Each of the problems is worth 20 points. You have two hours. **Neatness counts! Language counts!**

Problem 1. Consider the ring $\{0, 2, 4, 6, 8\}$ under addition and multiplication modulo 10. Does it have a unity?

Tip. This, of course, is not just a yes/no question. You are expected to fully justify your answer, whatever it is.

unity is 6

Problem 2.

5 1. Define "an integral domain".
   Commutative, unity, no zero divisors.

5 2. Define "a field".
   also, each non-zero has inverse

10 3. Prove: A finite integral domain is a field. -4 not clear where domain-ness was used.

Tip. As always in math exams, when proving a theorem you may freely assume anything that preceded it but you may not assume anything that followed it.

Field ⟺ Field.

Problem 3. Let R be a commutative ring with unity and let A be an ideal in R. Prove that R/A is a field if and only if A is maximal.

Tip. Don't forget! There are two directions to prove here!

Problem 4. Find all ring homomorphisms from $\mathbb{Z}/6$ to $\mathbb{Z}/10$.

5 1→0 or 15 1→5

Tip. Here, of course, you have to explain both why the homomorphisms you found really are homomorphisms and why there are no more.

-8 did not check well-definedness

Problem 5.

14 1. Let F be a field, let f be a polynomial in $F[x]$, and let a and b be two different elements of F. Prove that the remainder for the division of f by $(x - a)(x - b)$ is

$$\frac{f(b) - f(a)}{b - a}(x - a) + f(a).$$

6 2. Compute the remainder for the division exercise $\frac{x^{2008}}{x^2-1}$, done in $\mathbb{Q}[x]$.

Problem 6. Prove that the polynomial $f = 3x^5 + 15x^4 - 20x^3 + 10x + 20$ is irreducible over $\mathbb{Q}$. If you are using any major theorem, you need to quote it in full, but you don't need to prove it.

Good Luck!

Do not turn this page until instructed.

Math 401 Polynomial Equations and Fields

Term Test

University of Toronto, February 27, 2008

Solve 5 of the 6 problems on the other side of this page.
Each of the problems is worth 20 points.
You have two hours to write this test.

Notes.

- No outside material other than stationary and a basic calculator is allowed.
- The final exam date was posted by the faculty — it will take place on the *evening* of Tuesday April 28 between 7PM and 10PM, at BN2S (Large Gymnasium, South End, Benson Building, 320 Huron Street (south of Harbord Street), Second Floor).
- **Neatness counts! Language counts!** The *ideal* written solution to a problem looks like a proof from the textbook; neat and clean and made of complete and grammatical sentences. Definitely phrases like "there exists" or "for every" cannot be skipped. Lectures are mostly made of spoken words, and so the blackboard part of proofs given during lectures often omits or shortens key phrases. The ideal written solution to a problem does not do that.
- **New!** You will get 20% of the credit for any problem for which you will write explicitly "I don't know how to solve this problem" (whole problems only!).

Good Luck!

Math 401 Pols, Eqns, Fields, Feb 14 2007, Week 6

* APUS
* TT (at GB120?)
* Last class.

Today: 1. why care about irreducible poly?
  * related to Fields.
  * relatives of primes (fun game).
2. criteria for irreducibility

Theorem Let $p \in F[x]$. The $\langle p(x)\rangle$ is maximal iff p is irreducibl

Pf p maximal $\Rightarrow$ If reducible, contradiction

~~p irreducible $\Rightarrow$ $\langle p\rangle \subset I \subset F[x]$, $F[x]$ a pid $\Rightarrow$ $I = \langle g(x)\rangle$~~
~~$\Rightarrow p = g \cdot h$ for some h~~

if p is irred,

Cor $F[x]/\langle p(x)\rangle$ is a Field.

Cor $p, a, b \in F[x]$, p irred, $p \mid a \cdot b \Rightarrow p \mid a$ or $p \mid b$.

Examples 1. $\mathbb{Z}_2[x]/\langle x^3+x+1\rangle$ is a field w/ 8 elements.

2. $\mathbb{Z}_3[x]/\langle x^2+1\rangle$ is a field w/ 9 elements

Theorem Let $f \in \mathbb{Z}[x]$ be non-zero & non-unit. then

$$f = \underbrace{b_1 \cdots b_s}_{\substack{\text{irred of deg } 0 \\ \text{(i.e., primes)}}} \cdot \underbrace{p_1 \cdots p_m}_{\text{irred of deg} \geq 1}$$

& this is unique up to a permutation & up to signs.

Example The sicherman dice 122334 & 134568 and their uniqueness

Pf an irred factor of $(x^6+x^5+x^4+x^3+x^2+x)^2 = x^2(x+1)^2(x^2+x+1)^2(x^2-x+1)^2$

is of the form $x^q(x+1)^r(x^2+x+1)^t(x^2-x+1)^u$

where $0 \leq q, r, t, u \leq 2$.

$P = x^{a_1} + x^{a_2} + \cdots + x^{a_6}$

1. Evaluate $p(1)$ 2. evaluat $p(0)$.

…/cont.

$(x^2+x+1)^{2} = x^{1/4}$

$x^{1/4}$

401 Week 6 Page 2.

Theorem $\bar{f}$ red mod p of f, $\deg \bar{f} = \deg f$; if $\bar{f}$ is irred then f is irred.

Example $21x^3 - 3x^2 + 2x + 9$

1. irred over $\mathbb{Z}/2$
2. irred over $\mathbb{Z}/3$ but can't use thm.

Example $21x^3 - 3x^2 + 2x + 8$

1. red over $\mathbb{Z}/2$ 2. irred over $\mathbb{Z}/5$.

Eisenstein's Criterion: $a_n x^n + \ldots + a_0$

$p \nmid a_n, p \| a_{n-1} \ldots p \mid a_n, p^2 \nmid a_0$

$\Rightarrow$ irred over $\mathbb{Z}/\mathbb{Q}$

Cor $\dfrac{x^p - 1}{x - 1} = x^{p-1} \quad + 1$ is irred.

Today's agenda:
Polys & roots!
~~R~~educible polys:
$f = g \cdot h$
irreducible polys
– the rest.
(math is a dark art)

Math 401 Pols eqn's Fields, Feb 6 2008, week 5.

Thm If F is a field and $f, g \in F[x]$, $g \neq 0 \Rightarrow$

$$\exists ! \; q, r \in F[x] \text{ s.t. } f = gq + r \;\&\; \deg r < \deg g$$

q: "the quotient for f/g"
r: "the remainder for f/g".

Example $f = x^5 - 2$ $g = x^2 - x - 1$

$$x^3 + x^2 + 2x + 3$$
$$x^5 + 0x^4 + 0x^3 + 0x^2 + 0x - 2 \,\rceil\, x^2 - x - 1$$
$$x^5 - x^4 - x^3$$
$$x^4 + x^3$$
$$x^4 - x^3 - x^2$$
$$2x^3 + x^2$$
$$2x^3 - 2x^2 - 2x$$
$$0 \quad 3x^2 - 2x - 2$$
$$3x^2 - 3x - 3$$
$$5x + 5$$

Example

$$\begin{array}{l} \;\;275 \\ \overline{3579} \,|\, 13 \\ \underline{26} \\ \;97 \\ \;\underline{91} \\ \;\;69 \\ \;\;\underline{65} \\ \;\;\;4 \end{array}$$

$q = 275$
$r = 4$

$q = x^3 + x^2 + 2x + 3$
$r = 5x + 5$.

Cor $f \in F[x] \Rightarrow f(a)$ is the remainder for $f(x)/x-a$.

Cor $f \in F[x]$ $f(a) = 0 \iff (x-a) \mid f$

Def the mult. of a root.

Cor in $F[x]$ a poly of deg n has at most n zeros, counting with multiplicity.

1. **Example** $x^3 + x^2 - x - 1$
$= (x+1)^2(x-1)$

2. **Example** $x^2 + 3x + 2$ has 4 zeros in $\mathbb{Z}/6$.

3. **Example** $x^2 - 1$ in $\mathbb{C}$.

Def PID

Thm $F[x]$ is a PID; if A is an ideal, it is generated by any non-zero poly ~~idk~~ of min deg in it.

Example $f \mapsto f(i)$ in $\mathbb{R}[x]$ proves $\mathbb{R}[x]/\langle x^2+1 \rangle \cong \mathbb{C}$.

..... /cont.

not zero not unit. **Def** Unit in R

Cont: **Example:** $\mathbb{Z}[X]$, $F[X]$

Def f is irred in $D[X]$ if $f = gh \Rightarrow$ g or h is a unit. otherwise f is reducible.

Example $2x^2+4$ is irred over $\mathbb{Q}$
red over $\mathbb{Z}$.
irred over $\mathbb{R}$
red over $\mathbb{C}$.

x^2-2 irred over $\mathbb{Q}$
red over $\mathbb{C}$.

x^2+1 irred over $\mathbb{Z}/3$
red over $\mathbb{Z}/5$.

Thm $f \in F[X]$, $\deg f = 2,3 \Rightarrow f$ is irred iff it has a root.

Thm ($f \in \mathbb{Z}[x]$ if it is reducible over $\mathbb{Q}[X]$, it is also reducible over $\mathbb{Z}[X]$. (hence the contra-positive).

Example $6x^2+x-2 = (3x-3/2)(2x+4/3)$

Def primitive poly $\in \mathbb{Z}[X]$

Prop ~~Thm~~ Prod of primitives is primitive.

Prop ~~Thm~~/**Def** The content $c(f)$ of a poly $\in \mathbb{Q}[X]$ is well defined.

Prop ~~Thm~~ $c(fg) = c(f)c(g)$. **Pf** of Thm

07-401 Lect 5 1-6.

MAT401H1 LEC5 7/2/06.

① – COROLLARY: (Remainder theorem) If F is a field and $f(x) \in F[x]$. Then $f(a)$ is the remainder in the division of $f(x)$ by $x-a$.

PROOF

$f(x) = g(x)q(x) + r(x)$. $\deg(r) < \deg(g)$.

Let $g(x) = x-a$.

$f(x) = (x-a)q(x) + r(x)$.

Since degree of $x-a$ is 1, $\deg(g) > \deg(r)$ so $\deg(r) = 0$.

$r(x) = b$ a constant.

Let $x = a$

$f(a) = (a-a)q(a) + r(a)$

$f(a) = r(a)$

② – COROLLARY: If F is a field and $f(x) \in F[x]$, then a is a zero of $f(x)$ iff $x-a$ is a factor of $f(x)$.

③ – COROLLARY: Polynomials over a field of degree n have at most n zeros.

$f(x) = 0$ has at most n zeros counting multiplicity. $f(x) \in F[x]$ F is a field.

PROOF

Use induction on degree of $f(x)$.

If $n=1$, $f(x) = ax+b$ $a \neq 0$ $\Rightarrow x = -\frac{b}{a}$.

If for any polynomial $g(x)$ with $\deg(g) = k$, $g(x)$ has at most k zeros.

For $n = k+1$, $f(x)$ with $\deg f = k+1$

① $f(x)$ has no zero TRUE

② $f(x)$ has some zeros.

Choose any zero a of $f(x)$

$f(x) = (x-a)h(x)$ for some $h(x) \in F[x]$.

$\Rightarrow \deg h = k$.

By corrollary ②, we know $h(x)$ has at most k zeros.

$\Rightarrow f(x)$ has at most $k+1$ zeros.

* EXAMPLE

$f(x) = x^2 + 3x + 2 \in \mathbb{Z}_6[x]$
has 4 zero.

* EXAMPLE

The complex zeros of $x^n - 1 \in \mathbb{C}[x]$. Find all zeros of $x^n - 1$ over $\mathbb{C}$.

De Moivre's theorem: $(\cos\theta + i\sin\theta)^k = \cos k\theta + i\sin k\theta \quad k \in \mathbb{Z}$.

$x_1 = \cos(\frac{2\pi}{n}) + i\sin(\frac{2\pi}{n})$ is a zero of $x^n - 1$

$x_1^n = (\cos\frac{2\pi}{n} + i\sin\frac{2\pi}{n})^n = \cos n \times \frac{2\pi}{n} + i\sin n\times\frac{2\pi}{n} = 1$

$\Rightarrow x_1^n - 1 = 0$.

$x_k = x_1^k = \cos \frac{k2\pi}{n} + i\sin\frac{k2\pi}{n} \quad k = 0, 1, 2, \ldots, n-1$.

If $i \neq j$, $x_i \neq x_j$

$\Rightarrow x^n - 1 = \prod_{k=0}^{n-1} (x - x_k)$

- DEFINITION: Principal Ideal Domain (PID) is an integral domain R in which every ideal has the form $\langle a \rangle = \{ra \mid r \in R\}$ for some $a \in R$.

- $\mathbb{Z}$ is a PID.
  $\mathbb{Q}$ is a PID.

- THEOREM: If F is a field, $F[x]$ is a PID
  PROOF

Assume that I is an ideal of $F[x]$.

① $I = \{0\} = \langle 0 \rangle$.

② $I \neq \{0\}$, assume that $g(x) \in I$ st $\deg(g(x))$ is the smallest.

CLAIM: $I = \langle g(x) \rangle$

$\forall f(x) \in I$

$f(x) = g(x) \cdot q(x) + r(x)$ for some $q, r \in F[x]$ $\deg r(x) < \deg g(x)$.

$g(x) \in I \Rightarrow \underset{f(x)}{g(x)} \cdot \underset{f(x)}{q(x)} \in I. \Big\} \underbrace{f(x) - g(x)q(x)}_{r(x)} \in I$.

$\Rightarrow r(x) = 0$

$f(x) = g(x) \cdot q(x)$.

$\left.\begin{array}{l} \forall f(x) \in \langle g(x) \rangle \subseteq I \\ I \subseteq \langle g(x) \end{array}\right\} I = \langle g \rangle$.

③ $I \neq 0$, $g(x) \neq 0 \in I$ and $\deg(g(x)) = 0 \Rightarrow g(x) = a \neq 0$.

$\Rightarrow I = F[x] = \langle 1 \rangle$.

* EXAMPLE

$\phi : \mathbb{R}[X] \to \mathbb{C}$ where $f(x) \xrightarrow{\phi} f(i)$ $f(x) \in \mathbb{R}[x]$, $f(i) \in \mathbb{C}$.

$\ker\phi = \{ g(x) \in \mathbb{R}[x] \mid \phi(g(x)) = g(i) = 0 \}$

$x^2+1 \in \ker\phi$. since $i^2+1=0$.

$\ker\phi = \langle x^2+1 \rangle$.

know that $R/\ker\phi \cong Im\,\phi$.

$\mathbb{R}[X]/\langle x^2+1 \rangle \cong Im\,\phi = \mathbb{C}$.

- DEFINITION: R is an integral domain, $f(x) \in \mathbb{R}[x]$, $f(x) \neq 0$ or a unit in $R[x]$.
$f(x)$ is called irreducible if whenever $f(x) = g(x) \cdot h(x)$, $g(x), h(x) \in R[x]$ then $g(x)$ or $h(x)$ is a unit in $R[x]$.
If $f(x)$ is not irreducible, $f(x)$ is called reducible. $f(x) \neq 0$, $f(x)$ is not a unit

- NOTE: If $R = F$ a field, $f(x)$ is irreducible $\Leftrightarrow$ $f(x)$ cannot be written as $g(x)\,h(x)$ with $\deg(g) < \deg(f)$, $\deg(h) < \deg(f)$.

* EXAMPLE.

$f(x) = 2x^2+4$.

① $R = \mathbb{Q}$ $f(x)$ is irreducible.

② $R = \mathbb{Z}$ $f(x)$ is reducible.

$$f(x) = \underset{g(x)}{\overset{\shortparallel}{2}} \times \underset{h(x)}{\overset{\shortparallel}{(x^2+2)}}.$$

③ $R = \mathbb{R}, \mathbb{C}$ $f(x)$ is irreducible.

* EXAMPLE

$f(x) = x^2-2$.

① $R = \mathbb{Q}$ $f(x)$ is irreducible.

② $R = \mathbb{R}$ $f(x)$ is reducible.

$f(x) = (x^2-2) = (x-\sqrt{2})(x+\sqrt{2})$.

* EXAMPLE

$f(x) = x^2+1$.

① $R = \mathbb{R}$ $f(x)$ is irreducible.

② $R = \mathbb{C}$ $f(x)$ is reducible

$f(x) = (x+\sqrt{-1})(x-\sqrt{-1})$

③ $R = \mathbb{Z}_3 = \{0, 1, 2\}$. $f(x)$ is irreducible.

④ $R = \mathbb{Z}_5$ $f(x)$ is reducible.

⑤ $R = \mathbb{F}_p = \mathbb{Z}_p$ p is prime. $f(x)$ is $\begin{cases} \text{irreducible if } p \equiv 3 \bmod 4 \\ \text{reducible if } p \equiv 1 \bmod 4 \end{cases}$.

- THEOREM: (Reducibility Test for degree 2 and 3) Let F be a field. If $f(x) \in F[x]$ $\deg(f) = 2$ or 3, then $f(x)$ is reducible iff $f(x)$ has a zero over F.

PROOF

$f(x)$ is reducible $\Leftrightarrow f(x) = g(x) \cdot h(x)$ with $\deg(g), \deg(h) < \deg(f)$, $g(x), h(x) \in F[x]$

$\deg(f) = \underset{<3}{\deg(g)} + \underset{<3}{\deg(h)}$. one of the degrees must be 1.

$\Rightarrow$ If $\deg(g)=1$, $g(x) = ax+b \Rightarrow x = -\frac{b}{a} \in F$.

$-\frac{b}{a}$ is a zero of $g(x)$

$\Rightarrow -\frac{b}{a}$ is a zero of $f(x)$ since $f(x) = g(x) \cdot h(x)$.

* EXAMPLE

$f(x) = x^4+2x^2+1 = (x^2+1)^2$ is reducible over $\mathbb{R}$.

but $f(x)$ has no zeros over $\mathbb{R}$.

$\Rightarrow$ Theorem is not true for $\deg(f) > 3$.

- DEFINITION: The content of a nonzero polynomial $f(x) = a_n x^n + \ldots + a_1 x^1 + a_0$ over $\mathbb{Z}[x]$ is $\gcd(a_n, a_{n-1}, \ldots, a_1, a_0)$ $a_i \in \mathbb{Z}$. $i = 0, 1, 2, \ldots$

- DEFINITION: A primitive polynomial is an element in $\mathbb{Z}[x]$ with content 1.

* EXAMPLE

$2x^2+4$ content $= 2$. $\gcd(2,4) = 2$

$3x^2+5$ content $= 1$ $\gcd(3,5) = 1$.

- GAUSS'S LEMMA: The product of 2 primitive polynomials is primitive.

PROOF

Let $g(x)$, $h(x)$ be two primitive polynomials.

$f(x) = g(x) \cdot h(x)$.

GOAL: Prove $f(x)$ is primitive.

Cont(f): content of polynomial.

If $f(x)$ is not primitive, $\text{cont}(f) \neq 1$, $\text{cont}(f) \in \mathbb{N}$.

Let p be a prime divisor.

Let $\bar{f}(x)$, $\bar{g}(x)$ and $\bar{h}(x)$ be the polynomials obtained from $f(x)$, $g(x)$ and $h(x)$ by reducing the coefficients mod p.

$\overline{f(x)} = \overline{g(x)} \cdot \overline{h(x)}$

By $p \mid \text{cont}(f)$, $\overline{f(x)} \neq 0$.

But $\overline{g(x)} \cdot \overline{h(x)} \neq 0$ $(\Rightarrow\Leftarrow)$.

* EXAMPLE

$x^2 - 2$ is irreducible over $\mathbb{Z}$ and $\mathbb{Q}$.

$x^2 - 2$ is reducible over $\mathbb{Q}(\sqrt{2})$

$x^2 - 2 = (x - \sqrt{2})(x + \sqrt{2})$

\- THEOREM: Let $f(x) \in \mathbb{Z}[x]$. If $f(x)$ is reducible over $\mathbb{Q}$, $f(x)$ is reducible over $\mathbb{Z}$.

NOTE: The converse is not true. Eg $f(x) = 2x^2 + 4$.

PROOF

$f(x) = g(x) \cdot h(x)$, $g(x), h(x) \in \mathbb{Q}[x]$.

Let a be the least common multiple of the denominators of the coefficients of $g(x)$.

$g(x) = \sum_{i=0}^{m} \frac{b_i}{c_i} x^i$ $b_i, c_i \in \mathbb{Z}$. $\gcd(b_i, c_i) = 1$.

$a = [c_m, c_{n-1}, \ldots, c_1, c_0]$ = least common multiple.

$h(x) = \sum_{j=1}^{k} \frac{d_j}{e_j} x^d$ $d_j, c_j \in \mathbb{Z}$. $\gcd(d_j, c_j) = 1$.

$b' = [e_k, e_{k-1}, \ldots, e_1, e_0]$.

$$ab' \cdot f(x) = ab' g(x) \cdot h(x)$$
$$= \underset{\mathbb{Z}[x]}{\underset{\wedge}{(a\, g(x))}} \times \underset{\mathbb{Z}[x]}{\underset{\wedge}{(b' h(x))}}$$

$$\left.\begin{aligned} &\text{cont}(a\, g(x)) = c_1 \Rightarrow a \cdot g_1(x) = c_1 \cdot g(x) \\ &\text{cont}(b\, h(x) = c_2 \Rightarrow b \cdot h_1(x) = c_2 \cdot h(x) \end{aligned}\right\} g(x), h(x) \in \mathbb{Z}[x].$$

$\Rightarrow g_1(x)$, $h_1(x)$ are primitive.

Get ab. $\underbrace{f(x)}_{\text{primitive}} = [a g_1(x)][b h(x)] = c_1 g_1(x) \cdot c_2 h_1(x)$

$$= c_1 c_2 \underbrace{(g_1(x) h_1(x))}_{\text{primitive.}}$$

$\Rightarrow f(x) = g_1(x) \cdot h_1(x).$

1. HW2 returned
2. HW3 due
3. HW4 assigned
4. HW appeals
5. Happy birthday to me!

Math 401 Pols Eqns Fields, Jan 30 2008, week 4.

On board def of homomo, annoying properties.

Agenda
1. homomorphisms - annoying props & iso. thm.
2. An aside on domains an fields.
3. polynomials, division, roots...

Continue as on Jan 24, 2007

Continue as on Jan 31, 2007.

*Make up photo today!
*Web only HW4!

Math 401 Pols, Eqns, Fields, Jan 31 2007 Week 4

Reminder 1. If $\phi : R \to S$ is a ring homomorphism then $\ker \phi$ is an ideal in R

math "patterns":
1. set theoretic analog
2. rank-nullity thm. →

2. $R/\ker \phi \cong \phi(R)$ "The first iso. thm"

3. Every ideal is a kernel! If $A \subset R$ is an ideal, $\pi : R \to R/A$ has $\ker \pi = A$.

Claim R with unity $\Rightarrow \iota : \mathbb{Z} \to R$ by $n \mapsto n \cdot 1$ is a homom.

Cor char $R = n > 0 \Rightarrow R$ contains a subring iso. to $\mathbb{Z}/n$

char $R = 0 \Rightarrow R$ contains — " — $\mathbb{Z}$

Cor F a field: char $F = p \Rightarrow F \supset \mathbb{Z}/p$

2/3 hour

char $F = 0 \Rightarrow F \supset \mathbb{Q}$.

Thm If D is an integral domain, then there is a field F ("the field of quotients of D") that contains an isomorphic copy of D.

1½ hour.

Def $R[X]$ (R commutative); deg; evaluation.

Thm D a domain $\Rightarrow D[X]$ too; $\deg f \cdot g = \deg f + \deg g$.

Thm (F a field) $f, g \in F[X]$, $g \neq 0 \Rightarrow \exists!\ q, r \in F[X]$ s.t. $f = gq + r$ & $\deg r < \deg g$.

q: "The quotient for f/g"
r: "the remainder for f/g"

Cor 1 $f \in F[X] \Rightarrow f(a)$ is the remainder for $f/x-a$.

Cor 2 a is a root of $f \Leftrightarrow (x-a)$ is a factor of f.

Def Zero of a pol., multiplicity of a zero.

Cor. A polynomial of degree n over a field has at most n zeros, counting multiplicities

Def P.I.D.

Thm $F[x]$ is a PID; if I is a~~n~~ non-zero ideal in $F[x]$, $I = \langle g \rangle$ iff g is a non-zero element of I of minimal degree.

Example $\mathbb{R}[x]/\langle x^2+1\rangle \cong \mathbb{C}$ using $\phi : p \longmapsto p(i)$ & the first iso. thm.

Class photo today!
(At first break)

Math 401 Pols, Eqns, Fields, Jan 24 2007 Week 3

Ideal $A - A \subset A$, $RA \subset A$ $AR \subset A$; R/A makes sense!?

R commutative

Thm R/A is a domain iff A is prime.

Thm R/A is a field iff A is maximal.

Homomorphisms: philosophy: objects, games, sets, groups, rings

Examples 3. $a+bi \longmapsto \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$

4. $\mathbb{Z}_4 \xrightarrow{\cdot 5} \mathbb{Z}/_{10}$, 1. $\mathbb{Z} \xrightarrow{\text{parity}} \{0,1\}$

2. $\mathbb{R}[x] \xrightarrow{p \mapsto p(2)} \mathbb{R}$

} more in text

Annoying Properties
for $\phi: R \to S$
$\overset{\cup}{\text{subring } A}$ $\quad$ $\overset{\cup}{B \text{ ideal}}$

1. $\phi(nr) = n\phi$; $\phi(r^n) = \phi(r)^n$
2. $\phi(A)$ is a subring ($\text{im}\,\phi = \phi(R)$ too)
3. A ideal; is $\phi(A)$ an ideal? No, need ϕ is onto!
4. $\phi^{-1}(B)$ an ideal ($\ker\phi$ too)
5. R commut. $\Rightarrow$ $\phi(R)$ too
6. R has a unity, $S \neq \{0\}$, ϕ is onto $\Rightarrow$ $\phi(1)$ is the unity
7. ϕ is iso. iff $\ker\phi = \{0\}$ & $\text{im}\,\phi = S$
8. ϕ is an iso $\Rightarrow$ ϕ^{-1} is too

Thm $R/_{\ker\phi} \cong \phi(R)$

Thm Every ideal $A \subset R$ is a kernel; $\pi: R \to R/A$ $\ker \pi = A$.

Thm R with unity $\Rightarrow$ $\mathbb{Z} \to R$ by $n \longmapsto n \cdot 1$ is a homomorphism.

Cor char $R = n > 0$ $\Rightarrow$ R contains a subring iso. to $\mathbb{Z}/n$
char $R = 0$ $\Rightarrow$ R contains — " — $\mathbb{Z}$.

Cor F a field; char $F = p$ $\Rightarrow$ $F \supset \mathbb{Z}/p$
char $F = 0$ $\Rightarrow$ $F \supset \mathbb{Q}$

Thm If D is an integral domain, then there is a field F that contains D as a subring.

…/cont

Ring of polynomials.

Def $R[X]$ (R commutative) deg p

Thm D a domain $\Rightarrow$ $D[X]$ too.

Thm Long division in $F[X]$

1. Class photo @ break
2. APNS
3. HW 1 2 3

Math 401 Pols, Eqns, Fields, Jan 23 2008, Week 3

$$\mathbb{R}[x]/\langle x^2+1\rangle \text{ "="} \mathbb{C} = \left(\begin{matrix}\mathbb{R} \text{ "+" } ? \\ \text{root of } x^2+1=0\end{matrix}\right)$$

1. The meaning of the lhs.
2. Ideals & quotients.
3. The meaning of "=".

Ideals: $\underbrace{\underbrace{A-A\subset A,\ AA\subset A}_{\text{subring}},\ A\cdot R\subset A\ \ RA\subset A}_{\text{ideal.}}$; R/A makes sense!

Prime & maximal ideals as on Jan 17, 2007.

Homomorphisms as on Jan 24, 2007.

Office hours: Chao Li, TA: Tue 12-2, SS1071
DBN Wed 10:30-11:30, BA 6178

Math 401 Pols, Eqns, Fields, Jan 17 2007, Week 2.

Ring: + − Abelian group
× − Assoc, distributive. Sometimes "unity", "inverses"
If exist, they are unique.

Subring: A subset which is a ring ⇔ closed under ×, −

Zero divisor: $a \cdot b = 0$ but $a \neq 0, b \neq 0$
(in commutative rings)

integral domain: commutative with unity & no zero divisors; $ac = bc, c$ $\Downarrow$ $a = b$.

Field +, × both make abelian groups (+ distributive)

Thm A finite integral domain is a field; $\mathbb{Z}/p$ is a field.

---

char R; char R if R has a unity.

Thm: If D is an integral domain, char D is 0 or a prime.

Go over Table 13.2

Ideal (two sided) ⇔ 1. $\forall a, b \in A$ $a-b \in A$
2. $\forall a \in A, r \in R$ $ra \in A$ & $ar \in A$.

Examples $\{0\}$, $n\mathbb{Z}$, $\langle n \rangle$ $\langle x \rangle \in \mathbb{R}[x]$, $\langle x, 2 \rangle \in \mathbb{Z}$

R/A = Equiv classes and/or cosets.
Addition, mult.

Examples $R/\{0\}$, $\mathbb{Z}/n\mathbb{Z}$, $\mathbb{R}[x]/\langle x^2+1 \rangle$

(more examples in text)

Def prime & maximal ideals. (of commutative rings)

Example $n\mathbb{Z} \subset \mathbb{Z}$ is prime iff n is.

Example $\langle x^2+1 \rangle$ is maximal in $\mathbb{R}[x]$

Example
~~prime ⇒~~
maximal ⇒ pr
but not vice versa.

Thm R/A is an integral domain iff A is prime.

Thm R/A is a field iff A is maximal.

Cont.

Ring homomorphisms in brief.

TABLE 13.2 Summary of Rings and Their Properties

Ring	Form of Element	Unity	Commutative	Integral Domain	Field	Characteristic
Z	k	1	Yes	Yes	No	0
Z_n, n composite	k	1	Yes	No	No	n
Z_p, p prime	k	1	Yes	Yes	Yes	p
$Z[x]$	$a_nx^n + \cdots + a_1x + a_0$	$f(x) = 1$	Yes	Yes	No	0
$nZ, n > 1$	nk	None	Yes	No	No	0
$M_2(Z)$	$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	No	No	No	0
$M_2(2Z)$	$\begin{bmatrix} 2a & 2b \\ 2c & 2d \end{bmatrix}$	None	No	No	No	0
$Z[i]$	$a + bi$	1	Yes	Yes	No	0
$Z_3[i]$	$a + bi; a, b \in Z_3$	1	Yes	Yes	Yes	3
$Z[\sqrt{2}]$	$a + b\sqrt{2}; a, b \in Z$	1	Yes	Yes	No	0
$Q[\sqrt{2}]$	$a + b\sqrt{2}; a, b \in Q$	1	Yes	Yes	Yes	0
$Z \oplus Z$	(a, b)	$(1, 1)$	Yes	No	No	0

Math 401 Pols Eqns Fields, Jan 9 2007 week 1. (& 2)
Jan 16

About Galois; about this class.

Why rings? Alg. numbers form a ring
"formulas" form a ring

<u>Def</u> A ring R is a non-empty set w/ two binary ops $(a,b) \mapsto a+b$ and $(a,b) \mapsto a \cdot b$ s.t. for all a, b, c:

1. $a+b = b+a$ 3. an element $0 \in R$ is given,
2. $(a+b)+c = a+(b+c)$ such that $0+a=a$

4. $\forall a \; \exists (-a)$ s.t. $a+(-a)=0$.

5. $a(bc)=(ab)c$ 6. $a(b+c) =$
$(b+c)a =$.

<u>Def.</u> commutative ring, ring w/ unity.

Examples as in Table 13.2.

<u>Thm 1</u> 1. $a \cdot 0 = 0 \cdot a = 0$ 2. $a(-b) = (-a)b = -(ab)$

3. $(-a)(-b) = ab$ 4 $a(b-c) = ab - ac$

if unity, 5. $(-1)a = -a$ 6. $(-1)(-1) = 1$.

<u>Thm 2</u> If a ring has a unity, it is unique.
If an element of a ring has an inverse, it is unique.

Zero divisors, integral domains, cancellation,
(in commutative rings) commutative ring w/ unity & no 0-divisors

Back to Table 13.2 →

Field: commutative ring w/ unity in which every non-zero element has an inverse.

Thm A finite integral domain is a field.

Cor $\mathbb{Z}/p$ is a field.

Char R

Char R if R has a unity.

Thm If D is a domain, char D is 0 or a prime.

Week 2: repeat 07-401 / Week 2.

Notes HW2 on web

R/A is an integral domain iff A is prime.

08-401/About This Class

From Drorbn

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Crucial Information

Agenda: Follow Évariste Galois (http://en.wikipedia.org/wiki/Galois) to the top of mathematics' first mountain.

Classes: Wednesdays 6-9PM (OMG) at Sidney Smith 1086 (http://www.osm.utoronto.ca/cgi-bin/class_spec/spec03?bldg=SS&room=1086) .

Instructor: Dror Bar-Natan (http://www.math.toronto.edu/~drorbn/) , drorbn@math.toronto.edu, Bahen 6178, 416-946-5438. Office hours: Tuesdays 12:30-1:30 and most Thursdays 1-2 (lunchtime, at the math lounge or at my office), or by appointment.

Teaching Assistant: Yichao Zhang, zhangyichao2002@hotmail.com. Office hours: Tuesdays 1-3 at the Math Aid Centre, Sidney Smith 1071.

Grades. All grades will be on CCNet (http://ccnet.utoronto.ca/20081/mat401h1s/) .

URL: http://katlas.math.toronto.edu/drorbn/index.php?title=08-401.

Abstract

Taken from the Faculty of Arts and Science Calendar (http://www.artsandscience.utoronto.ca/ofr/calendar/crs_mat.htm) :

> Commutative rings; quotient rings. Construction of the rationals. Polynomial algebra. Fields and Galois theory: Field extensions, adjunction of roots of a polynomial. Constructibility, trisection of angles, construction of regular polygons. Galois groups of polynomials, in particular cubics, quartics. Insolvability of quintics by radicals.

- Prerequisite: MAT224H1 (Linear Algebra II), MAT235Y1/MAT237Y1 (Calculus II/Multivariable Calculus),

MAT246H1/MAT257Y1 (Concepts in Abstract Mathematics/Analysis II).
- Exclusion: MAT347Y1 (Groups, Rings and Fields).

Text Book(s)

- (Required) J. A. Gallian, "Contemporary Abstract Algebra", chapters 12-17, 20-23 and 31-33 (approx.).
- (Recommended) D. S. Dummit and R. M. Foote, "Abstract Algebra", chapters 7, 8, 9, 13, 14.
- (Suggested) T. Hungerford, "Abstract Algebra, an Introduction".

Plan

I will aim to cover the above-mentioned 13 chapters of Gallian's book at a bit faster than one per week, so as to leave us some time for extras at the end, but we may fall back to a rate of just one chapter a week or even less. If so, chapters 23 and 31 will be the first candidates for skipping.

Wiki

The class web site is a wiki, as in Wikipedia (http://www.wikipedia.org) - meaning that anyone can and is welcome to edit almost anything and in particular, students can post notes, comments, pictures, whatever. Some rules, though -

- This wiki is a part of my (Dror's) academic web page. All postings on it must be class-related (or related to one of the other projects I'm involved with).
- I (Dror) will allow myself to exercise editorial control, when necessary.
- The titles of all pages related to this class should begin with "08-401/" or with "08-401-", just like the title of this page.

To edit, you must have a wiki account. To get one email a request to Dror at drorbn@math.toronto.edu, and include:

- Your first and last name.
- Your preferred user id.
- Your email address, if different from the address you've used for this email.

Some further editing help is available at Help:Contents.

Marking Scheme

There will be one term test (25% of the total grade) and a final exam (50%), as well as about 10 homework assignments (25%).

The Term Test

The term test will take place in class on February 28. A student who misses the term test without providing a valid reason (for example, a doctor's note) within one week of the test will receive a mark of 0 on the term test. There will be no make-up term test. If a student misses the term test for a valid reason, the weight of the problem sets will increase to 35% and the weight of the final exam to 65%.

Homework

Assignments will be posted on the course web page approximately on the weeks shown in the class timeline. Typically an "in preparation" version of any assignment will be posted a bit before class and the "in preparation" tag will be removed shortly after class, once our progress in class is precisely measured. Assignments will be due in class a week after they are assigned and they will be marked by the TA, usually within another week. All students (including those who join the course late) will receive a mark of 0 on each assignment not handed in; though to allow you some leeway, in computing the homework grade your worst two assignments will not count. I encourage you to discuss the assignments with other students or even browse the web, so long as you do at least some of the thinking on your own and you write up your own solutions. Remember that cheating is always possible and may increase your homework grade a bit. But it will hurt your exam grades a lot more.

Good Deeds

Students will be able to earn up to 25 "good deeds" points throughout the year for doing services to the class as a whole. There is no pre-set system for awarding these points, but the following will definitely count:

- Drawing a beautiful picture to illustrate a point discussed in class and posting it on this site.
- Taking class notes in nice handwriting, scanning them and posting them here.
- Typing up or formatting somebody else's class notes, correcting them or expanding them in any way.
- Writing an essay expanding on anything mentioned in class and posting it here; correcting or expanding somebody else's article.
- Doing anything on our 08-401/To do list.
- Any other service to the class as a whole.

Good deed points will count towards your final grade! If you got n of those, they are solidly your and the formula for the final grade below will only be applied to the remaining $100-n$ points. So if you got 25 good deed points (say) and your final grade is 80, I will report your grade as $25+80(100-25)/100=85$. Yet you can get an overall 100 even without doing a single good deed.

Class Photo

To help me learn your names, I will take a class photo on the third week of classes. I will post the picture on the class' web site and you will be *required* to identify yourself on the Class Photo page of this wiki.

Galois at the age of fifteen from the pencil of a classmate. He was young-looking for his age and had black hair.

On Galois

The first paragraph of the Wikipedia entry (http://en.wikipedia.org/wiki/Galois) on Galois:

> **Évariste Galois** (IPA: [evaʁist gaˈlwa]; October 25, 1811 – May 31, 1832) was a French mathematician born in Bourg-la-Reine. While still in his teens, he was able to determine a necessary and sufficient condition for a polynomial to be solvable by radicals, thereby solving a long-standing problem. His work laid the foundations for Galois theory, a major branch of abstract algebra, and the subfield of Galois connections. He was the first to use the word "group" (French: groupe) as a technical term in mathematics to represent a group of permutations. A radical Republican during the monarchy of Louis Philippe in France, he died from wounds suffered in a duel under murky circumstances at the age of twenty.

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08-401/Homework Assignment 1

From Drorbn

In Preparation

The information below is preliminary and cannot be trusted! (v)

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11	Mar 26	HW9, Notes
12	Apr 2	HW10, Notes
13	Apr 9	Notes
S	Apr 14-18	Study Period
F		Final

Image:08-401 Class Photo.jpg
Add your name / see who's in!

Register of Good Deeds

Reading

Read chapters 12 and 13 of Gallian's book three times:

- First time as if you were reading a novel - quickly and without too much attention to detail, just to learn what the main keywords and concepts and goals are.
- Second time like you were studying for an exam on the subject - slowly and not skipping anything, verifying every little detail.
- And then a third time, again at a quicker pace, to remind yourself of the bigger picture all those little details are there to paint.

Doing

Solve problems 1, S2, 8, 12, S13, S19, 20, S22, 33, 47 and 50 in Chapter 12 of Gallian's book and problems 4, 7, 12, S13, 16, S24 and 33 in Chapter 13 of the ame book, but submit only the solutions of the problems marked with the letter "S".

Due Date

This assignment is due in class on Wednesday January 16, 2007.

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0: About this class. . . .

Math 401 Pols, Eqns, Fields, Jan 10, 2007 Week 1

Examples $\mathbb{Z}$, $\mathbb{Z}/n$, $\mathbb{Z}[x]$, $M_2(\mathbb{Z})$, $2\mathbb{Z}$, $\mathbb{Z}[i]$, $\mathbb{C}$

Def A ring R is a non-empty set with two binary ops $(a,b) \mapsto a+b$ and $(a,b) \mapsto ab$ s.t. for all a, b, c,

1. $a+b=b+a$
2. There is an additive identity . . .
3. $\forall a \, \exists (-a)$ s.t $a+(-a)=0$
5. $a(bc)=(ab)c$
6. $a(b+c)=ab+ac$ $(b+c)a=ba+ca$

Def direct sum.

Thm 1. $a0=0a=0$ 2. $a(-b)=(-a)b=-(ab)$ 3. $(-a)(-b)=ab$
4 $a(b-c)=ab-ac$ 5. if $\exists 1 \in R$ s.t $1a=a$, then $(-1)a=-a$, $(-1)(-1)=1$.

Thm If a ring has a unity, it is unique; if an element in a ring has an inverse, it is unique.

Subring: A subset which is a ring under same ops.
① non-empty subset
2. closed under subtraction & under mult.

Zero Divisors, integral domains; go over above examples
in commutative ring
commutative ring w/ unity s.t.
cancellation.

Field: Commutative ring with unity in which every non zero element is a unit.

--/

Thm A finite integral domain is a field.

Cor $\mathbb{Z}/p$ is a field.

---

char R

char R if R has a unity.

Thm If D is an an integral domain,
char D is 0 or a prime.

go over table 13.2.