

Take $D = ([x, y], o) \in \text{tder}(x, y)$. Then

$$\ell^D \cdot x = x + [x, y]x + \frac{1}{2} [[x, y], x], y]x \\ + \frac{1}{2} [[x, y], [x, y], x] + \dots$$

$$= \begin{array}{c} | \\ x \end{array} + \begin{array}{c} | \\ x \\ y \\ | \\ x \end{array} + \frac{1}{2} \begin{array}{c} | \\ x \\ y \\ | \\ x \\ y \\ | \\ x \end{array} + \frac{1}{2} \begin{array}{c} | \\ x \\ y \\ | \\ x \\ y \\ | \\ x \\ y \end{array}$$

$$y + [x, y] = \ell^{\text{ad } x} y$$

$\ell^{\text{ad } x}$

Back to $D = ([y, x], o)$:

$$\ell^D \cdot x = \begin{array}{c} | \\ x \end{array} + \begin{array}{c} | \\ x \\ y \\ | \\ x \end{array} + \frac{1}{2} \begin{array}{c} | \\ x \\ y \\ | \\ x \\ y \\ | \\ x \end{array} + \frac{1}{2} \begin{array}{c} | \\ x \\ y \\ | \\ x \\ y \\ | \\ x \end{array}$$

Q In Lie_n, how do I tell if an element is of the form $(\ell^P x_i) = \ell^P x_i; \ell^{-P}$, where ℓ is a Lie-series?

$$\mathbb{J}(\ell^P x_i; \ell^{-P}) = (\ell^P \otimes \ell^P)(x_i \otimes 1 + 1 \otimes x_i)(\ell^{-P} \otimes \ell^{-P}) = \\ = \ell^P x_i \ell^{-P} \otimes 1 + 1 \otimes \ell^P x_i \ell^{-P}$$