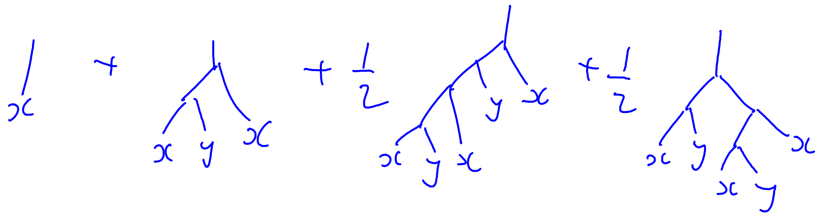
Take $D=([x,y],0)\in \mathrm{tder}(x,y)$. Then

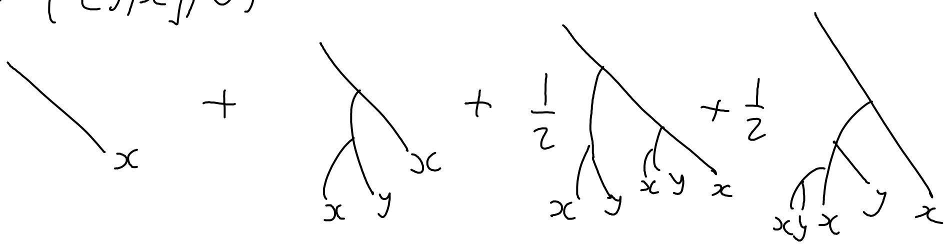
$$e^D x = x + [[x,y],x] + \tfrac12[[[[x,y],x],y],x] + \tfrac12[[x,y],[[x,y],x]] + \dots$$

$=$ 

$y + [x,y] = \cancel{e^{\mathrm{ad}\,x} y}$

$e^{\mathrm{ad}\,x}$

Back to $D=([y,x],0)$:

$e^D\cdot x =$ 

Q In Lie, how do I tell if an element is of the form $e^{\mathrm{ad}P}x_i = e^P x_i e^{-P}$, where is a Lie-series?

$$\Delta(e^P x_i e^{-P}) = (e^P\otimes e^P)(x_i\otimes 1 + 1\otimes x_i)(e^{-P}\otimes e^{-P}) =$$
$$= e^P x_i e^{-P}\otimes 1 + 1\otimes e^P x_i e^{-P}$$