Hain at SCGP

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S = surface of type g,n $\quad 2g-2+n>0$

$\pi_1(S,x)$ w/ ξ a non-vanishing v.F.

$\lambda(S)$: conjugacy classes $\equiv$ free homotopy classes of loops in S

$R\lambda(S) = |R\pi_1(S,x)|$

Goldman bracket: $\{,\}: R\lambda(S)\otimes R\lambda(S)\to R\lambda(S)$

Turaev cobracket: $\delta_\xi: R\lambda(S)\to R\lambda(S)\otimes R\lambda(S)$

I = aug ideal of $R\pi_1(S,x)$

$\rightsquigarrow$ I-adic topology

$\rightsquigarrow$ $R\hat{\pi}_1(S,x) = \varprojlim R\pi_1(S,x)/I^n$

Continuous dual:

$$\mathrm{Hom}^{cts}(R\hat{\pi}_1(S,x),R) = \varinjlim \mathrm{Hom}(R\pi_1/I^n, R)$$

descends to λ

Kawazumi-Kuno: $\{,\},\delta_\xi$ are cont.

<u>Thm</u> For every alg str. on (X,ξ), these are morphisms of MHS (mixed Hodge structures)

<u>Template for a Proof</u>

Template for a Proof

(1) Factor $\{,\}$ $\delta_{\{}$

(2) Give a homological description of cont. dual of each term

(3) show that each factor is a morphism of MHS.

Goldman first:

$S = \bar{S} - D$, $\quad \bar{S}$: compact Riemann surface of genus g.

D finite, $\#D = n$.

$\cdot\ \Lambda S = \{\text{piecewise smooth loops } \underset{[0,1]/0\sim1}{\overset{}{S^1}} \longrightarrow S\}$

$H_0(\Lambda S, R) = R\lambda(S)$

$\mathcal{L}_S$ = locally constant sheafs over S whose fiber over $x \in S$ is

$$H_0(\underset{\text{loops based at } x}{\Lambda_x S}) = R\pi_1(S, x)$$

Prop $H_j(\Lambda S) = H_j(S, \mathcal{L}_S)$