# Direct Exponentiation

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## Cheat Sheet $sl_2$-Portfolio (an implementat

$\mathcal{U}_{\gamma\epsilon;\hbar}$ **conventions.** "consolidate"

$q = \mathbb{e}^{\hbar\gamma\epsilon}$, $H = \langle a, x\rangle/([a,x] = \gamma x)$ with

$A = \mathbb{e}^{-\hbar\epsilon a}$, $xA = qAx$, $S_H(a, A, x) = (-a, A^{-1}, -A^{-1}x)$,

$$\Delta_H(a, A, x) = (a_1 + a_2, A_1A_2, x_1 + A_1x_2)$$

and dual $H^* = \langle b, y\rangle/([b,y] = -\epsilon y)$ with

$B = \mathbb{e}^{-\hbar\gamma b}$, $By = qyB$, $S_{H^*}(b, B, y) = (-b, B^{-1}, -yB^{-1})$,

$$\Delta_{H^*}(b, B, y) = (b_1 + b_2, B_1B_2, y_1B_2 + y_2).$$

Pairing by $(a,x)^* = \hbar(b,y)$ ($\Rightarrow \langle B, A\rangle = q$) making $\langle y^lb^i, a^jx^k\rangle = \delta_{ij}\delta_{kl}\hbar^{-(j+k)}j![k]_q!$ so $R = \sum \frac{\hbar^{j+k}y^kb^j\otimes a^jx^k}{j![k]_q!}$. Then $\mathcal{U} = H^{*cop}\otimes H$ with $(\phi f)(\psi g) = \langle \psi_1 S^{-1}f_3\rangle\langle\psi_3, f_1\rangle(\phi\psi_2)(f_2g)$ and

$$S(y, b, a, x) = (-B^{-1}y, -b, -a, -A^{-1}x),$$

$$\Delta(y, b, a, x) = (y_1 + y_2B_1, b_1 + b_2, a_1 + a_2, x_1 + A_1x_2).$$

$\hbar = \gamma = 1$

$[a,x] = x \quad ax = x(a+1)$

$Ax = e^{-\epsilon a}x = xe^{-\epsilon(a+1)} = e^{-\epsilon}xe^{-\epsilon a} = e^{-\epsilon}xA$

$[b,y] = -\epsilon y \quad by = y(b-\epsilon)$

$By = e^{-b}y = ye^{-b+\epsilon} = e^{\epsilon}yB$

$e^{\xi\Delta(x)} = e^{\xi(x_1 + A_1x_2)} = e^{\xi x_1 + \xi x_2 A_1}$

$e^{\eta\Delta(y)} = e^{\eta(y_1B_2 + y_2)} = e^{\eta y_1 B_2 + \eta y_2}$

$e^{\xi S(x)} = e^{-\xi A^{-1}x} = \mathbb{O}_{ax}\left(e^{-\xi x}\rho_3\right)$

$e^{\eta S(y)} = e^{-\eta y B^{-1}} = \mathbb{O}_{yb}\left(e^{-\eta B^{-1}y}\rho_4\right)$

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<u>Question</u> Suppose $yx = (1+\epsilon)xy$

Compute $P_1$ & $P_2$ where

$$e^{ax+by} = \mathbb{O}_{xy}\left(e^{ax+by}P_1\right)$$

and $e^{axy} = \mathbb{O}_{xy}\left(e^{axy}P_2\right)$,

as power series in $\epsilon$.

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Recast $[x,y] = -\epsilon xy = -\epsilon a$

$$[a,x] = [xy,x] = x[y,x] = \epsilon x a$$

$$[x, y+\delta xy] + \epsilon x(y+\delta xy)$$

$[x, xy] = x[x,y]$
$= -\epsilon x^2 y$

$$= -\epsilon xy - \epsilon\delta x^2 y + \epsilon xy + \epsilon\delta x^2 y = 0$$

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Re-recast suppose $sl = (1+\gamma)ls$

Compute $P_1$ & $P_2$ s.t.

1. $e^{\lambda l + \sigma s} = \mathbb{O}_{ls}\left(e^{\lambda l+\sigma s}P_1\right)$
2. $e^{\delta ls} = \mathbb{O}_{ls}\left(e^{\delta ls}P_2\right)$

<u>Sol'n</u> 1. Rewrite $e^{h(\lambda l+\sigma s)} = \mathbb{O}_{ls}\left(e^{h(\lambda l+\sigma s)}P_1\right)$

At $h=0$, get $P_1 = 1$. Now take $\partial/\partial h$:

$$e^{h(\lambda l+\sigma s)}(\lambda l + \sigma s) = \mathbb{O}_{ls}\left(e^{h(\lambda l+\sigma s)}\left((\lambda l + \cancel{\sigma s})P_1 + \partial_h P_1\right)\right)$$

$\parallel$

$$\Theta_{ls}\left(e^{h(\lambda l+\sigma s)}P_1\right)(\lambda l+\sigma s)$$

$$\|$$

$$\Theta_{ls}\left(\cancel{e^{h(\lambda l+\sigma s)}P_1\cdot\sigma s} + \lambda l e^{h(\lambda l+\sigma s)}P_1\big|_{s\mapsto(1+\gamma)s}\right)$$

So we need to solve

$$\lambda l e^{h\gamma\sigma s}\left[P_1\big|_{s\mapsto(1+\gamma)s}\right] = \lambda l P_1 + \partial_h P_1$$