

Pensieve header: Even better learning from experience.

In[*]:= **\$k = 3**

Out[*]= 3

In[*]:= **t2b_{ni_, \$k_} := Block[{i}, t2b_{i_, \$k} = E[α_i a_i - τ_i γ b_j, ξ_i x_i + η_i y_i, e^{ε τ_i a_i} + O[ε]^{\$k+1}]; t2b_{ni, \$k}];**
t2b_{i_} := t2b_{i, \$k}

In[*]:= **?? Subscript**

Subscript[x,y] is an object that formats as x_y.
 Subscript[x,y₁,y₂,...] formats as x_{y₁,y₂,...} >>

Attributes[Subscript] = {NHoldRest}

Subscript[t2b, ni_, \$k_] := Block[{i},
 Subscript[t2b, i_, \$k] = E[Subscript[α, i] Subscript[a, i] - Subscript[τ, i] γ Subscript[b, j],
 Subscript[ξ, i] Subscript[x, i] + Subscript[η, i] Subscript[y, i],
 e^{ε Subscript[τ, i] Subscript[a, i]} + O[ε]^{\$k+1}];
 Subscript[t2b, ni, \$k]

Subscript[t2b, i_] := Subscript[t2b, i, \$k]

In[*]:= **t2b_{i,2}**

Out[*]=
$$\mathbb{E} \left[a_i \alpha_i - \gamma b_j \tau_i, y_i \eta_i + x_i \xi_i, 1 + a_i \tau_i \epsilon + \frac{1}{2} a_i^2 \tau_i^2 \epsilon^2 + O[\epsilon]^3 \right]$$

In[*]:= **t2b_{i,2}**

Out[*]=
$$\mathbb{E} \left[a_i \alpha_i - \gamma b_j \tau_i, y_i \eta_i + x_i \xi_i, 1 + a_i \tau_i \epsilon + \frac{1}{2} a_i^2 \tau_i^2 \epsilon^2 + O[\epsilon]^3 \right]$$

In[*]:= **t2b₁**

Out[*]=
$$\mathbb{E} \left[a_1 \alpha_1 - \gamma b_j \tau_1, y_1 \eta_1 + x_1 \xi_1, 1 + a_1 \tau_1 \epsilon + \frac{1}{2} a_1^2 \tau_1^2 \epsilon^2 + \frac{1}{6} a_1^3 \tau_1^3 \epsilon^3 + O[\epsilon]^4 \right]$$

In[*]:= **?? Subscript**

Subscript[x, y] is an object that formats as x_y .

Subscript[x, y₁, y₂, ...] formats as $x_{y_1, y_2, \dots}$ >>

Attributes[Subscript] = {NHoldRest}

$$\text{Subscript}[t2b, i\$, 2] = \mathbb{E}[\text{Subscript}[a, i\$] \text{Subscript}[\alpha, i\$] - \gamma \text{Subscript}[b, j] \text{Subscript}[\tau, i\$], \\ \text{Subscript}[y, i\$] \text{Subscript}[\eta, i\$] + \text{Subscript}[x, i\$] \text{Subscript}[\xi, i\$], \\ 1 + \text{Subscript}[a, i\$] \text{Subscript}[\tau, i\$] \epsilon + \frac{1}{2} \text{Subscript}[a, i\$]^2 \text{Subscript}[\tau, i\$]^2 \epsilon^2 + \mathcal{O}[\epsilon]^3]$$

$$\text{Subscript}[t2b, i\$, 3] = \mathbb{E}[\text{Subscript}[a, i\$] \text{Subscript}[\alpha, i\$] - \gamma \text{Subscript}[b, j] \text{Subscript}[\tau, i\$], \\ \text{Subscript}[y, i\$] \text{Subscript}[\eta, i\$] + \text{Subscript}[x, i\$] \text{Subscript}[\xi, i\$], \\ 1 + \text{Subscript}[a, i\$] \text{Subscript}[\tau, i\$] \epsilon + \frac{1}{2} \text{Subscript}[a, i\$]^2 \text{Subscript}[\tau, i\$]^2 \epsilon^2 + \\ \frac{1}{6} \text{Subscript}[a, i\$]^3 \text{Subscript}[\tau, i\$]^3 \epsilon^3 + \mathcal{O}[\epsilon]^4]$$

$$\text{Subscript}[t2b, ni\$, \$k_] := \text{Block}[\{i\}, \\ \text{Subscript}[t2b, i\$, \$k] = \mathbb{E}[\text{Subscript}[\alpha, i] \text{Subscript}[a, i] - \text{Subscript}[\tau, i] \gamma \text{Subscript}[b, j], \\ \text{Subscript}[\xi, i] \text{Subscript}[x, i] + \text{Subscript}[\eta, i] \text{Subscript}[y, i], \\ \epsilon \in \text{Subscript}[\tau, i] \text{Subscript}[a, i] + \mathcal{O}[\epsilon]^{\$k+1}]; \\ \text{Subscript}[t2b, ni, \$k]]$$

$$\text{Subscript}[t2b, i_] := \text{Subscript}[t2b, i, \$k]$$

```
In[*]:= f[0] = f[1] = 1;
f[n_] := Once[Echo@n; f[n - 1] + f[n - 2]];
f[100]
```

```
Out[*]:= 573 147 844 013 817 084 101
```

```
In[*]:= $k = 2;
Boot[$k_] := Once[Block[{i, tu = TimeUsed[]},
  b2ti, $k =  $\mathbb{E}[\alpha_i a_i - \beta_i t_i / \gamma, \xi_i x_i + \eta_i y_i, e^{\epsilon \beta_i a_i / \gamma} + \mathcal{O}[\epsilon]^{\$k+1}];$ 
  Print["Booted @ $k=", $k, " in ", TimeUsed[] - tu, " sec."];
]];
b2ti := b2ti, $k; b2ti, $k := (Boot[$k]; b2ti, $k);
"more like that";
```

```
In[*]:= b2t3
Booted @ $k=2 in 0.015 sec.
```

$$\text{Out}[*] = \mathbb{E}\left[a_3 \alpha_3 - \frac{t_3 \beta_3}{\gamma}, y_3 \eta_3 + x_3 \xi_3, 1 + \frac{a_3 \beta_3 \epsilon}{\gamma} + \frac{a_3^2 \beta_3^2 \epsilon^2}{2 \gamma^2} + \mathcal{O}[\epsilon]^3\right]$$

```
In[*]:= b2t2
```

$$\text{Out}[*] = \mathbb{E}\left[a_2 \alpha_2 - \frac{t_2 \beta_2}{\gamma}, y_2 \eta_2 + x_2 \xi_2, 1 + \frac{a_2 \beta_2 \epsilon}{\gamma} + \frac{a_2^2 \beta_2^2 \epsilon^2}{2 \gamma^2} + \mathcal{O}[\epsilon]^3\right]$$

In[]:= **Block** [{ \$k = 2 } , **b2t2**]

$$\text{Out[]} = \mathbb{E} \left[a_2 \alpha_2 - \frac{t_2 \beta_2}{\gamma}, y_2 \eta_2 + x_2 \xi_2, 1 + \frac{a_2 \beta_2 \epsilon}{\gamma} + \frac{a_2^2 \beta_2^2 \epsilon^2}{2 \gamma^2} + O[\epsilon]^3 \right]$$

In[]:= **Block** [{ \$k = 4 } , **b2t2**]

Booted @ \$k=4 in 0. sec.

$$\text{Out[]} = \mathbb{E} \left[a_2 \alpha_2 - \frac{t_2 \beta_2}{\gamma}, y_2 \eta_2 + x_2 \xi_2, 1 + \frac{a_2 \beta_2 \epsilon}{\gamma} + \frac{a_2^2 \beta_2^2 \epsilon^2}{2 \gamma^2} + \frac{a_2^3 \beta_2^3 \epsilon^3}{6 \gamma^3} + \frac{a_2^4 \beta_2^4 \epsilon^4}{24 \gamma^4} + O[\epsilon]^5 \right]$$

In[]:= **\$k = 3**

Out[]:= 3

In[]:= **t2b_{i_}, \$k_** := (**Block** [{ **i** } , **t2b_{i_}, \$k** = $\mathbb{E} [\alpha_i a_i - \tau_i \gamma b_j, \xi_i x_i + \eta_i y_i, e^{\epsilon \tau_i a_i} + O[\epsilon]^{k+1}]$] ; **t2b_{i_}, \$k**) ;
t2b_{i_} := **t2b_{i_}, \$k**

RuleDelayed: Pattern **i_** appears on the right-hand side of rule +

$$t2b_{i_}, $k_ \rightarrow (\text{Block}[\{i\}, t2b_{i_}, $k = \mathbb{E}[\text{Subscript}[\llbracket 2 \rrbracket] \text{Subscript}[\llbracket 2 \rrbracket] - \text{Times}[\llbracket 3 \rrbracket], \text{Subscript}[\llbracket 2 \rrbracket] \text{Subscript}[\llbracket 2 \rrbracket] + \text{Subscript}[\llbracket 2 \rrbracket] \text{Subscript}[\llbracket 2 \rrbracket], e^{\text{Times}[\llbracket 3 \rrbracket]} + O[\llbracket 1 \rrbracket]^{\text{Plus}[\llbracket 2 \rrbracket]}]; t2b_{i_}, $k).$$

In[]:= **?? Subscript**

Subscript[x,y] is an object that formats as x_y .
 Subscript[x,y1,y2,...] formats as $x_{y_1, y_2, \dots}$ >>

Attributes[Subscript] = {NHoldRest}

Subscript[t2b, i_, \$k_] := (Block[{i},
 Subscript[t2b, i_, \$k] = $\mathbb{E} [\text{Subscript}[\alpha, i] \text{Subscript}[a, i] - \text{Subscript}[\tau, i] \gamma \text{Subscript}[b, j],$
 $\text{Subscript}[\xi, i] \text{Subscript}[x, i] + \text{Subscript}[\eta, i] \text{Subscript}[y, i],$
 $e^{\epsilon \text{Subscript}[\tau, i] \text{Subscript}[a, i]} + O[\epsilon]^{k+1}]$;
 Subscript[t2b, i, \$k])

Subscript[t2b, i_] := Subscript[t2b, i, \$k]

In[]:= **t2b_i, 2**

$$\text{Out[]} = \mathbb{E} \left[a_i \alpha_i - \gamma b_j \tau_i, y_i \eta_i + x_i \xi_i, 1 + a_i \tau_i \epsilon + \frac{1}{2} a_i^2 \tau_i^2 \epsilon^2 + O[\epsilon]^3 \right]$$

In[]:= **t2b_i, 2**

$$\text{Out[]} = \mathbb{E} \left[a_i \alpha_i - \gamma b_j \tau_i, y_i \eta_i + x_i \xi_i, 1 + a_i \tau_i \epsilon + \frac{1}{2} a_i^2 \tau_i^2 \epsilon^2 + O[\epsilon]^3 \right]$$

In[]:= **t2b₁**

- Block:** Local variable specification {1} contains 1, which is not a symbol or an assignment to a symbol. +
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- General:** Further output of Block::lvsym will be suppressed during this calculation. +
- \$RecursionLimit:** Recursion depth of 1024 exceeded during evaluation of
{Block::lvsym, Block::lvsym, Block::lvsym, General::stop}. +
- \$RecursionLimit:** Recursion depth of 1024 exceeded during evaluation of
Message[Message::msgl, Hold[{Block::lvsym, Block::lvsym, Block::lvsym, General::stop}]]. +
- \$RecursionLimit:** Recursion depth of 1024 exceeded during evaluation of {\$RecursionLimit::reclim2}. +
- \$RecursionLimit:** Recursion depth of 1024 exceeded during evaluation of {OutputStream[+ ... Name: stdout Unique ID: 1]}. +
- General:** Further output of \$RecursionLimit::reclim2 will be suppressed during this calculation. +

Out[]:= **Hold [t2b_{1,3}]**

In[]:= **?? Subscript**

Subscript[x,y] is an object that formats as x_y .
 Subscript[x,y₁,y₂,...] formats as $x_{y_1,y_2,\dots}$ >>

Attributes [Subscript] = {NHoldRest}

$$\text{Subscript}[t2b, i_, 2] = \mathbb{E}[\text{Subscript}[a, i] \text{Subscript}[\alpha, i] - \gamma \text{Subscript}[b, j] \text{Subscript}[\tau, i], \text{Subscript}[y, i] \text{Subscript}[\eta, i] + \text{Subscript}[x, i] \text{Subscript}[\xi, i], 1 + \text{Subscript}[a, i] \text{Subscript}[\tau, i] \epsilon + \frac{1}{2} \text{Subscript}[a, i]^2 \text{Subscript}[\tau, i]^2 \epsilon^2 + O[\epsilon]^3]$$

$$\text{Subscript}[t2b, i_, \$k_] := (\text{Block}[\{i\}, \text{Subscript}[t2b, i_, \$k] = \mathbb{E}[\text{Subscript}[\alpha, i] \text{Subscript}[a, i] - \text{Subscript}[\tau, i] \gamma \text{Subscript}[b, j], \text{Subscript}[\xi, i] \text{Subscript}[x, i] + \text{Subscript}[\eta, i] \text{Subscript}[y, i], \epsilon^{\in \text{Subscript}[\tau, i] \text{Subscript}[a, i]} + O[\epsilon]^{\$k+1}]]]; \text{Subscript}[t2b, i, \$k])$$

$$\text{Subscript}[t2b, i_] := \text{Subscript}[t2b, i, \$k]$$