Tradler on Equivariant Holonomy via Iterated Integrals

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Equivariant Holonomy via Iterated Integrals.

A. Higher Hochschild Complexes.

Let $A = \Omega_{DR}(M) = \Omega(M)$

$$CH^{(n)}(A) = A^{\otimes(n+2)} \xrightarrow{H} \Omega(PM)$$

by

$$a_0 \otimes \dots \otimes a_{n+1} \mapsto \int_{\Delta^n = \{t_1 \le \dots \le t_n\}} a_0(0) a_1(t_1) \dots a_{n+1}(t_{n+1})\, dt_1 \dots dt_n$$

under this,

Hochschild $\longrightarrow$ d

shuffle $\longrightarrow$ $\wedge$

Face maps $D_j : CH^{(n+1)}(A) \to CH^{(n)}(A)$

$$D_j(a_0 \dots a_{n+2}) = a_0 \dots \otimes (a_j \wedge a_{j+1}) \otimes \dots \otimes a_{n+2}$$

Degeneracy maps $S_j : CH^{(n-1)}(A) \longrightarrow CH^{(n)}(A)$

$\rightsquigarrow$ add a $t_{j+\frac12}$ & use the form $1 \in \Omega^0$ for it.

A more abstract version

Define a category Δ:

$$\mathrm{obj}(\Delta) = \{\underline{0}, \underline{1}, \underline{2}, \underline{3} \dots$$

$$\underline{n} = \{0, \dots, n\}$$

$\mathrm{mor}(\underline{n}, \underline{m}) \ni f \iff f$ is a set map s.t.

$$i \leq j \Rightarrow f(i) \leq f(j)$$

e.g. $\sigma_3 : 0 \dots 6 \to 0 \dots 5$ by merging 4,3.

$\delta_2 : \underline{3} \to \underline{4}$ by skipping 2.

(δ_j & σ_j generate $\mathrm{mor}(\Delta)$)

Def A functor from $\Delta^{op} \to \mathcal{C}$ is a "simplicial object in $\mathcal{C}$". To specify such F we need

$$F(\underline{n}) \in \mathcal{C}$$

$$\& \quad F(\delta_j) : F(\underline{n}) \to F(\underline{n-1}) \quad \text{etc.}$$

Example $F : \Delta^{op} \to \mathrm{vect}$ for some fixed assoc alg A;

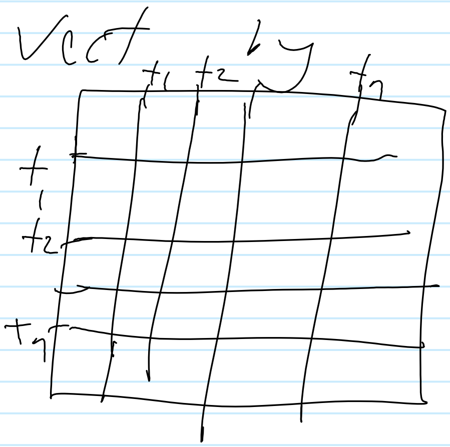
$$F(\underline{n}) = A^{\otimes(n+2)}$$

$\leadsto CH(A)$ is a simplicial v.s.

Example: Fix a commutative associative unital algebra A.

$F : \Delta^{op} \longrightarrow Vect$ by

$F(\underline{n}) =$



$\| \quad A^{\otimes (n+2)^2}$

Face & degeneracies are still defined.

Back to Iterated Integrals.

For any simplicial set $X : \Delta^{op} \longrightarrow set$ & any commutative assoc. algebra A

(A can be thought of as a functor $set \to VS$ by $X \to A^{\otimes X}$)

get $\Delta^{op} \xrightarrow{X} sets \xrightarrow{A} Vect.$

if $A = \Omega(M)$ get iterated integrals

$$\Delta^n \times Map(X, M) \longrightarrow M^X$$

$$\Omega(\Delta^n \times Map(I^?, M)) \xleftarrow{avd^*} (\Omega M)^{\otimes (n+2)^2}$$

$\downarrow$ integration

H

$\downarrow$ integration

$\Omega(\mathrm{Map}(X, M)) \xleftarrow{H}$

H is a map $CH^*(\Omega(M)) \longrightarrow \Omega(\mathrm{Map}(X, M))$

Holonomy In 1D iterated integrals give parallel transport.

In 2D get the same for a gerbe