Kapovitch's class, Wed Feb 25: Compactification of Teichmuller Space

February-25-15 11:14 AM

C: surface

Let S be the set of isotopy classes of simple closed curves in C

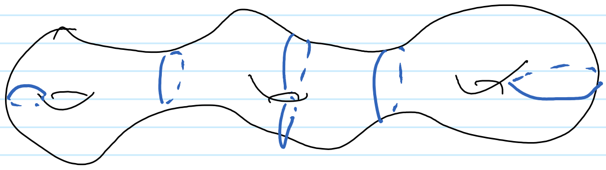
consider $\mathbb{R}_+^S$, $P\mathbb{R}_+^S = \mathbb{R}_+^S / x \sim \lambda x$

$\mathrm{Teich}(C) \hookrightarrow \mathbb{R}_+^S$

$MF(C) \hookrightarrow \mathbb{R}_+^S$ MF: Measured foliations.

$\mathrm{Teich}(C) = \left\{ \begin{matrix}\text{hyperbolic metrics} \\ \text{on } C\end{matrix} \right\} / \sim$

$m \sim m' \Leftrightarrow \exists$ isometry $C \to C$ s.t. φ is isotopic to id on C



$\to \mathrm{Teich} = \mathbb{R}^{3g-3} \times \mathbb{R}^{3g-3}$

The map $\mathrm{Teich} \hookrightarrow \mathbb{R}_+^S$ maps a curve α to the length of the unique geodesic isotopic to α.

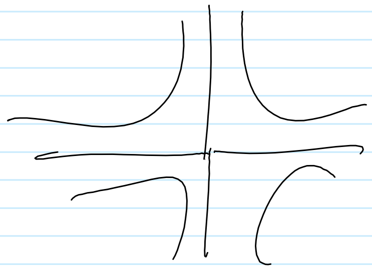
Measured Foliations on C:

<u>Def</u> a MF $\underbrace{\text{on}}_{(F,\mu)}$ C is a Foliation F along

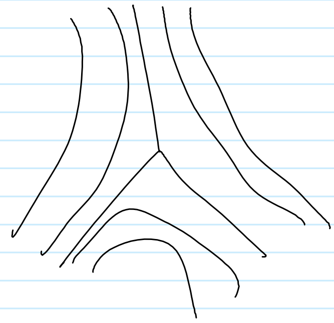
with a transverse measure μ: a measure on each arc transverse to the leafs, equivariant under shears along the foliation.

Regular points: Foliation given by dy, measure by $|dy|$.

Allowed singularities:



$z^2(dz)^2$



$z(dz)^2$

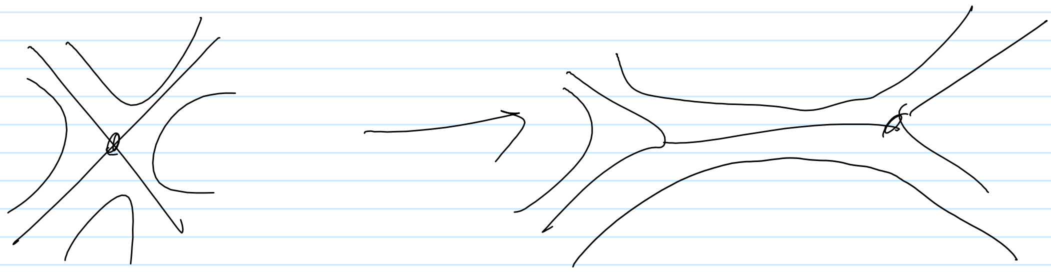
The map $i: MF(C) \hookrightarrow \mathbb{R}_+^S$

$$\int_\gamma (F,\mu) = \sup\left(\sum_i \mu(\alpha_i)\right)$$

where the α_i are mutually disjoint arcs in γ, transverse to F.

$$i(F,\mu)_s = \inf_{\gamma \in s} \int_\gamma (F,\mu)$$

Equivalence of MF's:



Teich is disjoint from MF in $\mathbb{R}_+^S$.

Pf By Poincare recurrence, there are always curves in C whose (F,μ) length is arbitrarily small. Not so for Teich.

Construction of γ: Teich $\to$ MF: