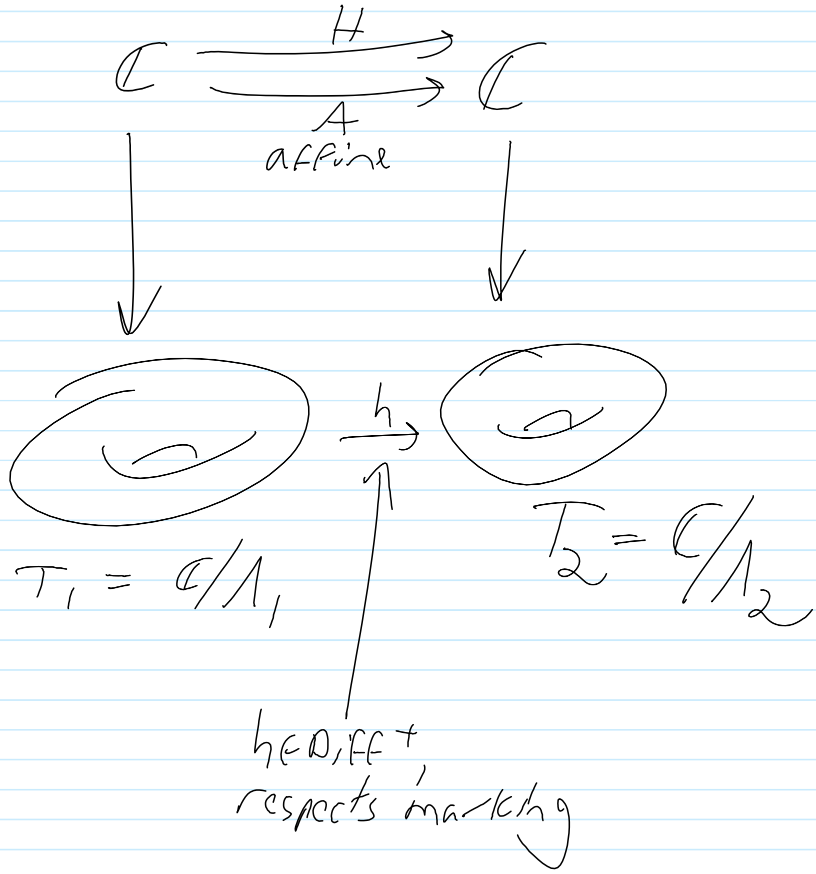
Yampolsky's Class, Wed Jan 14

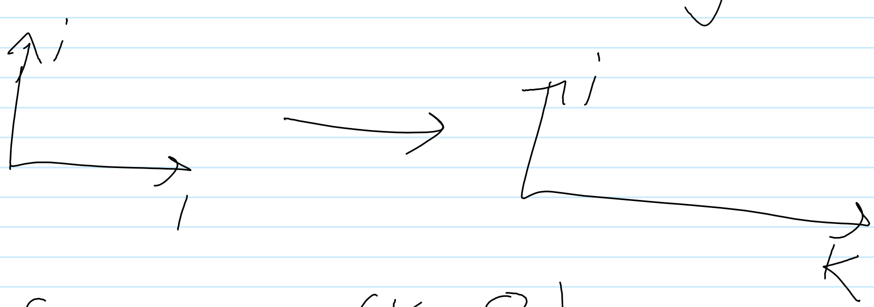
January-14-15 1:06 PM



$A: \mathbb{C} \to \mathbb{C}$ write $A = U \cdot P$

unitary positive def.

P:



So $P = \begin{pmatrix} K & 0 \\ 0 & 1 \end{pmatrix}$

the dilatation.

$\vdots$

<u>Exercise.</u> Verify

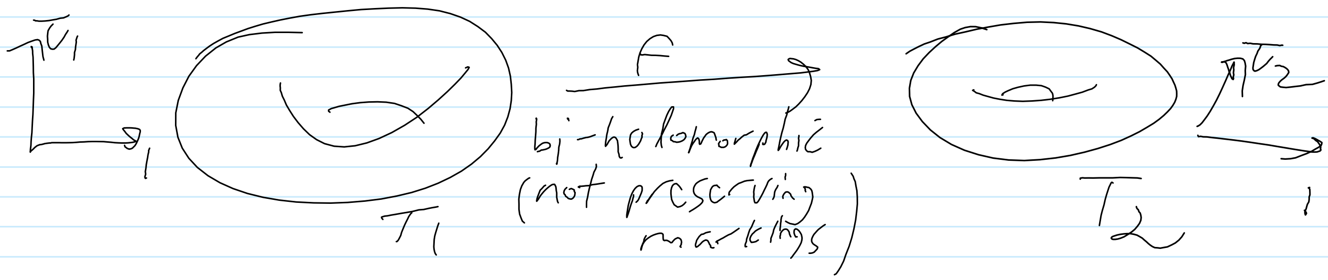
$\lim d(\tau, \tau') = \log \frac{|\tau' - \bar{\tau}| + |\tau' - \tau|}{\ldots}$

$$\mathrm{dist}_T(\tau, \tau') = \log \frac{|\tau' - \bar{\tau}| + |\tau' - \tau|}{|\tau' - \bar{\tau}| - |\tau' - \tau|}$$

--- the hyperbolic distance.

Moduli space of unmarked tori, $\mathcal{M}(\mathbb{T})$:

$$\begin{tikzcd} \mathbb{C} \arrow[rr, "0 \mapsto 0"', "F"', "\text{bi-holo}"'] \arrow[d] & & \mathbb{C} \arrow[d] \\ \, & & \, \end{tikzcd}$$



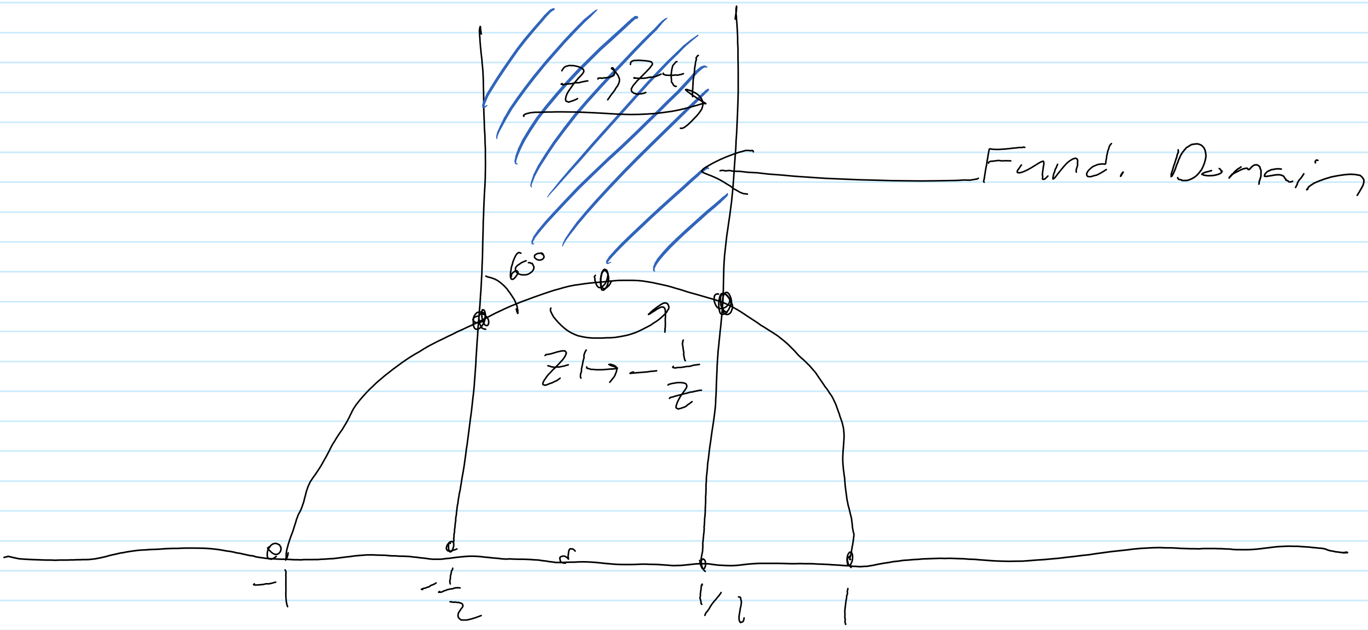
So $F(z) = \alpha z$, $\alpha \neq 0$.

$$F(\tau_1) = \alpha \tau_1 = a\tau_2 + b$$
$$F(1) = \alpha = c\tau_2 + d$$

So
$$\tau_1 = \frac{a\tau_2 + b}{c\tau_2 + d} \quad a, b, c, d \in \mathbb{Z}$$

So $T = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in PSL(2, \mathbb{Z})$ [inverse must have same properties]

The modular group of the torus



So $M(\Gamma) \cong \mathbb{C}$