Kapovitch's class, Wed Jan 28: Rafi on hyperbolic trinions

January-28-15 11:09 AM

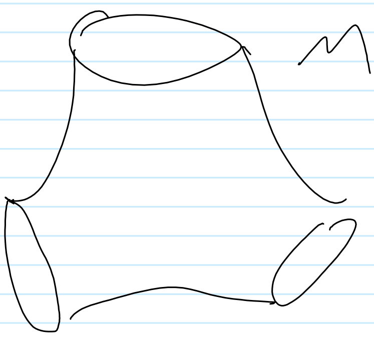
Thm (Thurston, defs later) Let M be a compact 3-manifold, Haken, $\pi_1(M)$ not Abelian, irreducible, atoroidal, then the interior of M admits a complete hyperbolic structure of finite volume.

Haken: $\pi_1(M)$ contain $\pi_1(S)$, S a surface $\chi(S)<0$

Atoroidal: Every $\mathbb{Z}\times\mathbb{Z} \hookrightarrow \pi_1(M)$ is conjugate to $\pi_1(\partial M)$

Irreducible: $\pi_2(M)=0$.

A bit on 2D hyperbolic geometry:



$\pi_1(M)=F_2$

Want injections

$F_2 \longrightarrow Isom(\mathbb{H}_2)$
$= Sl(2,\mathbb{R})$
$= \begin{bmatrix} a & b \\ c & d \end{bmatrix}$
$z \mapsto \frac{az+b}{cz+d}$

Classification of isometries by fixed points:

$$\frac{az+b}{cz+d} = z$$

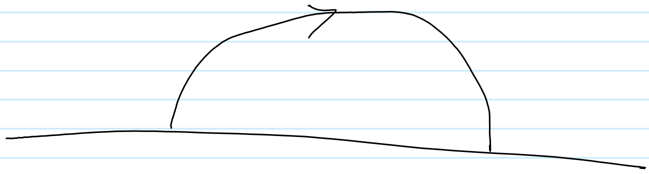
$\Longleftrightarrow cz^2+(d-a)z-b=0$

Possibilities:

$\iff cz^2+(d-a)z-b=0$

Possibilities:

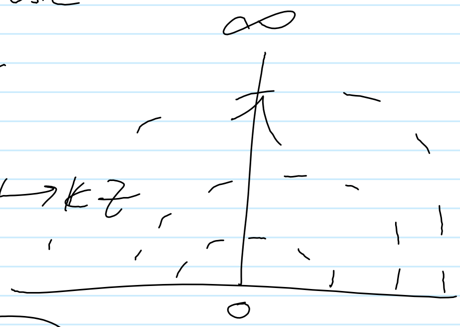
2 real sol'ns:



Translation along a geodesic

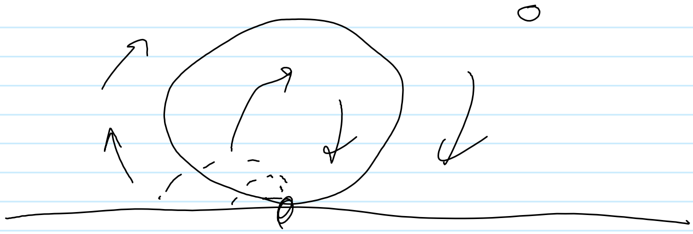
"loxodromic action"

conjugate to $z \mapsto kz$

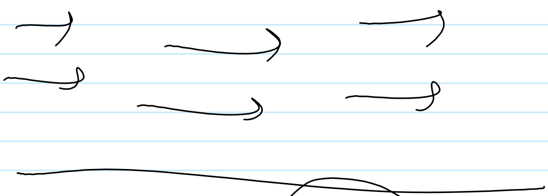


1 real sol'n

"parabolic

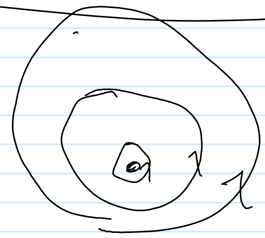


conjugate to $z \mapsto z+c$



Complex conjugate f.p.:

rotations



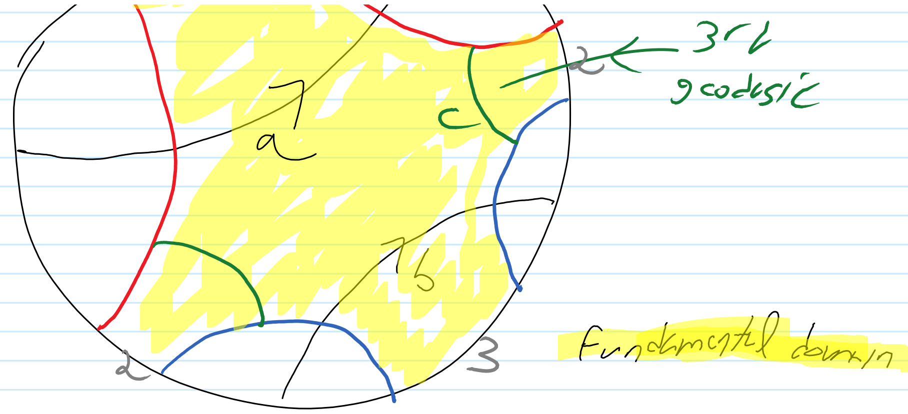
conjugate to $z \mapsto e^{i\theta}z$ in the disk model.

"elliptic elements"

In a group action on $\mathbb{H}^2$ for which the quotient is a mfld there can be no elliptic elements.

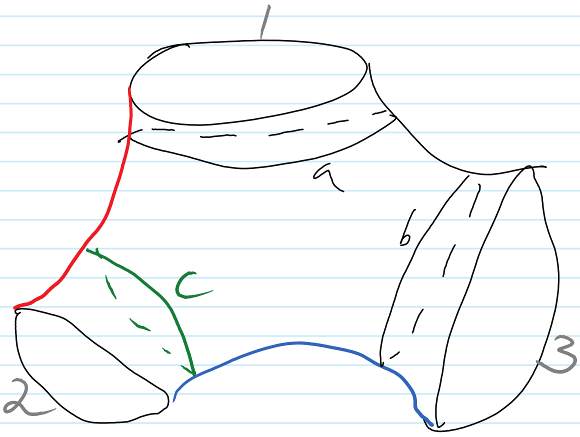
$F_2 = \langle a, b \rangle$

This is always a free group!
(if red/blue curves do not intersect)



Pf: "the ping-pong argument": ...

Easy to see that the quotient is



Limit set

* $S^1 \setminus$ (Domain of discontinuity)

* $\Lambda =$ given $x \in \mathbb{H}^2$,

$$\Lambda = \overline{F_2 x} \cap \partial \mathbb{H}^2$$

Λ is the set on infinite sequences in $a, \bar{a}, b, \bar{b}$ w/ no $a\bar{a}, b\bar{b}, \bar{a}a, \bar{b}b$.

$AH(M)$: all hyperbolic structures in the interior of M, with its algebraic topology, coming from

$$F_2 \to \mathrm{Isom}(\mathbb{H}) \Big/ \text{conjugation}.$$

In our case, $AH(M) \simeq \mathbb{R}_+^3$ (the lengths of the geodesics)