Kapovitch's class, Mon Jan 12: Towards the proof of the h-cobordism theorem, Whitney's trick

January-12-15 11:08 AM

Let (W^{n+1}, M_0, M_1), $n \geq 5$, $\pi_1(W)=0$ be an h-cobordism

$M_i \hookrightarrow W$ hom. equiv.

then $W \cong M_0 \times I$

step 1: Put a self-indexing Morse function on W, s.t.

$$f|_{M_0}=0 \qquad f|_{M_1}=n+1$$

wish to get rid of all crit. pt.

We know $H_i(W, M_0)=0 \ \forall i$

& Morse function gives a complex computing H_i

W is a "relative CW complex" — start w/ M_0 & attach handles.

chain complex:

$\mathcal{C}: \ C_i \xrightarrow{\partial} C_{i-1} \xrightarrow{\partial} \dots$

C_i is generated by crit. pts of index i

$\mathcal{C}$ is exact. Let $Z_i \subset C_i$ be $\ker \partial_i$.

$= \mathrm{im}\, \partial_{i+1}$

Pick bases $z^i_1, \dots z^i_l$ of Z_i

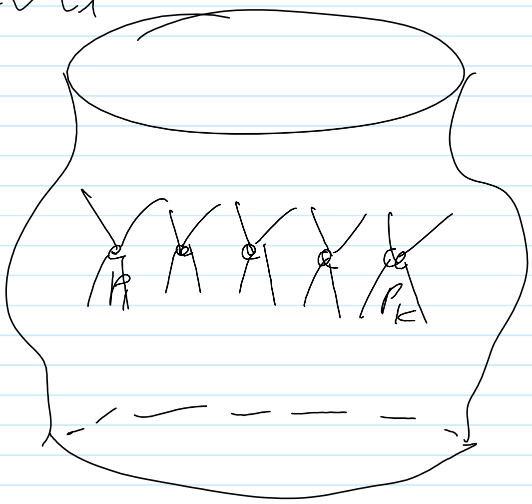
$z^{i+1}_1 \dots z^{i+1}_k$ of Z_{i+1}

$z^i_j = \partial B^{i+1}_j$

w.r.t. these bases, $\partial = \begin{pmatrix} 1 & & & \heartsuit \\ & 1 & & \\ & & 1 & \\ 0 & & & 0 \\ & & & & 0 \end{pmatrix}$

<u>Lemma</u> Can deform f to have the same crit. pts of same indices s.t. these base pts represent out bases.

<u>Idea</u> work within a nbd of the i-th level

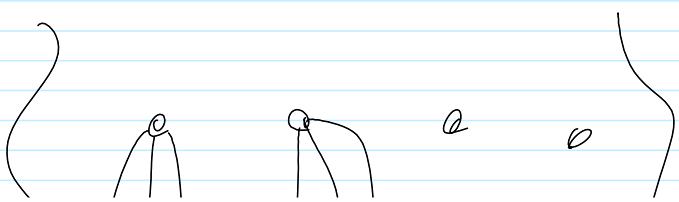


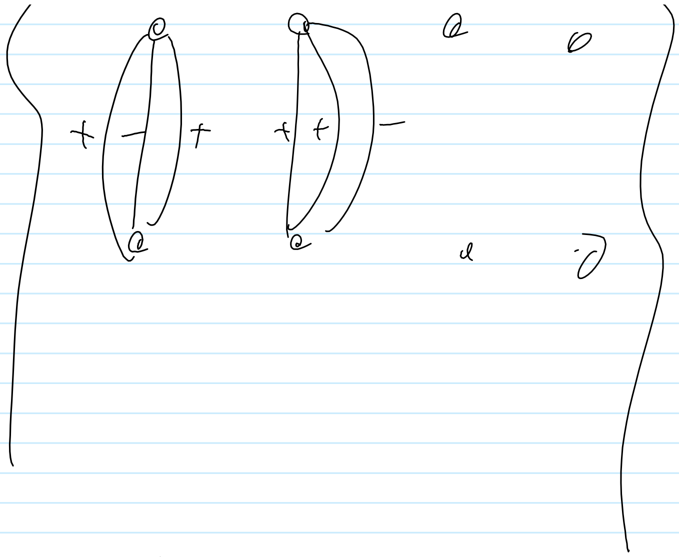
Enough to go

$[P_1] \dots [P_k] \longrightarrow$

$[P_1] + [P_2], [P_2], [P_3], \dots$

Changing to this function, we have





Now cancel points in pairs. — but first one has to eliminate pairs of opposite signs & equal ends. — this is:

<u>Lemma</u> (Whitney): Let $M_1^k, M_2^l \subset W^n$ (oriented, connected), $k+l \geq 3$, $k+l=n$, $M_1 \pitchfork M_2$, $\pi_1(W)=0$, suppose $p_1, p_2 \in M_1 \cap M_2$ have opposite signs the $\exists$ isotopy $h_t : W \times I \to I$ s.t.

$h_0 = Id$ &

$$h_1(M_2) \cap M_1 = (M_1 \cap M_2) \setminus \{p_1, p_2\}$$