

Kapovitch's class, Fri Jan 9: Alexander's trick

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From Morse Theory: $F: M^n \rightarrow \mathbb{R}$. If F has
 2 critical points, $M^n \cong S^n$. homeomorphism
only.

First - M^n is two discs glued along a
 diffeomorphism of the bndry "a twisted
 sphere".

Alexander's trick:

Lemma: a homeomorphism $F: S^{n-1} \rightarrow S^{n-1}$
 extends to a homeomorphism of the disk

$$\bar{F}: D^n \rightarrow D^n \text{ s.t. } \bar{F}|_{S^{n-1}} = F.$$

This lemma cannot be promoted to
 diffeomorphisms! (\Leftrightarrow Milnor's exotic
 spheres)

... Bounds on Betti numbers by numbers
 of critical points.