Kapovitch's class, Fri Jan 23:

January-23-15 11:09 AM

Given X^n, CW-complex

$S(X) = \{f : M^n \longrightarrow X\}/\sim$

M^n a closed manifold, f a homotopy equiv.

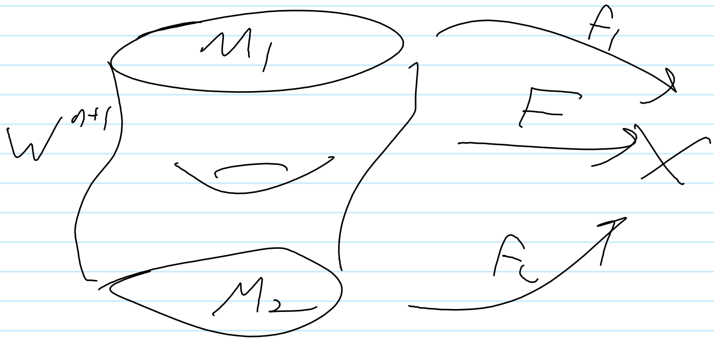
$$\begin{tikzcd} f_1 : M_1 \arrow[d, "\exists\varphi"'] \arrow[r, "f_1"] & X \\ f_2 : M_2 \arrow[ur, "f_2"'] & \end{tikzcd}$$

The equiv. relation $\sim$

enough $f_2 \circ \varphi \sim f_1$

By s-cobordism

$(f_1 \sim f_2) \iff \exists$ s-cobordism W^{n+1} s.t.



We want to understand $S(X)$

(If X is a sphere, $S(X)$ counts smooth structures on it)

For $S(X) \neq \emptyset$, X must be a poincare duality complex: $H^*(X, \mathbb{Z})$ satisfies P.D. for some n:

$H_n(X, \mathbb{Z}) = \mathbb{Z}$ for some n,

$H_k(X, \mathbb{Z}) \cong H^{n-k}(X, \mathbb{Z})$

Further conditions: If M^n is a manifold,

$\nu^{st}(M)$ = stable normal vect. bundle on M

e.g. $\nu^{st}(S^n)$ = trivial

note $\nu(M) \oplus T(M) \sim$ trivial

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$$\begin{tikzcd} & G/O \arrow[d] \\ X \arrow[r] \arrow[dr] & BO \arrow[d] \\ & BG \end{tikzcd}$$

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