Commutators talk post-mortem

January-16-15 6:37 PM

Definitions and Very Simple Examples

Definition. The commutator of two elements x and y in a group G is $[x, y] := xyx^{-1}y^{-1}$.

Example 1. In S_3, $[(12), (23)] = (12)(23)(12)^{-1}(23)^{-1} = (123)$ and in general in $S_{\geq 3}$, $[(ij),(jk)]=(ijk)$.

Example 2. In $S_{\geq 4}$, $[(ijk), (jkl)] = (ijk)(jkl)(ijk)^{-1}(jkl)^{-1} = (il)(jk)$.

Example 3. In $S_{\geq 5}$, $[(ijk), (klm)] = (ijk)(klm)(ijk)^{-1}(klm)^{-1} = (jkm)$.

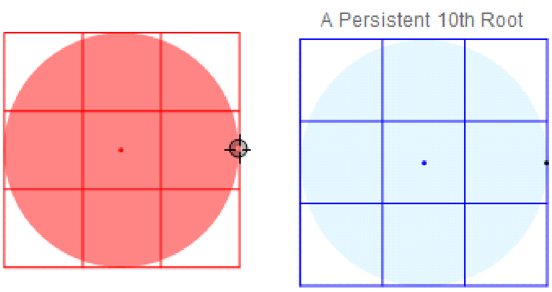
I should have added some de-commutations:

(123) = [(412),(253)] = [[(341),(152)],[(125),(543)]] = [[[(234),(451)],[(315),(542)]],[[(312),(245)],[(154),(423)]]] = [
 [[[(123),(354)],[(245),(531)]],[[(231),(145)],[(154),(432)]]],
 [[[(431),(152)],[(124),(435)]],[[(215),(534)],[(142),(253)]]]
]

as in Decommute.nb.

The 10th Root

Out[18]=



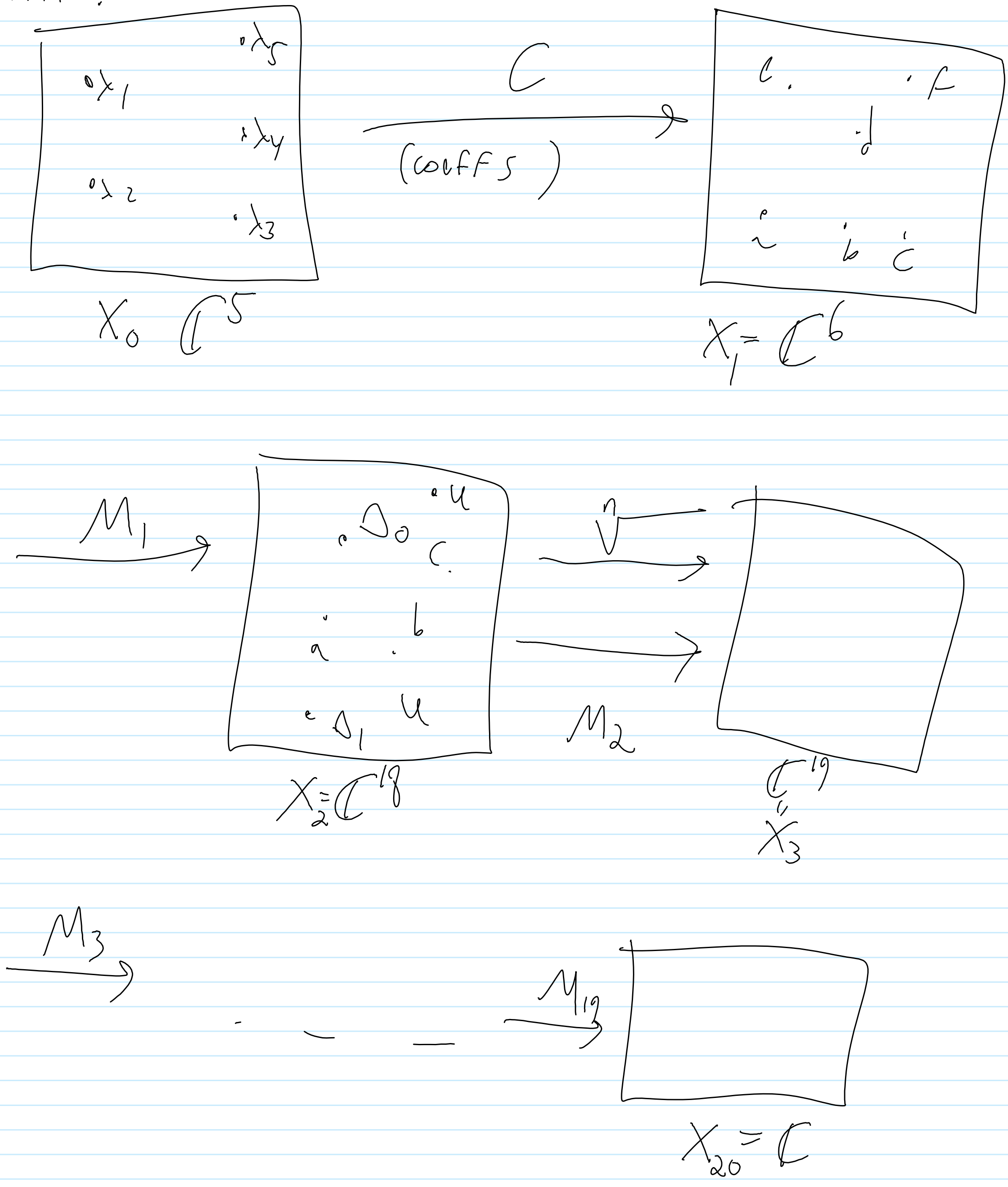
The Key Point

A

If a closed path is the comuutator of two closed paths, its persistent root is a closed path.

A: I should have put.
The root of a closed path is not necessarily a close path, yet .....

I should have shown more detail at the "main point".



If $\alpha, \beta_1, \beta_2, \gamma_{11}, \gamma_{12}, \gamma_{21}, \gamma_{22}, \delta_{111} \ldots$

are paths in X_0 that induce permutation of the roots and

$$\alpha = [\beta_1, \beta_2]$$

$$\beta_1 = [\gamma_{11}, \gamma_{12}] \qquad \beta_2 = [\gamma_{21}, \gamma_{22}]$$

etc., Then

$\alpha // C // M_0 // M_1 // M_2 // M_3 \ldots // M_{19}$ is a closed path.

none is closed

all are closed

δ's are closed

δ's may open, γ's are closed.

etc.

More "Commutators" comments:

1. After the deg 3 "testing", add: The phenomena observed, that the output always follows one of the λ's, is provable.

2. Unlabel the λ's on the input panes, but not in $\prod(x-\lambda_i)=1$.

3. Add advantages/disadvantages.