Commutators

December-13-14 3:52 PM

The commutator of two elements x and y in a group G is xyx^{-1}y^{-1} - x followed by y followed by the inverse of x followed by the inverse of y. In my talk I will tell you how commutators are related to the following four riddles:

1. Can you send a secure message to a friend you have never met before, using a messenger you do not trust?
2. Can you hang a picture on a string on the wall using n nails, so that if you remove any one of them the picture will fall?
3. Can you draw an n-component link (a knot made of n non-intersecting circles) so that if you remove any one of those n components, the remaining n-1 will fall apart?
4. Can you solve the quintic in radicals? Namely, is there a formula for the zeros of a degree 5 polynomial in terms of its coefficients, using only the operations on a scientific calculator?

If you want to think hard about these riddles before the talk, see their detailed formulations at …

Ideas.

- Quote the Princess Bride left-right hand story as an explanation for the delays.
- Quote the "Yes, PM" quantum politician story as an explanation of holonomy.

Sketch.

1. Definition of commutators, [(ijk),(klm)].
2. Solve the secure message problem.
3. Solve the hanging problem.
4. Skip the solution of the Brunnian problem yet show a picture of a Borromean link.
5. Explain the quintic problem.
6. The princess-bride quote. (YouTube?)
7. Explain complex powers.
8. Explain complex roots, show an animation, discuss "Yes, PM" (YouTube?) and path lifting.
9. Root functions lift commutators of closed paths to closed paths.
10. Solve the quintic problem with animations.

Some commutators in S_4:

$$[(12),(123)] = (12)(123)(21)(321) = (123)$$

$$[(12),(234)] = (12)(234)(21)(432) = (124)$$

$$[(12),(13)(24)] = (12)(13)(24)(12)(13)(24) = (12)(34)$$