

Treehouse Talks, Friday October 17, 2014, Beeton Auditorium, Toronto Reference Library, 789 Yonge Street, 6:30PM

**Abstract.** My goal is to get you hooked, captured and unreleased until you find all 17 in real life, around you.

We all know know that the plane can be filled in different periodic manners: floor tiles are often square but sometimes hexagonal, bricks are often laid in an interlaced pattern, fabrics often carry interesting patterns. A little less known is that there are precisely 17 symmetry patterns for tiling the plane; not one more, not one less. It is even less known how easy these 17 are to identify in the patterns around you, how fun it is, how common some are, and how rare some others seem to be.

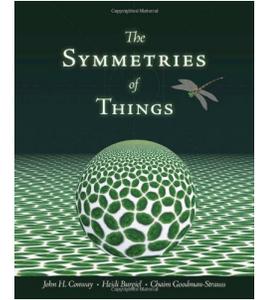
**Gotta catch 'em all!**

**Reading.** An excellent book on the subject is *The Symmetries of Things* by J. H. Conway, H. Burgiel, and C. Goodman-Strauss, CRC Press, 2008.

Another nice text is *Classical Tessellations and Three-Manifolds* by J. M. Montesinos, Springer-Verlag, 1987.

**Question.** In what ways can you make \$2 change, using coins denominated  $\frac{1}{2}$ ,  $\frac{2}{3}$ ,  $\frac{3}{4}$ ,  $\frac{4}{5}$ ,  $\frac{5}{6}$ , etc.?

**Answer.**  $2 = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{3}{4} + \frac{3}{4} + \frac{1}{2} = \frac{5}{6} + \frac{2}{3} + \frac{1}{2}$ , and that's it.



Video, handout, links at [drorbn.net/Treehouse](http://drorbn.net/Treehouse)

### The Basic Features.

3



rotation only

§



rotation-reflection

M



free mirror-reflection

G



free glide-reflection

**Gotta catch 'em all!**

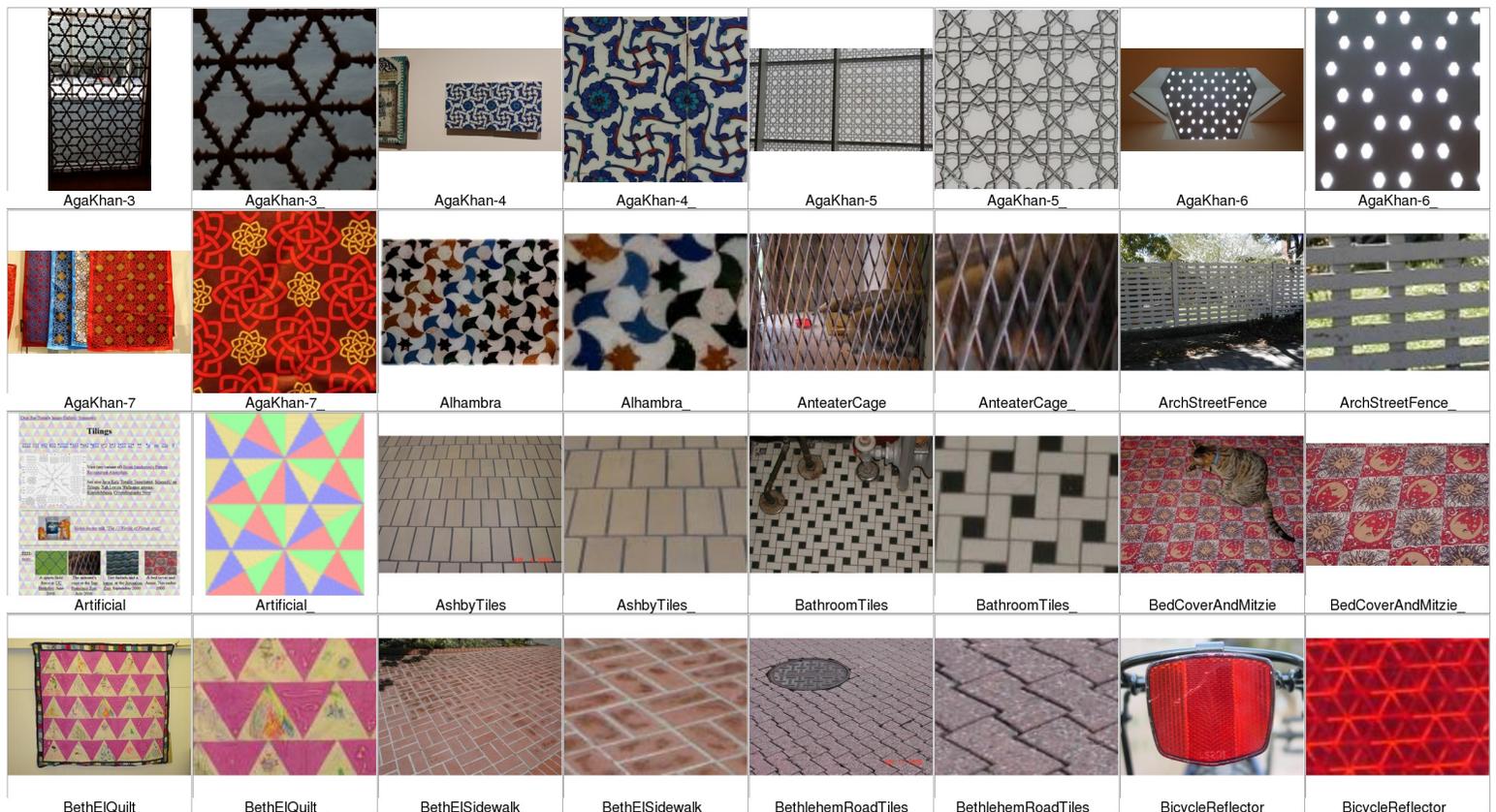


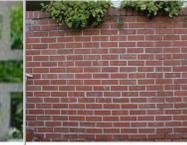
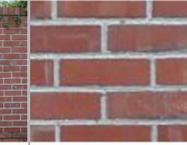
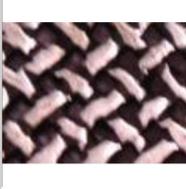
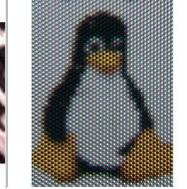
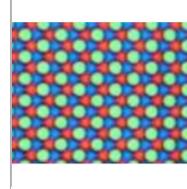
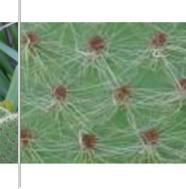
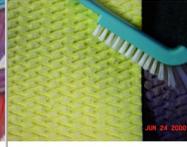
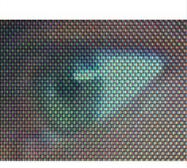
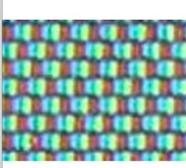
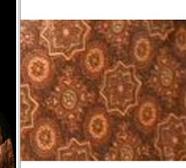
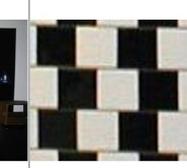
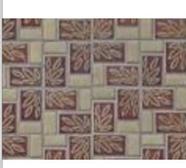
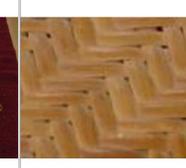
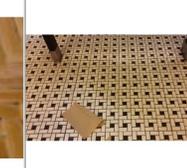
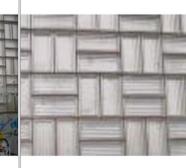
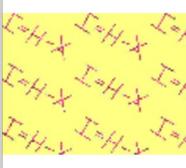
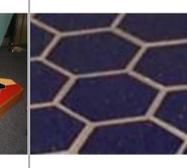

**Theorem.** There are precisely 17 patterns with which to tile the plane, no more, no less. They are all made of combinations of the 10 basic features, 2, 3, 4, 6, §, §, §, §, M, and G, as follows:

✓	Dror's	Conway's	crystallo-graphic	✓	Dror's	Conway's	crystallo-graphic
	2222	2222	p2		3§	3*3	p31m
	333	333	p3		2§§	2*22	cmm
	442	442	p4		22M	22*	pmg
	632	632	p6		MM	**	pm
	§§§§	*2222	pmm		MG	*o	cm
	§§§	*333	p3m1		GG	oo	pg
	44§	*442	p4m		22G	22o	pgg
	§§§	*632	p6m		∅	0	p1
	4§	4*2	p4g				

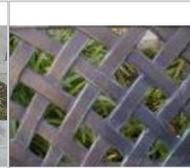
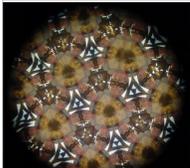
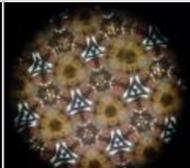
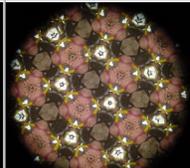
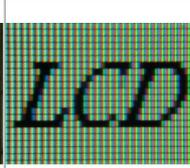
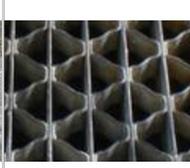
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**Tilings worksheet.** Classify the following pictures according to the following possibilities: 2222=2222, 333=333, 442=442, 632=632, §§§§=\*2222, §§§=\*333, 44§=\*442, §§§=\*632, 4§=4\*2, 3§=3\*3, 2§§=2\*22, 22M=22\*, MM=\*\*, MG=\*o, GG=oo, 22G=22o, and ∅=0 (the pictures come in {context, pattern} pairs).

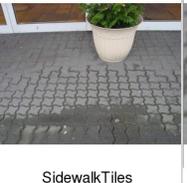
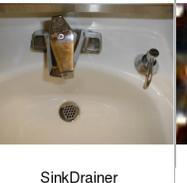
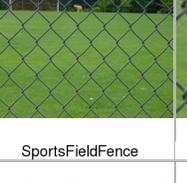
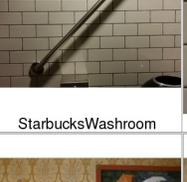
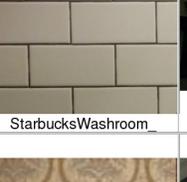
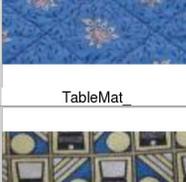
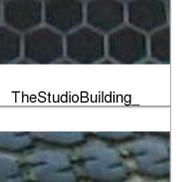
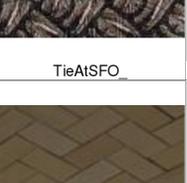
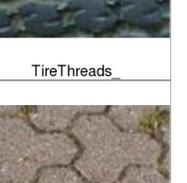
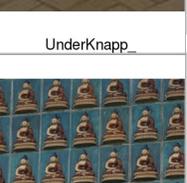
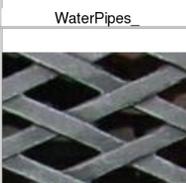


							
BWICarpet	BWICarpet_	BrakePedal	BrakePedal_	BrickPartition	BrickPartition_	BrickWall	BrickWall_
							
BriennesNecklace	BriennesNecklace_	CRTPixels	CRTPixels_	Cactus	Cactus_	CaesareaMosaic	CaesareaMosaic_
							
ChildsDrawing	ChildsDrawing_	CleaningCloth	CleaningCloth_	ClimbingNet	ClimbingNet_	CollegeStreetStarbucks	CollegeStreetStarbucks_
							
DSC-S70Pixels	DSC-S70Pixels_	DamascusTable	DamascusTable_	EowynRieke	EowynRieke_	ExploratoriumBathroom	ExploratoriumBathroom_
							
ExternalWallTiles	ExternalWallTiles_	FRACeiling	FRACeiling_	FloorTiles@Starbucks	FloorTiles@Starbucks_	FlowerBasket	FlowerBasket_
							
FlowersWallpaper	FlowersWallpaper_	FootstepInSand	FootstepInSand_	Footstool	Footstool_	FransRestaurant	FransRestaurant_
							
FrontGrill	FrontGrill_	FrozenFence	FrozenFence_	FutureShop	FutureShop_	GarageDoor	GarageDoor_
							
GatedDoorway	GatedDoorway_	GionTiles	GionTiles_	GlassWall	GlassWall_	HNLWall	HNLWall_
							
HTMLBackground	HTMLBackground_	HarHaMenuhotWall	HarHaMenuhotWall_	HebrewAcademy	HebrewAcademy_	HexGame	HexGame_

Gotta!

							
HotPad	HotPad_	IncentBurner	IncentBurner_	JCCBench	JCCBench_	JapanesePlate	JapanesePlate_
							
Kaleidoscope-2	Kaleidoscope-2_	Kaleidoscope	Kaleidoscope_	KauffmanTie	KauffmansTie	KleinBottleSofa	KleinBottleSofa_
							
KuramaguchiStation	KuramaguchiStation_	LCDPixels	LCDPixels_	LakeMerritBuilding	LakeMerritEnlarged	LalushkaFloorTiles	LalushkaFloorTiles_
							
Lego	Lego_	LittleItalyChair	LittleItalyChair_	MSRIBathroom	MSRIBathroom_	MayasTablecloth	MayasTablecloth_
							
MetalGrating	MetalGrating_	MoroccanTabletop	MoroccanTabletop_	NewGeometrics	NewGeometrics_	NicholasKZieveShirt	NicholasKZieveShirt_
							
NonSlid	NonSlid_	OldRadio	OldRadio_	OutsideWall	OutsideWall_	OxfordStreetEntryway	OxfordStreetEntryway_
							
PaperTowel	PaperTowel_	ParkingLotTiles	ParkingLotTiles_	PentagonalTiles	PentagonalTiles_	PicnicBasket	PicnicBasket_
							
PizzaUnoFloorTiles	PizzaUnoFloorTiles_	PlaygroundFence	PlaygroundFence_	PotholeCover	PotholeCover_	ProudSymbol	ProudSymbol_
							
PrudentialBuilding	PrudentialBuilding_	QueenOfHearts	QueenOfHearts_	RainbowMarketSquare	RainbowMarketSquare_	RenaissanceHotel	RenaissanceHotel_

Gotta!

							
ReznikoffShirt	ReznikoffShirt_	RicksCafeKensington	RicksCafeKensington_	SFGlassWindow	SFGlassWindow_	SableSpeaker	SableSpeaker_
							
SafewayCart	SafewayCart_	Shredder	Shredder_	SidewalkTiles	SidewalkTiles_	SinkDrainer	SinkDrainer_
							
SisInLawShirt	SisInLawShirt_	SpadinaSubway-1	SpadinaSubway-1_	SpadinaSubway-2	SpadinaSubway-2_	SportsFieldFence	SportsFieldFence_
							
StClairWSubway-1	StClairWSubway-1_	StClairWSubway-2	StClairWSubway-2_	StarbucksWashroom	StarbucksWashroom_	SugarPacket	SugarPacket_
							
TableMat	TableMat_	TanHall	TanHall_	ThaiWallpaper	ThaiWallpaper_	TheStudioBuilding	TheStudioBuilding_
							
TieAtFrys	TieAtFrys_	TieAtSFO	TieAtSFO_	TieAtSears	TieAtSears_	TireThreads	TireThreads_
							
ToiletPaper	ToiletPaper_	UnderKnapp	UnderKnapp_	WalnutStreetBuilding	WalnutStreetBuilding_	WalthamParkTiles	WalthamParkTiles_
							
WaterPipes	WaterPipes_	WhiteDagoba	WhiteDagoba_	WineBottles	WineBottles_	WovenBasket	WovenBasket_
WovenBenchBack	WovenBenchBack_	Yeshua	Yeshua_				

Gotta!