

Pensieve header: Understanding Rot120 / Triality.

Rot120 / Triality

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SetDirectory["C:\\drorbn\\AcademicPensieve\\2013-11\\DoubleTree"];
<< "../2013-05/FreeLie.m"
<< "muCalculus.m"
<< "../2013-10/WKOSolution8-1.m"
α = MakeLieSeries[{"1", "2"}, αs]; β = MakeLieSeries[{"1", "2"}, βs];
γ = MakeCWSeries[{"1", "2"}, γs];
V = M[{1 → α, 2 → β}, γ];
κs[d_, 1] := If[OddQ[d], 0, κs[d]]; κ = MakeCWSeries[{"1"}, κs];
Unprotect[C]; C = M[{1 → MakeLieSeries[0]}, κ];
$SeriesShowDegree = 5; $SeriesCompareDegree = 6;
{V, C // dc[1]}

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$$\left\{ M \left[\left\{ 1 \rightarrow \text{LS} \left[0, -\frac{\overline{12}}{24}, 0, \frac{\overline{71112}}{5760} - \frac{\overline{71122}}{5760} + \frac{\overline{12222}}{1440}, 0 \right], 2 \rightarrow \text{LS} \left[\frac{\overline{1}}{2}, -\frac{\overline{12}}{12}, 0, \right. \right. \right. \\
 \left. \left. \left. \frac{\overline{11112}}{5760} - \frac{1}{720} \frac{\overline{1122}}{1222} + \frac{1}{720} \frac{\overline{1222}}{22}, -\frac{\overline{11112}}{7680} + \frac{\overline{11122}}{3840} - \frac{\overline{11212}}{6912} \right] \right\}, \right. \\
 \left. \text{CWS} \left[0, -\frac{\overline{12}}{48}, 0, \frac{\overline{1112}}{2880} + \frac{\overline{1122}}{2880} + \frac{\overline{1212}}{5760} + \frac{\overline{1222}}{2880}, 0 \right], \right. \\
 \left. M \left[\{\}, \text{CWS} \left[0, -\frac{\overline{11}}{96}, 0, \frac{\overline{1111}}{11520}, 0 \right] \right] \right\}$$

V // Rot120

$$M \left[\left\{ 1 \rightarrow \text{LS} \left[-\frac{\overline{2}}{2}, \frac{\overline{12}}{12}, 0, \right. \right. \right. \\
 \left. \left. \left. -\frac{1}{720} \frac{\overline{11112}}{1122} + \frac{1}{720} \frac{\overline{1122}}{1222} - \frac{\overline{12222}}{5760}, -\frac{\overline{112222}}{3840} - \frac{\overline{121222}}{8640} + \frac{\overline{122222}}{7680} \right] \right\}, \right. \\
 \left. 2 \rightarrow \text{LS} \left[-\frac{\overline{2}}{2}, \frac{\overline{12}}{24}, -\frac{1}{96} \frac{\overline{1122}}{1222}, -\frac{\overline{11112}}{1440} + \frac{\overline{71122}}{5760} - \frac{\overline{12222}}{2880}, \right. \right. \\
 \left. \left. \frac{\overline{111222}}{5760} - \frac{\overline{7112222}}{23040} - \frac{13 \overline{121222}}{34560} - \frac{\overline{112122}}{5760} + \frac{\overline{7122222}}{23040} \right] \right\}, \\
 \left. \text{CWS} \left[-\frac{\overline{2}}{2}, \frac{\overline{12}}{48} + \frac{\overline{22}}{48}, 0, -\frac{\overline{1112}}{2880} - \frac{\overline{1122}}{2880} - \frac{\overline{1212}}{5760} - \frac{\overline{1222}}{2880} - \frac{\overline{2222}}{5760}, 0 \right] \right\}$$

V ≡ (V // Rot120 // Rot120 // Rot120)

True

$$\mathbf{quo} = (\mathbf{v} // \mathbf{dA}[1, 2]) ** (\mathbf{v} // \mathbf{Rot120})$$

$$\begin{aligned} M \left[\left\{ 1 \rightarrow \text{LS} \left[-\frac{\overline{2}}{2}, \frac{\overline{12}}{8}, -\frac{1}{48} \overline{112} + \frac{1}{96} \overline{122}, \frac{1}{384} \overline{1112} - \frac{1}{384} \overline{1122}, \right. \right. \\ \left. \left. -\frac{\overline{11112}}{3840} + \frac{\overline{111222}}{2560} - \frac{\overline{112222}}{23040} - \frac{\overline{121222}}{5760} - \frac{\overline{112122}}{7680} - \frac{\overline{122222}}{23040} \right], \right. \\ \left. 2 \rightarrow \text{LS} \left[-\frac{\overline{1}}{2} - \frac{\overline{2}}{2}, 0, 0, 0, 0 \right], \text{CWS} \left[-\frac{\overline{2}}{2}, \frac{\overline{12}}{24} + \frac{\overline{22}}{48}, 0, \right. \right. \\ \left. \left. -\frac{\overline{1112}}{1440} - \frac{\overline{1122}}{1440} - \frac{\overline{1212}}{2880} - \frac{\overline{1222}}{1440} - \frac{\overline{2222}}{5760}, 0 \right] \right] \end{aligned}$$

$$\mathbf{v} ** (\mathbf{R}^+[1, 2] // \mathbf{dP}[12, 3])$$

$$\begin{aligned} M \left[\left\{ 1 \rightarrow \text{LS} \left[0, -\frac{\overline{12}}{24}, 0, \frac{7}{5760} \overline{1112} - \frac{7}{5760} \overline{1122} + \frac{\overline{1222}}{1440}, 0 \right], 2 \rightarrow \right. \\ \left. \text{LS} \left[\frac{\overline{1}}{2}, -\frac{\overline{12}}{12}, 0, \frac{\overline{1112}}{5760} - \frac{1}{720} \overline{1122} + \frac{1}{720} \overline{1222}, -\frac{\overline{11112}}{7680} + \frac{\overline{111222}}{3840} - \frac{\overline{112122}}{6912} \right], \right. \\ \left. 3 \rightarrow \text{LS} [\overline{1} + \overline{2}, 0, 0, 0, 0], \text{CWS} \left[0, -\frac{\overline{12}}{48}, 0, \frac{\overline{1112}}{2880} + \frac{\overline{1122}}{2880} + \frac{\overline{1212}}{5760} + \frac{\overline{1222}}{2880}, 0 \right] \right] \end{aligned}$$

$$((\mathbf{R}^+[1, 2] // \mathbf{dP}[12, 3]) ** \mathbf{v}) \equiv (\mathbf{v} ** (\mathbf{R}^+[1, 2] // \mathbf{dP}[12, 3]))$$

$$\frac{\langle 12 \rangle}{2} = 0$$

$$((\mathbf{R}^+[1, 2] // \mathbf{dP}[12, 3]) ** \mathbf{quo}) \equiv (\mathbf{quo} ** (\mathbf{R}^+[1, 2] // \mathbf{dP}[12, 3]))$$

True

$$\mathbf{e}[1, 2, -1/2]$$

$$\begin{aligned} M \left[\left\{ 1 \rightarrow \text{LS} \left[-\frac{\overline{2}}{4}, \frac{\overline{12}}{32}, -\frac{1}{384} \overline{112} + \frac{1}{768} \overline{122}, \frac{1}{6144} \overline{1112} - \frac{1}{6144} \overline{1122}, \right. \right. \\ \left. \left. -\frac{\overline{11112}}{122880} + \frac{\overline{111222}}{81920} - \frac{\overline{112222}}{737280} - \frac{\overline{121222}}{184320} - \frac{\overline{112122}}{245760} - \frac{\overline{122222}}{737280} \right], \right. \\ \left. 2 \rightarrow \text{LS} \left[-\frac{\overline{1}}{4}, -\frac{\overline{12}}{32}, \frac{1}{768} \overline{112} - \frac{1}{384} \overline{122}, \frac{1}{6144} \overline{1122} - \frac{1}{6144} \overline{1222}, -\frac{\overline{11112}}{737280} - \right. \right. \\ \left. \left. \frac{\overline{111222}}{737280} + \frac{\overline{112222}}{81920} + \frac{\overline{121222}}{122880} - \frac{\overline{112122}}{245760} - \frac{\overline{122222}}{122880} \right], \text{CWS} [0, 0, 0, 0, 0] \right] \end{aligned}$$

$$\mathbf{e}[1, 2, -1/2] // \mathbf{dm}[1, 2, 2]$$

$$M \left[\left\{ 2 \rightarrow \text{LS} \left[-\frac{\overline{2}}{2}, 0, 0, 0, 0 \right], \text{CWS} \left[-\frac{\overline{2}}{4}, \frac{\overline{22}}{32}, 0, -\frac{\overline{2222}}{3072}, 0 \right] \right] \right.$$

Vpp =

(v ** e[1, 2, -1] ** (de[1] U (e[1, 2, -1/2] // dm[1, 2, 2]))) // Rot120 // Rot120

$$M\left[\left\{1 \rightarrow LS\left[0, -\frac{\overline{12}}{24}, 0, \frac{7\overline{1112}}{5760} - \frac{7\overline{1122}}{5760} + \frac{\overline{1222}}{1440}, 0\right], 2 \rightarrow\right.\right.$$

$$LS\left[\frac{\overline{1}}{2}, -\frac{\overline{12}}{12}, 0, \frac{\overline{1112}}{5760} - \frac{1}{720}\overline{1122} + \frac{1}{720}\overline{1222}, -\frac{\overline{11112}}{7680} + \frac{\overline{11122}}{3840} - \frac{\overline{11212}}{6912}\right],$$

$$\left. CWS\left[\frac{\overline{1}}{4}, \frac{5\overline{11}}{96} + \frac{\overline{12}}{48}, 0, -\frac{23\overline{1111}}{46080} - \frac{\overline{1112}}{2880} - \frac{\overline{1122}}{2880} - \frac{\overline{1212}}{5760} - \frac{\overline{1222}}{2880}, 0\right]\right\}$$

\$SeriesShowDegree = 6; (v // dA[1, 2]) ** Vpp

$$M\left[\left\{1 \rightarrow LS[0, 0, 0, 0, 0, 0], 2 \rightarrow LS[0, 0, 0, 0, 0, 0]\right\},\right.$$

$$CWS\left[\frac{\overline{1}}{4}, \frac{5\overline{11}}{96} + \frac{\overline{12}}{24}, 0, -\frac{23\overline{1111}}{46080} - \frac{\overline{1112}}{1440} - \frac{\overline{1122}}{1440} - \frac{\overline{1212}}{2880} - \frac{\overline{1222}}{1440},\right.$$

$$0, \frac{19\overline{111111}}{2322432} + \frac{\overline{111112}}{60480} + \frac{\overline{111122}}{60480} + \frac{\overline{111212}}{60480} + \frac{\overline{111222}}{60480} + \frac{\overline{112112}}{120960} +$$

$$\left.\left.\frac{\overline{112122}}{60480} + \frac{\overline{112212}}{60480} + \frac{\overline{112222}}{60480} + \frac{\overline{121212}}{181440} + \frac{\overline{121222}}{60480} + \frac{\overline{122122}}{120960} + \frac{\overline{122222}}{60480}\right]\right]$$

(v // dA[1, 2]) ** Vpp ** (C ** C) // dP[12]

$$M\left[\left\{1 \rightarrow LS[0, 0, 0, 0, 0, 0], 2 \rightarrow LS[0, 0, 0, 0, 0, 0]\right\},\right.$$

$$CWS\left[\frac{\overline{1}}{4}, \frac{\overline{11}}{32} - \frac{\overline{22}}{48}, 0, -\frac{\overline{1111}}{3072} + \frac{\overline{2222}}{5760}, 0, \frac{\overline{111111}}{184320} - \frac{\overline{222222}}{362880}\right]$$

C ** C

$$M\left[\left\{1 \rightarrow LS[0, 0, 0, 0, 0, 0]\right\}, CWS\left[0, -\frac{\overline{11}}{48}, 0, \frac{\overline{1111}}{5760}, 0, -\frac{\overline{111111}}{362880}\right]\right]$$

$$\left\{\frac{48}{32}, \frac{5760}{3072}, \frac{362880}{184320}\right\}$$

$$\left\{\frac{3}{2}, \frac{15}{8}, \frac{63}{32}\right\}$$