Pensieve header: Formula testing for the future double-tree paper; continues pensieve://2013-10/.

Double-Tree Formula Testing

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2013-11\\DoubleTree"];
<< "../../2013-05/FreeLie.m"
<< "muCalculus.m"
<< "../../2013-10/WKOSolution8-1.m"
α = MakeLieSeries[{"1", "2"}, αs]; β = MakeLieSeries[{"1", "2"}, βs];
γ = MakeCWSeries[{"1", "2"}, γs];
V = M[{1 → α, 2 → β}, γ];
κs[d_, 1] := If[OddQ[d], 0, κs[d]]; κ = MakeCWSeries[{"1"}, κs];
Unprotect[C]; C = M[{1 → MakeLieSeries[0]}, κ];
Ci = M[{1 → MakeLieSeries[0]}, -κ];
$SeriesShowDegree = 5; $SeriesCompareDegree = 6;
{V, C // dc[1]}
```

$$\left\{M\left[\left\{1 \to LS\left[0, -\frac{\overline{12}}{24}, 0, \frac{7\,\overline{1\,\overline{1\,\overline{12}}}}{5760} - \frac{7\,\overline{1\,\overline{12\,2}}}{5760} + \frac{\overline{\overline{12}\,\overline{2\,2}}}{1440}, 0\right], 2 \to LS\left[\frac{\overline{1}}{2}, -\frac{\overline{12}}{12}, 0, \frac{\overline{1\,\overline{1\,\overline{12}}}}{5760} - \frac{1}{720}\,\overline{1\,\overline{\overline{12}\,2}} + \frac{1}{720}\,\overline{\overline{\overline{12}\,2}\,2}, -\frac{\overline{1\,1\,\overline{1\,\overline{12}}}}{7680} + \frac{\overline{1\,1\,\overline{\overline{12}\,2}}}{3840} - \frac{\overline{\overline{1\,\overline{12}}\,\overline{12}}}{6912}\right]\right\}, CWS\left[0, -\frac{\widehat{12}}{48}, 0, \frac{\widehat{1112}}{2880} + \frac{\widehat{1122}}{2880} + \frac{\widehat{1212}}{5760} + \frac{\widehat{1222}}{2880}, 0\right]\right], M\left[\{\}, CWS\left[0, -\frac{\widehat{11}}{96}, 0, \frac{\widehat{1111}}{11\,520}, 0\right]\right]\right\}$$

```
Vb = (C * (C // dP[2])) ** V ** (Ci // dP[12]);
```

General Equations

1. The Hard R4 Equation

```
R⁺[2, 3] ** R⁺[1, 3] ** V ≡ V ** (R⁺[1, 2] // dP[12, 3])
```

```
True
```

2. The Twist Equation

```
V ** Θ[1, 2] ≡ R⁺[1, 2] ** (V // dP[2, 1])
```

```
True
```

3. The Non-Degeneracy Equations

```
{(V // dη[1]) ≡ dε[2], (V // dη[2]) ≡ dε[1]}
```

```
{True, True}
```

4. Cup / Cap

```
C ≡ (C // dS[1])
```

```
True
```

5. The Asosciator Φ

```
Φ = Module[{Vi = V // dA[1, 2]},
  (Vi // dP[12, 3]) ** Vi ** (V // dP[2, 3]) ** (V // dP[1, 23])]
```

```
M[{1 → LS[0, 23/24, 0, -1123/1440 + 7123/5760 + 1233/5760 - 72223/5760 + 72233/5760 +
      1/480 1213 - 1323/1920 + 1/640 1232 - 13222/1152 - 13322/1152 - 23332/1440, 0],
  2 → LS[0, -13/24, 0, 1113/1440 - 1123/1152 + 7123/1920 - 1/480 1132 - 1133/5760 +
      1233/1152 + 71213/5760 + 191323/5760 + 71232/1920 + 71322/5760 + 71332/5760 + 13332/1440, 0],
  3 → LS[0, 12/24, 0, -1112/1440 + 1123/5760 + 7123/5760 + 7112/5760 - 1132/1440 - 1233/1440 +
      1213/5760 + 1323/1440 - 12232/1152 - 7122/5760 - 7132/5760 - 13322/1440, 0]}, CWS[0, 0, 0, 0, 0]]
```

6. The Pentagon

```
Φ ** (Φ // dP[1, 23, 4]) ** (Φ // dP[2, 3, 4]) ≡ (Φ // dP[12, 3, 4]) ** (Φ // dP[1, 2, 34])
```

```
True
```

7. TBD

8. Horizontal Flip for Φ

```
Φ ** (Φ // dP[3, 2, 1]) ≡ dϵ[1, 2, 3]
```

```
True
```

9. Vertical Flip for Φ

```
Φ ** (Φ // dS[1, 2, 3]) ≡ dϵ[1, 2, 3]
```

```
True
```

10. The Overhand Equation

```
(Φ // dP[{{0, 1}, {2}, {3}}] // dS[2, 3] // dm[0, 3, 0] // dm[1, 2, 1]) ≡ dϵ[0, 1]
```

```
True
```

11. The definition of *v*

```
v = Φ // dS[2] // dm[3, 2, 1, 1]
```

```
M[{1 → LS[0, 0, 0, 0, 0]}, CWS[0, 11/24, 0, -1111/2880, 0]]
```

12. C and *v*

```
C ** C ** C ** C ** v ≡ dϵ[1]
```

```
True
```

13. V with left cup and right puncture

```
((V // hη[1] // tη[2]) dϵ[1] // dm[2, 1, 1]) ≡ R[1, 1, 1/2]
```
```
True
```

14. V with right cup and left puncture

```
((V // hη[2]) hϵ[2] // tη[1] // dm[1, 2, 1]) ≡ dϵ[1]
```
```
True
```

```
((Vb // hη[2]) hϵ[2] // tη[1] // dm[1, 2, 1]) ≡ dϵ[1]
```
```
True
```

```
V // tη[1]
```
```
M[{1 → LS[0, 0, 0, 0, 0, 0, 0, 0], 2 → LS[0, 0, 0, 0, 0, 0, 0, 0]},
 CWS[0, 0, 0, 0, 0, 0, 0, 0]]
```

```
V // tη[2]
```
$$\mathrm{M}\left[\left\{1 \to \mathrm{LS}[0, 0, 0, 0, 0, 0, 0, 0],\ 2 \to \mathrm{LS}\left[\overline{1}, 0, 0, 0, 0, 0, 0, 0\right]\right\},\ \mathrm{CWS}[0, 0, 0, 0, 0, 0, 0, 0]\right]$$

15. TBD

16. V with top cap and left puncture

```
(V // tη[1] // dS[2] // hm[1, 2, 1]) ≡ dϵ[1]
```
```
True
```

```
V // tη[1]
```
```
M[{1 → LS[0, 0, 0, 0, 0], 2 → LS[0, 0, 0, 0, 0]}, CWS[0, 0, 0, 0, 0]]
```

```
{(V // tη[1])[1]@{8}, (V // tη[1])[2]@{8}, (V // tη[1])[W]@{8}}
```
```
{LS[0, 0, 0, 0, 0, 0, 0, 0], LS[0, 0, 0, 0, 0, 0, 0, 0], CWS[0, 0, 0, 0, 0, 0, 0, 0]}
```

17. The Buckle

```
ZB = (Φ // dP[4, 2, 13]) ** (Φ // dP[1, 3, 2]) **
     Θ[3, 2] ** (Φ // dP[3, 2, 1]) ** (Φ // dP[12, 3, 4]);
```

18. Buckle 2 V

```
VB = (ZB // tη[1] // hη[2]) hϵ[2] // dm[1, 2, 1];
VB = (VB // tη[3] // hη[4]) hϵ[4] // dm[3, 4, 2]
```
$$\mathrm{M}\left[\left\{1 \to \mathrm{LS}\left[0, -\frac{\overline{12}}{24}, 0, \frac{7\,\overline{1\,\overline{1\,\overline{12}}}}{5760} - \frac{7\,\overline{1\,\overline{12\,2}}}{5760} + \frac{\overline{\overline{12}\,2\,2}}{1440}, 0\right],\right.$$
$$2 \to \mathrm{LS}\left[\overline{1}, -\frac{\overline{12}}{12}, 0, \frac{\overline{1\,1\,\overline{12}}}{5760} - \frac{1}{720}\,\overline{1\,\overline{12\,2}} + \frac{1}{720}\,\overline{\overline{12}\,2\,2},\right.$$
$$\left.\left.-\frac{\overline{1\,1\,\overline{1\,\overline{12}}}}{7680} + \frac{\overline{1\,1\,\overline{12\,2}}}{3840} - \frac{\overline{1\,\overline{12}\,\overline{12}}}{6912}\right]\right\}, \mathrm{CWS}[0, 0, 0, 0, 0]\Big]$$

Equations for Unitary V

1. TBD
2. The Unitarity Equation

```
V ** (V // dA[1] // dA[2]) ≡ dε[1] ⋃ dε[2]
```

```
True
```

3. The Vertical Flip Equation

```
V ** (V // dS[1] // dS[2]) ≡ R^+[1, 2]
```

```
True
```

4. The Cap Equation

```
(V ** (C // dP[12]) // dc[1] // dc[2]) ≡ (C ⋃ (C // dP[2]) // dc[1] // dc[2])
```

```
True
```

5. TBD
6. TBD
7. TBD
8. The Bucke Equation V~VB

```
C ** (C // dP[2]) ** V ≡ VB ** (C // dP[12])
```

```
True
```

```
V ** (C ** (C // dP[2])) ≡ VB ** (C // dP[12])
```

```
True
```

```
$SeriesShowDegree = 6;
{VB // dS[1] // dm[1, 2, 1] // dc[1], C * C * C * C // dc[1]}
```

$$\left\{M\left[\{\}, CWS\left[0, -\frac{\widehat{11}}{24}, 0, \frac{\widehat{1111}}{2880}, 0, -\frac{\widehat{111111}}{181\,440}\right]\right],\ M\left[\{\}, CWS\left[0, -\frac{\widehat{11}}{24}, 0, \frac{\widehat{1111}}{2880}, 0, -\frac{\widehat{111111}}{181\,440}\right]\right]\right\}$$

```
$SeriesShowDegree = 8;
{V // dS[1] // dm[1, 2, 1] // dc[1], C * C // dc[1]}
```

$$\left\{M\left[\{\}, CWS\left[0, -\frac{\widehat{11}}{48}, 0, \frac{\widehat{1111}}{5760}, 0, -\frac{\widehat{111111}}{362\,880}, 0, \frac{\widehat{11111111}}{19\,353\,600}\right]\right],\ M\left[\{\}, CWS\left[0, -\frac{\widehat{11}}{48}, 0, \frac{\widehat{1111}}{5760}, 0, -\frac{\widehat{111111}}{362\,880}, 0, \frac{\widehat{11111111}}{19\,353\,600}\right]\right]\right\}$$

```
C // dP[12] // dS[1] // dm[1, 2, 1] // dc[1]
```

```
M[{}, CWS[0, 0, 0, 0, 0, 0, 0, 0]]
```

Equations with Simple Buckle

1. TBD
2. The Unitarity Equation

```
Vb ** ((C * C) // dP[12]) ** (Vb // dA[1] // dA[2]) ≡ C * C * (C // dP[2]) * (C // dP[2])
```

```
True
```

3. The Vertical Flip Equation

```
Vb ** ((C * C) // dP[12]) ** (Vb // dS[1] // dS[2]) ≡
  R⁺[1, 2] ** (C * C * (C // dP[2]) * (C // dP[2]))
```

```
True
```

4. The Cap Equation

```
(Vb ** (C * C // dP[12]) // dc[1] // dc[2]) ≡
  ((C * C) ⋃ ((C * C) // dP[2]) // dc[1] // dc[2])
```

```
True
```

5. TBD
6. TBD
7. TBD
8. The Buckle Equation V~VB

```
Vb ≡ VB
```

```
True
```

Equations with Trivial Caps