Humbert@GRT: Beak diagrams and surface tangles

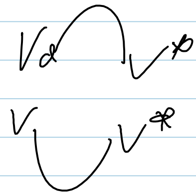
May-02-13
9:59 AM

* Relate "surface tangles" to some algebraic structures [monoidal categories]

$(\mathcal{C}, \otimes, \mathbb{1})$ with $(X \otimes Y) \otimes Z = X \otimes (Y \otimes Z)$

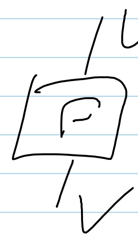
+ duality $V \rightsquigarrow V^*$

$$\begin{cases} V^* \otimes V \longrightarrow \mathbb{1} \\ \mathbb{1} \longrightarrow V \otimes V^* \end{cases}$$

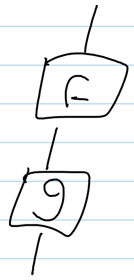


Graphical calculus:

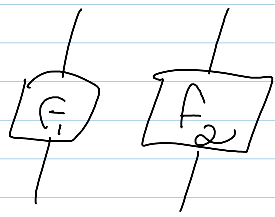
$f : U \to V \quad \longleftrightarrow$



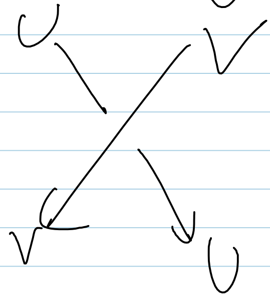
$g \circ f \longleftrightarrow$



$f_1 \otimes f_2 \longleftrightarrow$

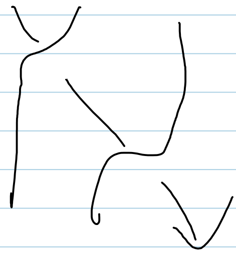


<u>Def</u> $\mathcal{C}$ is "ribbon" if it is equipped w/ braiding & twist:

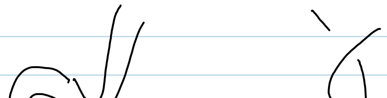


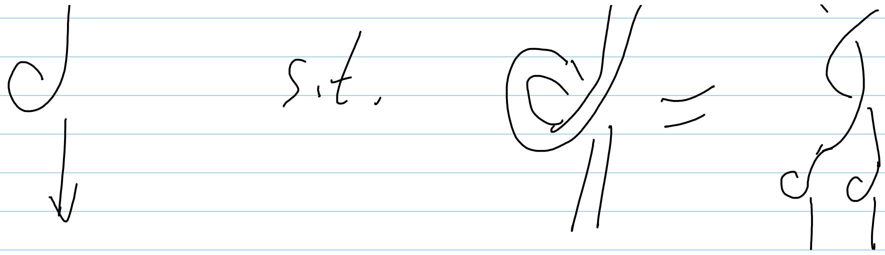
s.t.

=

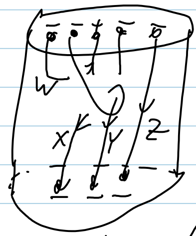


s.t.





Thm (Resh-Tur, Yetter ...)



The free ... $T_{\mathcal{C}}$
is the category of framed tangles whose objects
are coloured w/ $\mathcal{C}$

This language used to intimidate me and I'm sure it intimidates many still.

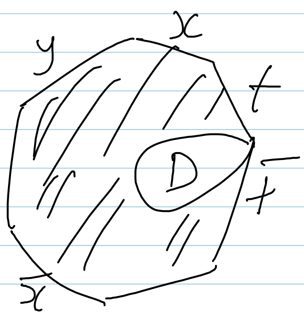
The category of tangles on a surface

Combinatorial surface:

an ordered finite set w/ f.p. free involution

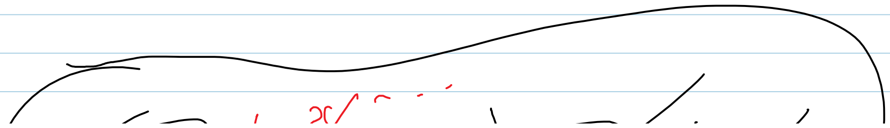
$(\Gamma, \le)$ & $(x \longmapsto \bar{x})$

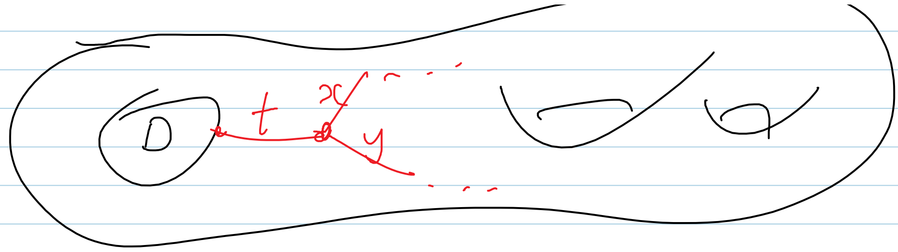
s.t. if $t = \min(\Gamma)$ then $\bar{t} = \max(\Gamma)$



"a once-bordered fat graph" represents:

S_Γ



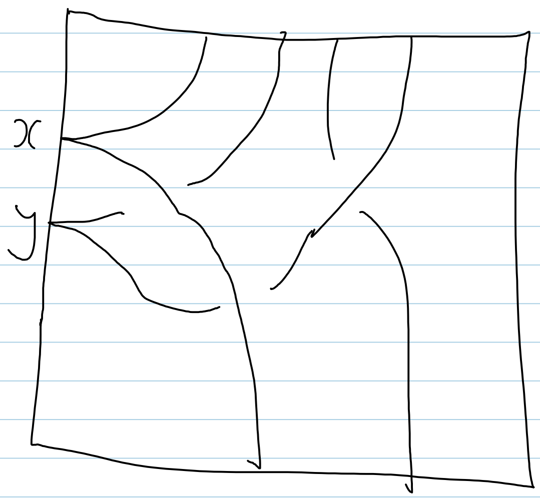


the vertices of the graph are the cycles of σ, where $\sigma(e) = \overline{\mathrm{pred}(e)}$

$T(\Gamma) :=$ tangles on $S_\Gamma \cup \mathrm{Disc}$
$(\text{in } (S_\Gamma \cup D) \times I)$

There is a functor $T \to T(\Gamma)$ by inclusion of the disk.

Representing surface tangles by "beak diagrams"



where $x, y \in \Gamma$

/ isotopy

obvious composition

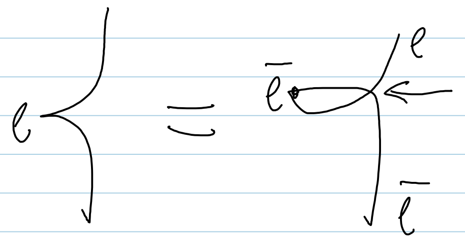
$D(\Gamma)$: beak diagrams category.

Functor

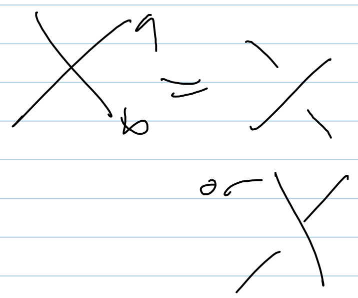
$$\varphi : D(\Gamma) \longrightarrow T(\Gamma)$$

by sending an x-beak to a path dual to x. We want to describe ker φ.

* usual Reid-moves. (R2 & R3 & $\circ = 1$)
* S-moves (slide)



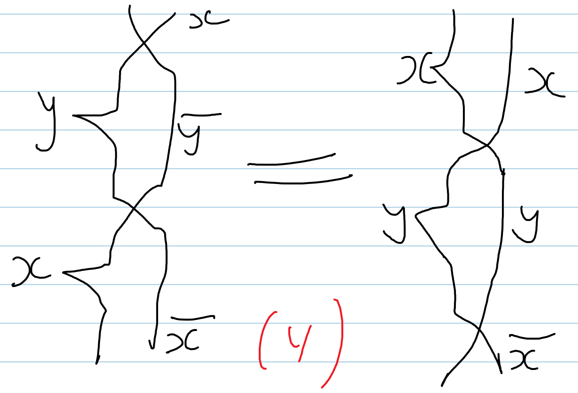
xing with sign, according to



if $a > b$

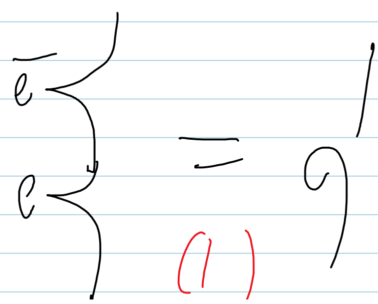
or

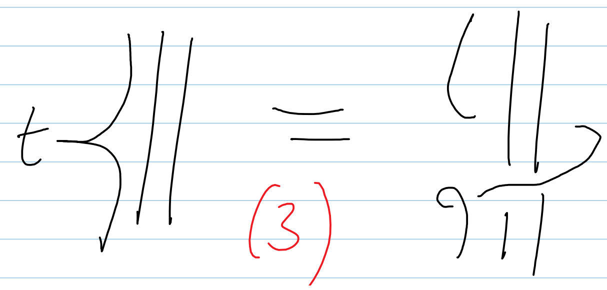
if $a < b$

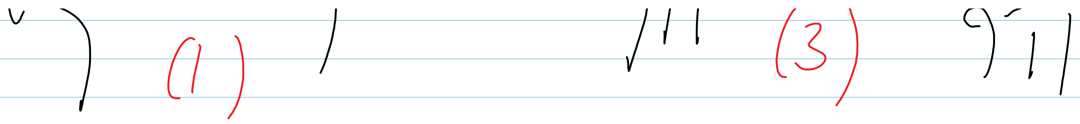


Same convention.

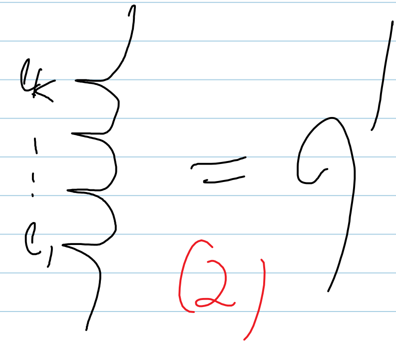
* T-moves (topology)







if $e_1 \dots e_k$ is a vertex,



<u>Thm</u> $$\frac{D(\Gamma)}{RST(\Gamma)} \xrightarrow{\sim} T(\Gamma)$$

<u>Def</u> A Γ-structure on $\mathcal{C}$ is a functor

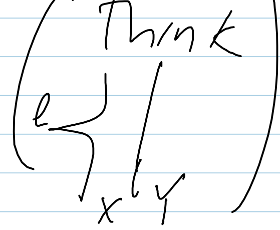
$\mathcal{C} \xrightarrow{\{\ \}} \mathcal{C}_\Gamma$ equipped with

$e \in \Gamma \rightsquigarrow (e) \in Aut(\{\cdot \otimes \cdot\})$

intimidation at its best ☺

meaning, for any $X, Y \in obj(\mathcal{C})$

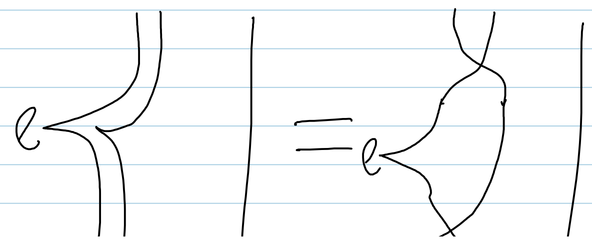
$(e)_{X,Y} \in Aut(\{X \otimes Y\})$ (think



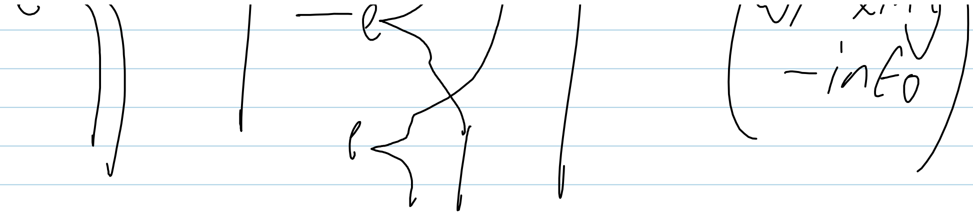
)

s.t. (1)-(3) hold, and

(5):



(w/ xing-info)



Poor marketing when it comes to applications.