Cheat Sheet β

http://drorbn.net/AcademicPensieve/2013-05/
initiated 24/3/13; continues 2013-04; modified 9/5/13, 10:15am

The original β-calculus: With $\epsilon := 1+\alpha$, $\langle\alpha\rangle := \sum_v \alpha_v$, and $\langle\gamma\rangle := \sum_{v\neq u}\gamma_v$,

$$\begin{array}{c|c}\omega_1 & H_1\\\hline T_1 & A_1\end{array} * \begin{array}{c|c}\omega_2 & H_2\\\hline T_2 & A_2\end{array} \underset{\beta}{=} \begin{array}{c|cc}\omega_1\omega_2 & H_1 & H_2\\\hline T_1 & A_1 & 0\\ T_2 & 0 & A_2\end{array}$$

$$\begin{array}{c|c}\omega & H\\\hline u & \alpha\\ v & \beta\\ T & \gamma\end{array} \xrightarrow[\beta]{tm^{uv}_w} \begin{array}{c|c}\omega & H\\\hline w & (\alpha+\beta) /\!\!/ \left(\begin{smallmatrix}u,v\\ \to w\end{smallmatrix}\right)\\ T & \gamma\end{array}$$

$$R^\pm_{ux} \underset{\beta}{=} \begin{array}{c|c}1 & x\\\hline u & t_u^{\pm1}-1\end{array}$$

$$\begin{array}{c|ccc}\omega & x & y & H\\\hline T & \alpha & \beta & \gamma\end{array} \xrightarrow[\beta]{hm^{xy}_z} \begin{array}{c|cc}\omega & z & H\\\hline T & \alpha+\beta+\langle\alpha\rangle\beta & \gamma\end{array}$$

$$\begin{array}{c|cc}\omega & x & H\\\hline u & \alpha & \beta\\ T & \gamma & \delta\end{array} \xrightarrow[\beta]{sw^{ux}_{th}} \begin{array}{c|cc}\omega\epsilon & x & H\\\hline u & \alpha(1+\langle\gamma\rangle/\epsilon) & \beta(1+\langle\gamma\rangle/\epsilon)\\ T & \gamma/\epsilon & \delta-\gamma\beta/\epsilon\end{array}$$

Constraints. • Column sums are monomials minus 1.

A

β-better calculus:

Constraints. • Sum of column x is $(\sigma_x-1)w$. • $\omega^{k-1} \mid \Lambda^k A$.

$$\begin{array}{c|c}\omega_1 & H_1\\ - & \sigma_1\\\hline T_1 & A_1\end{array} * \begin{array}{c|c}\omega_2 & H_2\\ - & \sigma_2\\\hline T_2 & A_2\end{array} \underset{\beta_b}{=} \begin{array}{c|cc}\omega_1\omega_2 & H_1 & H_2\\ - & \sigma_1 & \sigma_2\\\hline T_1 & \omega_2 A_1 & 0\\ T_2 & 0 & \omega_1 A_2\end{array}$$

$$\begin{array}{c|c}\omega & H\\ - & \sigma\\\hline u & \alpha\\ v & \beta\\ T & \gamma\end{array} \xrightarrow[\beta_b]{tm^{uv}_w} \begin{array}{c|c}\omega & H\\ - & \sigma\\\hline w & (\alpha+\beta) /\!\!/ \left(\begin{smallmatrix}u,v\\ \to w\end{smallmatrix}\right)\\ T & \gamma\end{array}$$

$$R^\pm_{ux} \underset{\beta_b}{=} \begin{array}{c|c}1 & x\\ - & t_u^{\pm1}\\\hline u & t_u^{\pm1}-1\end{array}$$

$$\begin{array}{c|ccc}\omega & x & y & H\\ - & \sigma_x & \sigma_y & \sigma\\\hline T & \alpha & \beta & \gamma\end{array} \xrightarrow[\beta_b]{hm^{xy}_z} \begin{array}{c|cc}\omega & z & H\\ - & \sigma_x\sigma_y & \sigma\\\hline T & \alpha+\sigma_x\beta & \gamma\end{array}$$

$$\begin{array}{c|cc}\omega & x & H\\ - & \sigma_x & \sigma\\\hline u & \alpha & \beta\\ T & \gamma & \delta\end{array} \xrightarrow[\beta_b]{sw^{ux}_{th}} \begin{array}{c|cc}\omega+\alpha & x & H\\ - & \sigma_x & \sigma\\\hline u & \sigma_x\alpha & \sigma_x\beta\\ T & \gamma & \delta+\frac{\alpha\delta-\gamma\beta}{\omega}\end{array} =: \begin{array}{c|c}\cdot & -\\ \cdot & -\\\hline | & \begin{pmatrix}\sigma_x\\ 1\end{pmatrix}\cdot A^{ux}\end{array}$$

Note. $A^{ux} = \begin{pmatrix}\alpha & \beta\\ \gamma & \delta+\frac{\alpha\delta-\gamma\beta}{\omega}\end{pmatrix} = \begin{pmatrix}\alpha & \beta\\ \gamma & \frac{(\omega+\alpha)\delta-\gamma\beta}{\omega}\end{pmatrix} = \frac{1}{\omega}\left[(\omega+\alpha)\begin{pmatrix}\alpha & \beta\\ \gamma & \delta\end{pmatrix} - \begin{pmatrix}\alpha\\ \gamma\end{pmatrix}\begin{pmatrix}\alpha & \beta\end{pmatrix}\right] = \frac{1}{\omega}\left[(\omega+a_{ux})A - a_{*;x}a_{u;*}\right]$.

Claim. $\omega^{k-1} \mid \Lambda^k A$ and $\omega^k \mid \Lambda^{k+1}A$ implies $(\omega+\alpha)^{k-1} \mid \Lambda^k A^{ux}$. **Proof.** With $\bar u$ $\cancel{:= u_1,\dots,u_k}$ $\in T^k$ and $\bar x$ $\cancel{:= x_1,\dots,x_l}$ $\in H^k$, ω^k divides $\begin{vmatrix}a_{ux} & a_{u\bar x}\\ a_{\bar u x} & a_{\bar u\bar x}\end{vmatrix}$ swap and $\begin{vmatrix}\omega & 0\\ 0 & a_{\bar u\bar x}\end{vmatrix}$ ~~so it divides~~ and hence their sum, $\begin{vmatrix}\omega+\alpha & a_{u\bar x}\\ a_{\bar u\bar x} & a_{\bar u\bar x}\end{vmatrix} = (\omega+\alpha)\begin{vmatrix}1 & 0\\ 0 & a_{\bar u\bar x}-\frac{1}{\omega+\alpha}a_{\bar u x}a_{u\bar x}\end{vmatrix} = \frac{1}{(\omega+\alpha)^{k-1}}\left|(\omega+\alpha)a_{\bar u\bar x}-a_{\bar u x}a_{u\bar x}\right|$, so $\frac{1}{(\omega+\alpha)^{k-1}}\left|\frac{1}{\omega}\left[(\omega+\alpha)a_{\bar u\bar x}-a_{\bar u x}a_{u\bar x}\right]\right|$ is integral. □

This proof is too short.

To do. • Consider a verification program. • Add dm formulas. • Add Burau calculus. • Add the conjugation relation. • Add the MVA formula

A. at $t_* = 1$, $w = 1$ & $A = 0$.