Cheat Sheet β

http://drorbn.net/AcademicPensieve/2013-05/
initiated 24/3/13; continues 2013-04; modified 7/5/13, 7:55pm

The original β-calculus: With $\epsilon := 1+\alpha$, $\langle\alpha\rangle := \sum_v \alpha_v$, and $\langle\gamma\rangle := \sum_{v\neq u}\gamma_v$,

$$\begin{array}{c|c}\omega_1 & H_1\\ \hline T_1 & A_1\end{array} * \begin{array}{c|c}\omega_2 & H_2\\ \hline T_2 & A_2\end{array} \underset{\beta}{=} \begin{array}{c|cc}\omega_1\omega_2 & H_1 & H_2\\ \hline T_1 & A_1 & 0\\ T_2 & 0 & A_2\end{array} \qquad \begin{array}{c|c}\omega & H\\ \hline u & \alpha\\ v & \beta\\ T & \gamma\end{array} \xrightarrow[\beta]{tm^{uv}_w} \begin{array}{c|c}\omega & H\\ \hline w & (\alpha+\beta) \,/\!\!/\, (\substack{u,v\\ \to w})\\ T & \gamma\end{array} \qquad R^{\pm}_{ux} \underset{\beta}{=} \begin{array}{c|c}1 & x\\ \hline u & t_u^{\pm1}-1\end{array}$$

$$\begin{array}{c|ccc}\omega & x & y & H\\ \hline T & \alpha & \beta & \gamma\end{array} \xrightarrow[\beta]{hm^{xy}_z} \begin{array}{c|cc}\omega & z & H\\ \hline T & \alpha+\beta+\langle\alpha\rangle\beta & \gamma\end{array} \qquad \begin{array}{c|cc}\omega & x & H\\ \hline u & \alpha & \beta\\ T & \gamma & \delta\end{array} \xrightarrow[\beta]{sw^{ux}_{th}} \begin{array}{c|cc}\omega\epsilon & x & H\\ \hline u & \alpha(1+\langle\gamma\rangle/\epsilon) & \beta(1+\langle\gamma\rangle/\epsilon)\\ T & \gamma/\epsilon & \delta-\gamma\beta/\epsilon\end{array}$$

Constraints. • Column sums are monomials minus 1.

β-better calculus:

$$\begin{array}{c|c}\omega_1 & H_1\\ - & \sigma_1\\ \hline T_1 & A_1\end{array} * \begin{array}{c|c}\omega_2 & H_2\\ - & \sigma_2\\ \hline T_2 & A_2\end{array} \underset{\beta_b}{=} \begin{array}{c|cc}\omega_1\omega_2 & H_1 & H_2\\ - & \sigma_1 & \sigma_2\\ \hline T_1 & \omega_2 A_1 & 0\\ T_2 & 0 & \omega_1 A_2\end{array} \qquad \begin{array}{c|c}\omega & H\\ - & \sigma\\ \hline u & \alpha\\ v & \beta\\ T & \gamma\end{array} \xrightarrow[\beta_b]{tm^{uv}_w} \begin{array}{c|c}\omega & H\\ - & \sigma\\ \hline w & (\alpha+\beta) \,/\!\!/\, (\substack{u,v\\ \to w})\\ T & \gamma\end{array} \qquad R^{\pm}_{ux} \underset{\beta_b}{=} \begin{array}{c|c}1 & x\\ - & t_u^{\pm1}\\ \hline u & t_u^{\pm1}-1\end{array}$$

$$\begin{array}{c|ccc}\omega & x & y & H\\ - & \sigma_x & \sigma_y & \sigma\\ \hline T & \alpha & \beta & \gamma\end{array} \xrightarrow[\beta_b]{hm^{xy}_z} \begin{array}{c|cc}\omega & z & H\\ - & \sigma_x\sigma_y & \sigma\\ \hline T & \alpha+\sigma_x\beta & \gamma\end{array} \qquad \begin{array}{c|cc}\omega & x & H\\ - & \sigma_x & \sigma\\ \hline u & \alpha & \beta\\ T & \gamma & \delta\end{array} \xrightarrow[\beta_b]{sw^{ux}_{th}} \begin{array}{c|cc}\omega+\alpha & x & H\\ - & \sigma_x & \sigma\\ \hline u & \sigma_x\alpha & \sigma_x\beta\\ T & \gamma & \delta+\frac{\alpha\delta-\gamma\beta}{\omega} = \frac{(\omega+\alpha)\delta-\gamma\beta}{\omega}\end{array} =: \begin{array}{c|c}\omega+\alpha & -\\ \hline 1 & \begin{pmatrix}\sigma_x\\ 1\end{pmatrix} A^{ux}\end{array}$$

Constraints. • Sum of col. x is $(\sigma_x-1)w$. • Likely, $\omega^{k-1} \mid \Lambda^k A$. Note. $\cancel{\begin{pmatrix}\alpha & \beta\\ \gamma & \frac{(\omega+\alpha)\delta-\gamma\beta}{\omega}\end{pmatrix}}$ A^{ux} $= \cancel{\frac{\omega+\alpha}{\omega}\begin{pmatrix}\alpha & \beta\\ \gamma & \delta\end{pmatrix} - \frac{1}{\omega}\begin{pmatrix}\alpha\\ \gamma\end{pmatrix}}(\alpha \quad \beta)$

$= \frac{1}{w}\left[(w+\alpha)\begin{pmatrix}\alpha & \beta\\ \gamma & \delta\end{pmatrix} - \begin{pmatrix}\alpha\\ \gamma\end{pmatrix}(\alpha \quad \beta)\right]$

$= \frac{1}{w}\left[(w+\alpha)A - c_x r_u\right]$

~~From 2012-05/A Higher Minors Experiment:~~

$$\frac{1}{w^2}\begin{vmatrix}(w+\alpha)\delta_{11}-\gamma_1\beta_1 & (w+\alpha)\delta_{12}-\gamma_1\beta_2\\ (w+\alpha)\delta_{21}-\gamma_2\beta_1 & (w+\alpha)\delta_{22}-\gamma_2\beta_2\end{vmatrix} = \qquad \begin{matrix}\alpha & \beta_1 & \beta_2\\ \gamma_1 & \delta_{11} & \delta_{12}\\ \gamma_2 & \delta_{21} & \delta_{22}\end{matrix}$$

starting A

$$\frac{1}{w^2}\left[-\gamma_1(w+\alpha)\begin{vmatrix}\beta_1 & \beta_2\\ \delta_{21} & \delta_{22}\end{vmatrix} - \gamma_2(w+\alpha)\begin{vmatrix}\delta_{11} & \delta_{12}\\ \beta_1 & \beta_2\end{vmatrix} + (w+\alpha)^2\begin{vmatrix}\delta_{11} & \delta_{12}\\ \delta_{21} & \delta_{22}\end{vmatrix}\right]$$

$$\underbrace{= \frac{w+\alpha}{w^2}\left[\underbrace{w\underbrace{\begin{vmatrix}\delta_{11} & \delta_{12}\\ \delta_{21} & \delta_{22}\end{vmatrix}}_{\text{divisible by } w}}_{\text{div by } w^2} + \underbrace{\begin{vmatrix}\alpha & \beta_1 & \beta_2\\ \gamma_1 & \delta_{11} & \delta_{12}\\ \gamma_2 & \delta_{21} & \delta_{22}\end{vmatrix}}_{\text{div by } w^2}\right]}_{\text{div by } w+\alpha.}$$

This is the "generic" case

To do. • Consider a verification program. • Add dm formulas. • Add Burau calculus. • Add the conjugation relation. • Add the MVA formula

<u>claim</u> $\forall k\ w^{k-1} \mid \Lambda^k A$ implies $\forall k\ (w+\alpha)^{k-1} \mid \Lambda^k A^{ux}$

<u>proof</u> Using $M_{u_1\dots u_k;x_1\dots x_k}$ to denote minors, and with $\bar{u} := u_1\dots u_k$, $\bar{x} := x_1\dots x_k$

$$w^k \mid A_{uu_1\dots u_k;xx_1\dots x_k} = \begin{vmatrix}a_{ux}=\alpha & (a_{ux_j})=(\beta_{x_j})\\ (\gamma_{u_i})=(a_{u_ix}) & (a_{u_ix_j})\end{vmatrix} =: M_1 \text{ and}$$

$w^k \,|\, w \cdot A_{u_1 \dots u_k; x_1 \dots x_k} = w \cdot |(a_{u_i; x_i})| =: \mu_2$, so

$$w^k \,|\, \mu_1 + \mu_2 = \begin{vmatrix} w+\alpha & (\beta_{x_j}) \\ (\gamma_{u_i}) & (a_{u_i; x_j}) \end{vmatrix} = \text{ by row \& col. reduction}$$

$$= (w+\alpha) \left| a_{u_i x_j} - \frac{1}{w+\alpha} (\gamma_{u_i})(\beta_{x_j}) \right|$$

$$= \frac{1}{(w+\alpha)^{k-1}} \cdot \left| (w+\alpha) a_{u_i x_j} - \gamma_{u_i} \beta_{x_j} \right|$$

So $\frac{1}{(w+\alpha)^{k-1}} \left| \frac{1}{w} (\; \dots \;) \right|$ is integral.

Now as $c_x r_u$ is rank 1,

$$A^{ux}_{u_1 \dots u_k; x_1 \dots x_k} = \frac{1}{w^k} \cdot \left[(w+\alpha) A - c_x r_u \right]_{u_1 \dots u_k; x_1 \dots x_k}$$

$$= \frac{1}{w^k} \left((w+\alpha)^k A_{\bar{u}; \bar{x}} - \dots \right.$$

If $B = c \otimes r^*$ is a rank 1 matrix, then

$$\Lambda(A - B) = \Lambda A - i_{r^*} /\!/ \Lambda A /\!/ e_c$$

$$c \wedge (\Lambda A)(i_{r^*}(\cdot))$$

$$\Lambda\left(\frac{1}{w}\left[(w+\alpha) A - c_x r_u\right]\right) = \Lambda\left(\frac{w+\alpha}{w} A\right) -$$

Perhaps I should first attempt the case

$$A = wB + \gamma\rho$$

<u>claim</u> $\det\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} = \alpha \det\begin{pmatrix} 1 & \beta/\alpha \\ \gamma & \delta \end{pmatrix} =$

$$= \det(\delta) + \ldots\ldots = \alpha \det\left(\delta - \frac{\gamma\beta}{\alpha}\right)$$