**Referee Report:** Dror Bar-Natan and Hernando Burgos-Soto, *Khovanov Homology for Alternating Tangles*

By a Theorem of Lee, the Khovanov Homology of a (non-split) alternating link is supported in two diagonals. The authors give a local version of this theorem within the framework introduced by the first author in [BN2]. To do so, they define a chain complex associated to a tangle to be *coherently diagonal* if it is (along with all its partial closures) supported in a single diagonal. They then show that single crossings give rise to such complexes and that the property of a chain complex being coherently diagonal is closed under a subset of planar algebra compositions that allows them to build any alternating tangle. Lee's Theorem can easily be recovered from this statement.

The result is interesting, giving a more explicit proof about the structure of the Khovanov homology of alternating links that does not rely on a long exact sequence argument. The proof has the potential to work with integer coefficients and to be extended to equivariant Khovanov Homology. I could recommend the publication of this article in JKTR, provided that the following issues are addressed.

**Basepoints:** The definition of a planar algebra that you are using contains a basepoint on every input and output disk. This is only mentioned in passing, but is in fact important in the proof: The definition of the rotation number depends on the choice of basepoint (at least the way I understand it, please give a more precise definition of the rotation number on page 3 than just referring to a picture). This is vitally important in section 3 when the rotation number of an element of the planar algebra is defined.

**Generalizations:** The only version of Khovanov Homology that you are considering is Khovanov's original theory with coefficients in $\mathbb{Z}[\frac{1}{2}]$. This seems unnecessary restrictive: You can work in $\mathbb{Z}$ by considering dots to be formal objects rather than one half of a handle, as is done in [BN3]. I also do not see where the relation that two dots on a component make a cobordism 0 is used in the proof[1]. The result is much more appealing in this generality.

**Perturbed Double Complexes:** These appear to simply be filtered chain complexes. It would be much more useful to the reader if the standard notation were used. The observation that Gaussian elimination preserves the "filteredness" of the chain complex then is a consequence of the homological perturbation lemma.

[1]In fact, I think everything should generalize to equivariant Khovanov Homology over $\mathbb{Z}[h, t]$, associated to the polynomial $x^2 - hx - t$.

**General Comments:**

1. Second-to-last line of abstract: “lines” should be “diagonals”
2. p 1, last paragraph: maybe say “non-trivial” instead of “no-zero dimension”
3. p 2, second paragraph (starting with “This local”): The paragraph is difficult to understand. Please reword unclear passages. “that last” $\rightarrow$ “this”, “fore-mentioned” $\rightarrow$ “aforementioned”, “consists in” $\rightarrow$ “consists of”, no comma after “elimination”. Be more precise about what you mean by “isomorphisms”.
4. p 3, first paragraph: Double-check the definition (see below). “as this” $\rightarrow$ “like this”.
5. p 3, second paragraph: As mentioned earlier, the picture isn’t entirely clear. Is the basepoint always supposed to have an outgoing edge on the left and an incoming edge on the right? That would contradict the picture on page 2. “smoothing” $\rightarrow$ “smoothings”.
6. p 4, paragraph below Definition 1.1: “signifies” $\rightarrow$ “signify”.
7. p 4, Theorem 1: “such as” $\rightarrow$ “such that”. The remark below is inconsistent with the Theorem: Do you mean to say that it is *coherently* $C$-diagonal, or is the Theorem a consequence rather than a restatement of the statement that “that the Khovanov homology $Kh(T)$ of a non-split alternating tangle is coherently diagonal”?
8. p 4, remarks below Theorem 2: “introduce” $\rightarrow$ “introducing”, “no-diagonal” $\rightarrow$ “non-diagonal”, “the Lee Theorem” $\rightarrow$ “Lee’s Theorem”.
9. p 5, second paragraph: “denote” $\rightarrow$ “denote by”, no comma after $Cob^3(B)$. third paragraph: “mod out” $\rightarrow$ “considered modulo”.
10. p 6, (3): Does this really hold definitionally? I would use $\cong$ rather than $=$.
11. p 7, first paragraph of section 3: “subject out” should be reworded, no comma after “categories”.
12. next paragraph: This paragraph could be clearer. What exactly is denoted by $S_o$? “orientation in” $\rightarrow$ “orientation of”, “denoted with” $\rightarrow$ “denoted by”, no comma after “arcs” and “this”.
13. next paragraph: I do not see where the condition that there can’t be an arc from the output disk to itself comes from. Did you forget to list that in the definition of a type-$\mathcal{A}$ diagram? Should the first sentence be “compute the Khovanov homology of”? “its ends” $\rightarrow$ “their ends”.

14. p 8, Proposition 3.2: I'm a little uncomfortable with stating this without proof, since the whole paper rests on this observation. Do you have to consider basepoints in the definitions of the basic operators? As a courtesy to the reader, even trivial statements should have a qed at the end of the statement.

15. p 8, paragraph below Prop 3.2: "same standard closure" → "same as the standard closure"

16. p 9, Example 5.1: If necessary, the basepoint should be marked in the diagram.

17. p 9, section 4.1: "and then apply" doesn't match the previous sentence.

18. p 9, Definition 4.3: "appropriated" → "appropriate".

19. p 9, (1) and (2): "closure" → "closures", "negative shifted-degree" is hard to parse.

20. p 10: "as" → "such as", "embedded" → "embed it", two periods missing at ends of paragraphs.

21. p 11, proof: "as" → "such as", "disk" → "disks".

22. p 14, Proposition 6.1: Is the statement meant to be universally quantified over $\sigma$? The way the statement is used below suggests that the conclusion should be stronger: the complex should be *coherently* ...-diagonal.

23. p 16, last paragraph: "1 open arc" → maybe "single arc".