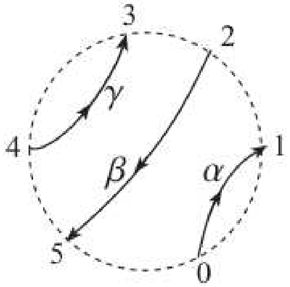
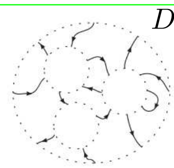
Khovanov Homology for Alternating Tangles

Rotation Numbers. $R(\alpha) := \frac{1}{2k}\left[t_\alpha - h_\alpha\right]_{2k} - \frac{1}{2}$, where $[j]_{2k} := \begin{cases} j & \text{if } j > 0 \\ j+2k & \text{if } j < 0 \end{cases}$, and $R(\circlearrowleft) = +1$ and $R(\circlearrowright) = -1$. Also, $R(\alpha\{q\}) := R(\alpha) + q$. Examples:



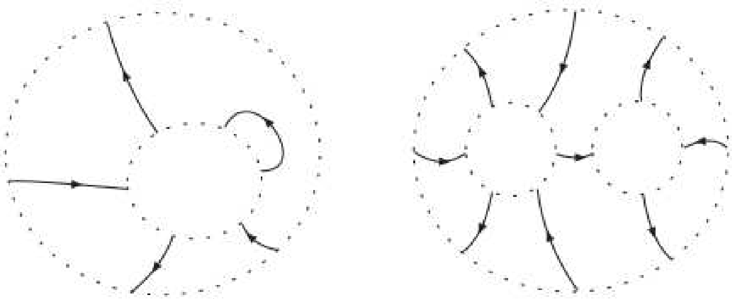
$$R(\alpha) = \tfrac{1}{6} - \tfrac{1}{2} = -\tfrac{1}{3}$$
$$R(\beta) = \tfrac{1}{2} - \tfrac{1}{2} = 0$$
$$R(\gamma) = \tfrac{5}{6} - \tfrac{1}{2} = \tfrac{1}{3}$$

Alternating Planar Algebra. All input/output boundaries are connected via the arcs, "in" and "out" strands alternate on all boundaries. A "rotation number" R_D can be defined.



Proposition 3.2. $R(D(\sigma_1,\dots,\sigma_d)) = R_D + \sum_{i=1}^{d} R(\sigma_i)$.

The Basic Operations.



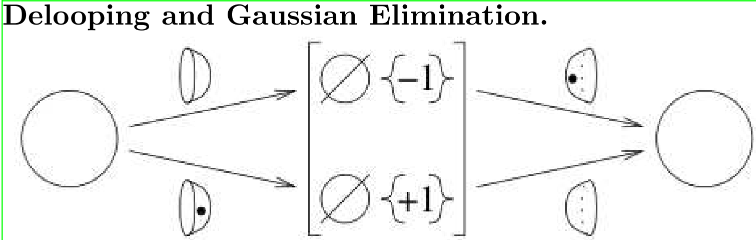
$R_D = \mp\frac{1}{2}$ $\qquad$ $R_D = 0$

Diagonal Complexes. Ω: $\cdots \to \left[\sigma_j^r\right]_j \to \left[\sigma_j^{r+1}\right]_j \to \cdots$ such that $2r - R(\sigma_j^r)$ is a constant $C(\Omega)$.

Coherently Diagonal Complexes. All partial closures can be *reduced* to diagonal, with $C(U(\Omega)) = C(\Omega) - C(D_U)$.

"Main" Lemma 6.2. The pairing $D(\Omega_1, \Omega_2)$ via an arc diagram that has at least one boundary arc coming from its first input of a coherently diagonal complex Ω_1 and a diagonal complex Ω_2 is coherently diagonal.

Delooping and Gaussian Elimination.



$$\cdots\ [C] \xrightarrow{\begin{pmatrix}\alpha\\ \beta\end{pmatrix}} \begin{bmatrix} b_1 \\ D \end{bmatrix} \xrightarrow{\begin{pmatrix}\phi & \delta \\ \gamma & \epsilon\end{pmatrix}} \begin{bmatrix} b_2 \\ E \end{bmatrix} \xrightarrow{\begin{pmatrix}\mu & \nu\end{pmatrix}} [F]\ \cdots$$

$$\cdots\ [C] \xrightarrow{\begin{pmatrix}0\\ \beta\end{pmatrix}} \begin{bmatrix} b_1 \\ D \end{bmatrix} \xrightarrow{\begin{pmatrix}\phi & 0 \\ 0 & \epsilon-\gamma\phi^{-1}\delta\end{pmatrix}} \begin{bmatrix} b_2 \\ E \end{bmatrix} \xrightarrow{\begin{pmatrix}0 & \nu\end{pmatrix}} [F]\ \cdots$$

Theorem 1. If T is non-split alternating, $Kh(T)$ is coherently diagonal.

Theorem 2. If $\{\Omega_i\}$ are coherently diagonal and D is alternating planar, then $D(\Omega_1, \Omega_2, \dots)$ is coherently diagonal.

Gravity and Smoothings.