

```

SetDirectory["C:\\drorbn\\AcademicPensieve\\2012-12"];
<< FreeLie.m;
BCH[n_Integer] := BCH[{"x"}, {"y"}][n];
BCHWords[n_Integer] := Cases[BCH[n], _LW, Infinity];
AllLyndonWords[n_Integer] := AllLyndonWords[n, {"x", "y"}];
    
```

BCH[7]

$$\begin{aligned}
 & \frac{\langle \text{xxxxxxy} \rangle}{30\,240} - \frac{\langle \text{xxxxxyy} \rangle}{5040} + \frac{\langle \text{xxxxyxy} \rangle}{10\,080} + \frac{\langle \text{xxxxyyy} \rangle}{3780} + \frac{\langle \text{xxxxyxy} \rangle}{10\,080} + \frac{\langle \text{xxxxyxy} \rangle}{1680} + \\
 & \frac{\langle \text{xxxxyxy} \rangle}{1260} + \frac{\langle \text{xxxxyyy} \rangle}{3780} + \frac{\langle \text{xyxyxy} \rangle}{2016} - \frac{\langle \text{xyxyxy} \rangle}{5040} + \frac{13 \langle \text{xyxyxy} \rangle}{15\,120} + \frac{\langle \text{xyxyxy} \rangle}{10\,080} - \\
 & \frac{\langle \text{xyxyxy} \rangle}{1512} - \frac{\langle \text{xyxyxy} \rangle}{5040} + \frac{\langle \text{xyxyxy} \rangle}{1260} - \frac{\langle \text{xyxyxy} \rangle}{2016} - \frac{\langle \text{xyxyxy} \rangle}{5040} + \frac{\langle \text{xyxyxy} \rangle}{30\,240}
 \end{aligned}$$

Expand[2 BCH[8]]

$$\begin{aligned}
 & \frac{\langle \text{xxxxxxy} \rangle}{30\,240} - \frac{\langle \text{xxxxxyxy} \rangle}{7560} - \frac{\langle \text{xxxxxyyy} \rangle}{5040} + \frac{\langle \text{xxxxxyxy} \rangle}{10\,080} - \frac{\langle \text{xxxxxyxy} \rangle}{10\,080} + \frac{\langle \text{xxxxxyxy} \rangle}{1260} + \\
 & \frac{23 \langle \text{xxxxxyyy} \rangle}{60\,480} + \frac{\langle \text{xxxxyxy} \rangle}{2016} - \frac{\langle \text{xxxxyxy} \rangle}{5040} + \frac{13 \langle \text{xxxxyyy} \rangle}{15\,120} + \frac{\langle \text{xxxxyxy} \rangle}{10\,080} - \\
 & \frac{\langle \text{xxxxyxy} \rangle}{1512} - \frac{\langle \text{xxxxyyy} \rangle}{5040} + \frac{\langle \text{xyxyxy} \rangle}{1260} - \frac{\langle \text{xyxyxy} \rangle}{2016} - \frac{\langle \text{xyxyxy} \rangle}{5040} + \frac{\langle \text{xyxyxy} \rangle}{30\,240}
 \end{aligned}$$

2 BCH[8] - b[{"x"}, BCH[7]] // Expand

$$-\frac{\langle \text{xxxxxxy} \rangle}{30\,240} + \frac{\langle \text{xxxxxyy} \rangle}{4320} - \frac{\langle \text{xxxxxyxy} \rangle}{4320} - \frac{\langle \text{xxxxxyyy} \rangle}{2160} - \frac{\langle \text{xxxxxyxy} \rangle}{1440} + \frac{\langle \text{xxxxxyyy} \rangle}{8640}$$

b[{"y"}, BCH[7]] // Expand

$$\begin{aligned}
 & -\frac{\langle \text{xxxxxxy} \rangle}{30\,240} + \frac{\langle \text{xxxxxyxy} \rangle}{7560} + \frac{\langle \text{xxxxxyyy} \rangle}{5040} - \frac{\langle \text{xxxxxyxy} \rangle}{10\,080} + \frac{\langle \text{xxxxxyxy} \rangle}{10\,080} - \\
 & \frac{\langle \text{xxxxyxy} \rangle}{1260} - \frac{\langle \text{xxxxyyy} \rangle}{3780} - \frac{\langle \text{xxxxyxy} \rangle}{2016} + \frac{\langle \text{xxxxyxy} \rangle}{5040} - \frac{13 \langle \text{xxxxyyy} \rangle}{15\,120} - \\
 & \frac{\langle \text{xxxxyxy} \rangle}{1260} - \frac{\langle \text{xxxxyxy} \rangle}{3780} - \frac{\langle \text{xxxxyyy} \rangle}{3780} - \frac{\langle \text{xyxyxy} \rangle}{4320} - \frac{\langle \text{xyxyxy} \rangle}{10\,080} + \frac{\langle \text{xyxyxy} \rangle}{1440} - \\
 & \frac{17 \langle \text{xyxyxy} \rangle}{15\,120} + \frac{\langle \text{xyxyxy} \rangle}{720} - \frac{29 \langle \text{xyxyxy} \rangle}{30\,240} + \frac{\langle \text{xyxyxy} \rangle}{2160} + \frac{\langle \text{xyxyxy} \rangle}{1080} + \\
 & \frac{\langle \text{xyxyxy} \rangle}{5040} - \frac{\langle \text{xyxyxy} \rangle}{432} + \frac{\langle \text{xyxyxy} \rangle}{1440} + \frac{\langle \text{xyxyxy} \rangle}{1440} + \frac{\langle \text{xyxyxy} \rangle}{1440} - \frac{\langle \text{xyxyxy} \rangle}{30\,240}
 \end{aligned}$$


```

SymmetricDifference[A_List, B_List] := {
  Complement[A, B], Complement[B, A]
};
Table[
  SymmetricDifference[
    b[<"x">, #] & /@ BCHWords[2 n - 1],
    BCHWords[2 n]
  ],
  {n, 2, 8}
] // ColumnForm

{{<xxx>, {}}
{{<xxxx>, {}}
{{<xxxxx>, {}}
{{<xxxxxxx>, {}}
{{<xxxxxxxx>, {<xxxxxxxxxy>}}
{{<xxxxxxxxxy>, {}}
{{<xxxxxxxxxy>, {}}
{{<xxxxxxxxxy>, {}}
{{<xxxxxxxxxy>, {<xxxxxxxxxyxy>}}

```