

$$hm_z^{xy} : \begin{array}{c|ccc} \omega & h_x & h_y & \dots \\ \hline \vdots & \alpha & \beta & \gamma \end{array} \mapsto \begin{array}{c|ccc} \omega & h_z & & \dots \\ \hline \vdots & \alpha + \beta + \langle \alpha \rangle \beta & & \gamma \end{array},$$

$$sw_{xy}^{th} : \begin{array}{c|cc} \omega & h_y & \dots \\ \hline t_x & \alpha & \beta \\ \vdots & \gamma & \delta \end{array} \mapsto \begin{array}{c|cc} \omega \epsilon & h_y & \dots \\ \hline t_x & \alpha(1 + \langle \gamma \rangle / \epsilon) & \beta(1 + \langle \gamma \rangle / \epsilon) \\ \vdots & \gamma / \epsilon & \delta - \gamma \beta / \epsilon \end{array}$$

$$\epsilon = 1 + \alpha$$

$$h_1 t_1 h_2 t_2 \rightarrow h_1 h_2' t_1' t_2$$

$$\begin{array}{c|ccc} \omega & h_1 & h_2 & h_3 \\ \hline t_1 & \alpha_{11} & \alpha_{12} & \alpha_{13} \\ t_2 & \alpha_{21} & \alpha_{22} & \alpha_{23} \\ t_3 & \alpha_{31} & \alpha_{32} & \alpha_{33} \end{array}$$

sw_{12}

$$\begin{array}{c|ccc} \omega(1+\alpha_{12}) & h_1 & h_2 & h_3 \\ \hline t_1 & \alpha_{11}(-) & \alpha_{12}(1 + \frac{\alpha_{22} + \alpha_{32}}{1 + \alpha_{12}}) & \alpha_{13}(-) \\ t_2 & \alpha_{21} - \frac{\alpha_{22}\alpha_{11}}{1 + \alpha_{12}} & \alpha_{22}/(1 + \alpha_{12}) & \alpha_{23} - \frac{\alpha_{22}\alpha_{13}}{1 + \alpha_{12}} \\ t_3 & \alpha_{31} - \frac{\alpha_{32}\alpha_{11}}{1 + \alpha_{12}} & \alpha_{32}/(1 + \alpha_{12}) & \alpha_{33} - \frac{\alpha_{32}\alpha_{13}}{1 + \alpha_{12}} \end{array}$$

$$\frac{\omega(1+\alpha_{12})}{}$$

$tm \rightarrow$

$$\alpha_{11} \left(1 + \frac{\alpha_{22} + \alpha_{32}}{1 + \alpha_{12}} \right) + \alpha_{21} - \frac{\alpha_{22}\alpha_{11}}{1 + \alpha_{12}} =$$

$$\frac{\alpha_{11} + \alpha_{11}(\alpha_{12} + \alpha_{22} + \alpha_{32}) + \alpha_{21} + \alpha_{21}\alpha_{12} - \alpha_{22}\alpha_{11}}{1 + \alpha_{12}} =$$

$$\alpha_{11} + \alpha_{21} + \alpha_{11}\alpha_{12} + \alpha_{11}\alpha_{32} + \alpha_{21}\alpha_{12}$$

$$\frac{\alpha_{11} + \alpha_{21} + \alpha_{11}\alpha_{12} + \alpha_{11}\alpha_{32} + \alpha_{21}\alpha_{12}}{1 + \alpha_{12}}$$

$$\langle h_1 \rangle = \text{above} + \alpha_{31} - \frac{\alpha_{32}\alpha_{11}}{1 + \alpha_{12}} =$$

$$\frac{\alpha_{31} + \alpha_{31}\alpha_{12} - \alpha_{32}\alpha_{11} + \alpha_{11} + \alpha_{21} + \alpha_{11}\alpha_{12} + \alpha_{11}\alpha_{32} + \alpha_{21}\alpha_{12}}{1 + \alpha_{12}}$$

$$= \frac{\alpha_{31} + \alpha_{11} + \alpha_{21} + \alpha_{12}(\alpha_{31} + \alpha_{11} + \alpha_{21})}{1 + \alpha_{12}} = \alpha_{11} + \alpha_{21} + \alpha_{31}$$

$$\alpha_{31} - \frac{\alpha_{32}\alpha_{11}}{1 + \alpha_{12}} + (1 + \alpha_{11} + \alpha_{21} + \alpha_{31}) \left(\frac{\alpha_{32}}{1 + \alpha_{12}} \right)$$

$$= \frac{\alpha_{31} + \alpha_{31}\alpha_{12} - \alpha_{32}\alpha_{11} + \alpha_{32} + \alpha_{32}\alpha_{11} + \alpha_{32}\alpha_{21} + \alpha_{32}\alpha_{31}}{1 + \alpha_{12}}$$

$$= \frac{\alpha_{31} + \alpha_{32} + \alpha_{31}\alpha_{12} + \alpha_{32}\alpha_{21} + \alpha_{32}\alpha_{31}}{1 + \alpha_{12}}$$

$$= \alpha_{31} + \alpha_{32} \frac{1 + \alpha_{21} + \alpha_{31}}{1 + \alpha_{12}}$$