

**Meta-Groups, Meta-Bicrossed-Products, and the Alexander Polynomial, I**

Dror Bar-Natan at Knots in Washington XXXIV  
<http://www.math.toronto.edu/~drorbn/Talks/GWU-1203/>  
 Foots & refs on PDF version



**Abstract.** A straightforward proposal for a group-theoretic invariant of knots fails if one really means groups, but works once generalized to meta-groups (to be defined). We will construct one complicated but elementary meta-group as a meta-bicrossed-product (to be defined), and explain how the resulting invariant is a not-yet-understood generalization of the Alexander polynomial, while at the same time being a specialization of a somewhat-understood “universal finite type invariant of w-knots” and of an elusive “universal finite type invariant of v-knots”.

Define meta-group

Knot diagram  $\delta_{17}$  | Tangle diagram  $\left[ \begin{array}{c} \text{---} \\ \text{---} \end{array} \right]$

Knot/tangle invariant  $Z: \left\{ \begin{array}{l} \text{knots} \\ \text{tangles} \end{array} \right\} \rightarrow \text{easy set}$

st.  $Z(\text{---}) = Z(\text{---})_1 \dots$   
 grayed out.

Example  $\left( \begin{array}{c} \text{---} \\ \text{---} \end{array} \right) \begin{matrix} \text{row k} \\ \text{col.} \\ \text{ops.} \end{matrix}$

Bicrossed products

Idea. Group  $G \begin{matrix} \nearrow \text{g}_a \\ \searrow \text{g}_b \end{matrix} \begin{matrix} \nearrow \text{g}'_a \\ \searrow \text{g}'_b \end{matrix}$   
 $R^p = (g_a g_b)$   $R^m = (g'_a g'_b)$

$\text{---} \rightarrow EG$

$\begin{matrix} \nearrow \text{g}'_a \\ \searrow \text{g}'_b \end{matrix} \rightarrow \text{g}_a g'_a = 1 = g_b g'_b \text{ so } ' = -!$

$\begin{matrix} \nearrow \text{g}'_a \\ \searrow \text{g}'_b \end{matrix} \begin{matrix} \nearrow \text{g}'_c \\ \searrow \text{g}'_d \end{matrix} \mapsto R_{16}^p R_{24}^p R_{53}^p = (g_a g_b) \dots$   
 $\xrightarrow{\text{mfm}} (\quad)$

$\text{---} \rightarrow \dots$

meta Bicrossed prod.

Meta-Groups, Meta-Bicrossed-Products, and the Alexander Polynomial, 2

$\beta(HIT)$   
& ops

What is it  
good for

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Program  
& comps.

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Where is it  
coming  
from.