Pensieve header: Perturbative β-calculations - Dror's 120328 fork.

In[1]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2012-03"];
<< betaCalculus.m
```

In[3]:=
```
Clear[ℏ];
$PerturbativeDegree = 3;
βSimplify[expr_] := Replace[
   Series[Normal[expr], {ℏ, 0, $PerturbativeDegree}],
   sd_SeriesData :> MapAt[Expand, sd, 3]
  ];
βCollect[B[ω_, μ_]] := B[
   βSimplify[ω],
   βSimplify[μ]
  ];
```

The Knot-Theoretic Equations

In[41]:=
```
{
  V0 = βCollect[
      B[ω[ℏ c1, ℏ c2], α[ℏ c1, ℏ c2] t[1] h[1] +
        β[ℏ c1, ℏ c2] t[1] h[2] + γ[ℏ c1, ℏ c2] t[2] h[1] + δ[ℏ c1, ℏ c2] t[2] h[2]]
    ] /. {
      (ϵ : (α | β | γ | δ | ω | ϰ))[___] :> ϵ0, (ϵ : (α | β | γ | δ | ω | ϰ))^(k__)[___] :> ϵ_FromDigits[{k}]
    },
  C0 = βCollect[B[ϰ[ℏ c1], 0]] /. {
      (ϵ : (α | β | γ | δ | ω | ϰ))[___] :> ϵ0, (ϵ : (α | β | γ | δ | ω | ϰ))^(k__)[___] :> ϵ_FromDigits[{k}]
    },
  eqns1 = HardR4[V0],
  eqns2 = TwistEq[V0],
  eqns3 = And[(V0 // dη[1]) == B[1, 0], (V0 // dη[2]) == B[1, 0]],
  eqns4 = V0 ** (V0 // dA[1] // dA[2]) == B[1, 0],
  eqns5 = CapEquation[V0, C0],
  eqns6 = (C0 // tη[1]) == B[1, 0],
  eqns7 = (V0 == Rot120[V0]),
  eqns8 = (R[1, 2] == (V0 // dP[2, 1]) ** (V0 // Inverse)),
  eqns9 = (R[2, 1, -1] == (V0 // dP[2, 1]) ** (V0 // Inverse))
 } // ColumnForm
```

Out[41]=

A very large output was generated. Here is a sample of it:

<<1>>

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In[42]:=
```
eqns = eqns1 && eqns2 && eqns3 && eqns4 && eqns5 && eqns6 && eqns9;
```

In[43]:=
```
sol = SolveAlways[eqns, {ℏ, c1, c2}]
```

Out[43]= { }

In[10]:= `{V0, C0} /. sol[[1]]`

Out[10]= $\left\{\begin{pmatrix} 1+\frac{1}{16}c_1c_2\left(-1+8\beta_0^2+16\delta_{10}\right)\hbar^2+\left(\frac{1}{32}c_1^2c_2\left(3\kappa_1-24\beta_0^2\kappa_1-48\delta_{10}\kappa_1-16\kappa_1^3+16\kappa_3\right)+\frac{1}{32}c_1c_2^2\left(3\kappa\right.\right. \\ t[1] \\ t[2]\end{pmatrix}\right.$

In[11]:= `(V0 /. sol[[1]]) // Rot120 // dη[1] // dP[2 → 1]`

Out[11]= $\begin{pmatrix} 1-c_1\beta_0\hbar+\left(\frac{c_1^2}{48}+\frac{1}{2}c_1^2\beta_0^2\right)\hbar^2+\left(\frac{c_1^3}{24}+\frac{13}{48}c_1^3\beta_0-\frac{3}{2}c_1^3\beta_0^3+c_1^3\beta_1+2c_1^3\beta_0\beta_1+c_1^3\beta_{11}-c_1^3\gamma_{20}-2c_1^3\beta_0\delta_{10}+c\right. \\ t[1]\end{pmatrix}$

In[12]:= `(V0 /. sol[[1]]) // Rot120 // dη[1] // dP[2 → 1] // dA[1] // dcap[1]`

Out[12]= $\begin{pmatrix} 1-c_1\beta_0\hbar+\left(\frac{c_1^2}{48}+\frac{1}{2}c_1^2\beta_0^2\right)\hbar^2+\left(\frac{c_1^3}{24}+\frac{13}{48}c_1^3\beta_0-\frac{3}{2}c_1^3\beta_0^3+c_1^3\beta_1+2c_1^3\beta_0\beta_1+c_1^3\beta_{11}-c_1^3\gamma_{20}-2c_1^3\beta_0\delta_{10}+c\right. \\ t[1]\end{pmatrix}$

In[13]:= `indvars = Flatten[Union[Cases[Last /@ #, ϵ_k_ :> ϵk, Infinity]] & /@ sol]`

Out[13]= $\{\beta_0, \beta_1, \beta_{11}, \gamma_3, \gamma_{12}, \gamma_{20}, \gamma_{21}, \gamma_{30}, \delta_{10}, \delta_{12}, \delta_{20}, \delta_{21}, \delta_{30}, \kappa_1, \kappa_3\}$

In[14]:=
```
sol1 = Union[
  sol[[1]] /. Thread[indvars → 0],
  Thread[indvars → 0]
 ]
```

Out[14]= $\{\alpha_0\to 0, \alpha_1\to -\frac{1}{8}, \alpha_2\to -\frac{1}{8}, \alpha_3\to -\frac{3}{64}, \alpha_{10}\to 0, \alpha_{11}\to -\frac{1}{48}, \alpha_{12}\to -\frac{5}{192},$
$\alpha_{20}\to 0, \alpha_{21}\to \frac{1}{64}, \alpha_{30}\to 0, \beta_0\to 0, \beta_1\to 0, \beta_2\to -\frac{1}{24}, \beta_3\to \frac{3}{64}, \beta_{10}\to \frac{1}{24}, \beta_{11}\to 0,$
$\beta_{12}\to -\frac{1}{192}, \beta_{20}\to -\frac{1}{24}, \beta_{21}\to -\frac{1}{64}, \beta_{30}\to \frac{1}{64}, \gamma_0\to \frac{1}{2}, \gamma_1\to \frac{1}{6}, \gamma_2\to \frac{1}{24}, \gamma_3\to 0,$
$\gamma_{10}\to \frac{1}{8}, \gamma_{11}\to \frac{1}{16}, \gamma_{12}\to 0, \gamma_{20}\to 0, \gamma_{21}\to 0, \gamma_{30}\to 0, \delta_0\to 0, \delta_1\to 0, \delta_2\to 0, \delta_3\to 0,$
$\delta_{10}\to 0, \delta_{11}\to 0, \delta_{12}\to 0, \delta_{20}\to 0, \delta_{21}\to 0, \delta_{30}\to 0, \kappa_0\to 1, \kappa_1\to 0, \kappa_2\to -\frac{1}{16}, \kappa_3\to 0,$
$\omega_0\to 1, \omega_1\to 0, \omega_2\to 0, \omega_3\to 0, \omega_{10}\to 0, \omega_{11}\to -\frac{1}{16}, \omega_{12}\to 0, \omega_{20}\to 0, \omega_{21}\to 0, \omega_{30}\to 0\}$

In[15]:= `V1 = V0 /. sol1`

Out[15]= $\begin{pmatrix} 1-\frac{1}{16}(c_1c_2)\hbar^2+O[\hbar]^4 & h[1] & \\ t[1] & -\frac{c_2\hbar}{8}+\left(-\frac{1}{48}c_1c_2-\frac{c_2^2}{16}\right)\hbar^2+\left(\frac{1}{128}c_1^2c_2-\frac{5}{384}c_1c_2^2-\frac{c_2^3}{128}\right)\hbar^3+O[\hbar]^4 & \frac{c_1\hbar}{24}+\left(-\frac{c}{4}\right. \\ t[2] & \frac{1}{2}+\left(\frac{c_1}{8}+\frac{c_2}{6}\right)\hbar+\left(\frac{c_1c_2}{16}+\frac{c_2^2}{48}\right)\hbar^2+O[\hbar]^4 & \end{pmatrix}$

In[16]:= `C1 = C0 /. sol1`

Out[16]= $\begin{pmatrix} 1-\frac{1}{32}c_1^2\hbar^2+O[\hbar]^4 \\ t[1]\end{pmatrix}$

In[17]:= `HardR4[V1]`

Out[17]= True

In[18]:=
```
TwistEq[V1]
```
Out[18]=
```
True
```
In[19]:=
```
V1 ** (V1 // dA[1] // dA[2])
```
Out[19]=
```
( 1 )
```
In[20]:=
```
CapEquation[V1, C1]
```
Out[20]=
```
True
```
In[21]:=
```
Φ1 = Φ[V1]
```
Out[21]=

$$\begin{pmatrix} 1 & h[1] \\ t[1] & \left(\frac{1}{192} c_2^2 c_3 - \frac{1}{576} c_2 c_3^2\right) \hbar^3 + O[\hbar]^4 \\ t[2] & \frac{c_3 \hbar}{24} + \left(-\frac{1}{24} c_2 c_3 - \frac{c_3^2}{48}\right) \hbar^2 + \left(-\frac{1}{384} c_1^2 c_3 - \frac{1}{64} c_1 c_2 c_3 + \frac{7 c_2^2 c_3}{1152} - \frac{11 c_1 c_3^2}{1152} + \frac{7 c_2 c_3^2}{1152} + \frac{c_3^3}{1152}\right) \hbar^3 + O[\hbar]^4 \\ t[3] & \frac{c_2 \hbar}{24} + \left(\frac{c_1 c_2}{24} - \frac{c_2 c_3}{24}\right) \hbar^2 + \left(-\frac{5}{192} c_1^2 c_2 - \frac{7}{192} c_1 c_2^2 - \frac{c_2^3}{144} - \frac{1}{144} c_1 c_2 c_3 + \frac{1}{96} c_2^2 c_3 + \frac{1}{72} c_2 c_3^2\right) \hbar^3 + O[\hbar]^4 \end{pmatrix}$$

In[22]:=
```
Pentagon[Φ1]
```
Out[22]=
```
True
```
In[23]:=
```
Hexagon[+1, Φ1]
```
Out[23]=
```
True
```
In[24]:=
```
Hexagon[-1, Φ1]
```
Out[24]=
```
True
```
In[25]:=
```
Φ1 ** (Φ1 // dP[3, 2, 1])
```
Out[25]=
```
( 1 )
```
In[26]:=
```
Φ1 ** (Φ1 // dS[1] // dS[2] // dS[3])
```
Out[26]=

$$\begin{pmatrix} 1 & h[1] & h[2] & h[3] \\ t[1] & 0 & \left(-\frac{1}{12} c_2 c_3 - \frac{c_3^2}{24}\right) \hbar^2 + O[\hbar]^4 & \left(\frac{c_1 c_2}{12} - \frac{c_2 c_3}{12}\right) \hbar^2 + O[\hbar]^4 \\ t[2] & \left(-\frac{1}{12} c_2 c_3 - \frac{c_3^2}{24}\right) \hbar^2 + O[\hbar]^4 & 0 & \left(\frac{c_1^2}{24} + \frac{c_1 c_2}{12}\right) \hbar^2 + O[\hbar]^4 \\ t[3] & \left(\frac{c_1 c_2}{12} - \frac{c_2 c_3}{12}\right) \hbar^2 + O[\hbar]^4 & \left(\frac{c_1^2}{24} + \frac{c_1 c_2}{12}\right) \hbar^2 + O[\hbar]^4 & 0 \end{pmatrix}$$

In[27]:=
```
(R[1, 3] ** R[2, 3] ** V1) == (V1 ** (R[1, 3] // dΔ[1, 1, 2]))
```
Out[27]=
```
True
```
In[28]:=
```
((V1 // Inverse // dP[2, 1]) ** R[2, 3] ** R[1, 3]) ==
  ((R[1, 3] // dΔ[1, 1, 2]) ** (V1 // Inverse // dP[2, 1]))
```
Out[28]=
```
True
```
In[29]:=
```
R[1, 3] ** R[2, 3] ** V1 ** (V1 // Inverse // dP[2, 1]) ==
  V1 ** (V1 // Inverse // dP[2, 1]) ** R[2, 3] ** R[1, 3]
```
Out[29]=
```
True
```
In[30]:=
```
(V1 // dP[2, 1]) ** (V1 // Inverse) ** R[1, 3] ** R[2, 3] ==
  R[2, 3] ** R[1, 3] ** (V1 // dP[2, 1]) ** (V1 // Inverse)
```
Out[30]=
```
True
```

In[31]:= **(V1 // dP[2, 1]) ** (V1 // Inverse)**

Out[31]=
$$\begin{pmatrix} 1 & h[1] & \\ t[1] & \frac{c_2 \hbar}{8} - \frac{1}{16} c_2^2 \hbar^2 + \left(-\frac{1}{384} c_1^2 c_2 - \frac{1}{128} c_1 c_2^2 + \frac{5 c_2^3}{384}\right) \hbar^3 + O[\hbar]^4 & \frac{1}{2} + \left(\frac{c_1}{8} - \frac{c_2}{8}\right) \hbar + \\ t[2] & -\frac{1}{2} + \left(-\frac{c_1}{8} + \frac{c_2}{8}\right) \hbar + \left(\frac{c_1 c_2}{16} - \frac{c_2^2}{48}\right) \hbar^2 + \left(\frac{c_1^3}{384} + \frac{1}{128} c_1^2 c_2 - \frac{5}{384} c_1 c_2^2 + \frac{c_2^3}{384}\right) \hbar^3 + O[\hbar]^4 & \frac{c_1 \hbar}{8} + \end{pmatrix}$$

In[36]:= **R[1, 2] ** R[1, 3] ** R[2, 3] == R[2, 3] ** R[1, 3] ** R[1, 2]**

Out[36]= True

In[38]:= **R[2, 1, -1] ** R[1, 3] ** R[2, 3] == R[2, 3] ** R[1, 3] ** R[2, 1, -1]**

Out[38]= True

In[33]:= **R[1, 2]**

Out[33]=
$$\begin{pmatrix} 1 & h[2] \\ t[1] & 1 + \frac{c_1 \hbar}{2} + \frac{1}{6} c_1^2 \hbar^2 + \frac{1}{24} c_1^3 \hbar^3 + O[\hbar]^4 \end{pmatrix}$$

In[39]:= **R[2, 1, -1] ** R[1, 2] ** R[1, 3] ** R[2, 3] == R[2, 3] ** R[1, 3] ** R[2, 1, -1] ** R[1, 2]**

Out[39]=
$$1 + \frac{\hbar c_1}{2} + \frac{1}{6} \hbar^2 c_1^2 + \frac{1}{24} \hbar^3 c_1^3 ==$$
$$1 + \hbar \left(\frac{c_1}{2} - c_2\right) + \hbar^2 \left(\frac{c_1^2}{6} - \frac{3 c_1 c_2}{2} - \frac{c_2^2}{2}\right) + \hbar^3 \left(\frac{c_1^3}{24} - \frac{7}{6} c_1^2 c_2 - \frac{3}{4} c_1 c_2^2 - \frac{c_2^3}{6}\right) \&\&$$
$$1 + \hbar \left(c_1 + \frac{c_2}{2}\right) + \hbar^2 \left(\frac{c_1^2}{2} + \frac{c_1 c_2}{2} + \frac{c_2^2}{6}\right) + \hbar^3 \left(\frac{c_1^3}{6} + \frac{1}{4} c_1^2 c_2 + \frac{1}{6} c_1 c_2^2 + \frac{c_2^3}{24}\right) ==$$
$$1 + \hbar \left(2 c_1 + \frac{c_2}{2}\right) + \hbar^2 \left(2 c_1^2 + c_1 c_2 + \frac{c_2^2}{6}\right) + \hbar^3 \left(\frac{4 c_1^3}{3} + c_1^2 c_2 + \frac{1}{3} c_1 c_2^2 + \frac{c_2^3}{24}\right)$$

In[34]:= **R[1, 2] == (V1 // dP[2, 1]) ** (V1 // Inverse)**

Out[34]=
$$0 == \frac{\hbar c_2}{8} - \frac{1}{16} \hbar^2 c_2^2 + \hbar^3 \left(-\frac{1}{384} c_1^2 c_2 - \frac{1}{128} c_1 c_2^2 + \frac{5 c_2^3}{384}\right) \&\& 1 + \frac{\hbar c_1}{2} + \frac{1}{6} \hbar^2 c_1^2 + \frac{1}{24} \hbar^3 c_1^3 ==$$
$$\frac{1}{2} + \hbar \left(\frac{c_1}{8} - \frac{c_2}{8}\right) + \hbar^2 \left(\frac{c_1^2}{48} - \frac{c_1 c_2}{16}\right) + \hbar^3 \left(\frac{c_1^3}{384} - \frac{5}{384} c_1^2 c_2 + \frac{1}{128} c_1 c_2^2 + \frac{c_2^3}{384}\right) \&\&$$
$$0 == -\frac{1}{2} + \hbar \left(-\frac{c_1}{8} + \frac{c_2}{8}\right) + \hbar^2 \left(\frac{c_1 c_2}{16} - \frac{c_2^2}{48}\right) + \hbar^3 \left(\frac{c_1^3}{384} + \frac{1}{128} c_1^2 c_2 - \frac{5}{384} c_1 c_2^2 + \frac{c_2^3}{384}\right) \&\&$$
$$0 == \frac{\hbar c_1}{8} + \frac{1}{16} \hbar^2 c_1^2 + \hbar^3 \left(\frac{5 c_1^3}{384} - \frac{1}{128} c_1^2 c_2 - \frac{1}{384} c_1 c_2^2\right)$$