Pensieve header: Perturbative β-calculations - Dror’s 120328 fork.

In[1]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2012-03"];
<< betaCalculus.m
```

In[3]:=
```
Clear[ℏ];
$PerturbativeDegree = 3;
βSimplify[expr_] := Replace[
   Series[Normal[expr], {ℏ, 0, $PerturbativeDegree}],
   sd_SeriesData :> MapAt[Expand, sd, 3]
  ];
βCollect[B[ω_, μ_]] := B[
   βSimplify[ω],
   βSimplify[μ]
  ];
```

The Knot-Theoretic Equations

In[7]:=
```
{
  V0 = βCollect[
      B[ω[ℏ c1, ℏ c2], α[ℏ c1, ℏ c2] t[1] h[1] +
        β[ℏ c1, ℏ c2] t[1] h[2] + γ[ℏ c1, ℏ c2] t[2] h[1] + δ[ℏ c1, ℏ c2] t[2] h[2]]
    ] /. {
     (ϵ : (α | β | γ | δ | ω | κ))[___] :> ϵ0, (ϵ : (α | β | γ | δ | ω | κ))^(k__)[___] :> ϵFromDigits[{k}]
    },
  C0 = βCollect[B[κ[ℏ c1], 0]] /. {
     (ϵ : (α | β | γ | δ | ω | κ))[___] :> ϵ0, (ϵ : (α | β | γ | δ | ω | κ))^(k__)[___] :> ϵFromDigits[{k}]
    },
  eqns1 = HardR4[V0],
  eqns2 = TwistEq[V0],
  eqns3 = And[(V0 // dη[1]) == B[1, 0], (V0 // dη[2]) == B[1, 0]],
  eqns4 = V0 ** (V0 // dA[1] // dA[2]) == B[1, 0],
  eqns5 = CapEquation[V0, C0],
  eqns6 = (C0 // tη[1]) == B[1, 0],
  eqns7 = (V0 == Rot120[V0]),
  eqns8 = (R[1, 2] == (V0 // dP[2, 1]) ** (V0 // Inverse)),
  eqns9 = (R[2, 1, -1] == (V0 // dP[2, 1]) ** (V0 // Inverse))
 } // ColumnForm
```

Out[7]=

A very large output was generated. Here is a sample of it:

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Show Less | Show More | Show Full Output | Set Size Limit...

In[8]:=
```
eqns = eqns1 && eqns2 && eqns3 && eqns4 && eqns5 && eqns6;
```

In[9]:= `sol = SolveAlways[eqns, {ℏ, c1, c2}]`

Out[9]=
$$\{\{\alpha_{30} \to 0,\ \beta_{30} \to \frac{1}{64}(1 + 8\beta_0 + 48\beta_1 + 64\gamma_3 - 96\gamma_{20}),$$
$$\beta_{21} \to \frac{1}{192}(-3 - 8\beta_0 - 48\beta_1 - 192\beta_{11} + 192\gamma_{12} - 96\gamma_{20}),$$
$$\beta_{12} \to \frac{1}{192}(-1 - 48\beta_1 - 192\beta_{11} - 96\gamma_{20} + 192\gamma_{21}),\ \beta_3 \to \frac{1}{64}(3 + 16\beta_0 + 48\beta_1 - 96\gamma_{20} + 64\gamma_{30}),$$
$$\delta_3 \to 0,\ \alpha_{21} \to \frac{1}{192}(3 + 16\beta_0 + 48\beta_1 - 96\gamma_{20} + 192\delta_{12}),$$
$$\alpha_{12} \to \frac{1}{192}\left(-5 - 32\beta_0 + 192\beta_0^3 - 144\beta_1 - 384\beta_0\beta_1 - 192\beta_{11} + 192\beta_0\delta_{10} - 96\delta_{20} + 192\delta_{21} + 36\kappa_1 - 288\beta_0^2\kappa_1 - 576\delta_{10}\kappa_1 - 192\kappa_1^3 + 192\kappa_3\right),$$
$$\alpha_3 \to \frac{1}{64}(-3 - 8\beta_0 - 96\gamma_{20} + 48\delta_{10} + 96\delta_{20} + 64\delta_{30}),\ \gamma_{11} \to \frac{1}{16}(1 + 4\beta_0 + 16\beta_1 + 16\beta_{11}),$$
$$\omega_{12} \to \frac{1}{16}\left(3\kappa_1 - 24\beta_0^2\kappa_1 - 48\delta_{10}\kappa_1 - 16\kappa_1^3 + 16\kappa_3\right),$$
$$\omega_{21} \to \frac{1}{16}\left(3\kappa_1 - 24\beta_0^2\kappa_1 - 48\delta_{10}\kappa_1 - 16\kappa_1^3 + 16\kappa_3\right),\ \omega_3 \to 0,\ \omega_{30} \to 0,$$
$$\beta_{20} \to \frac{1}{24}(-1 - 4\beta_0 - 24\beta_1 + 24\gamma_{20}),\ \beta_2 \to \frac{1}{24}(-1 - 6\beta_0 - 24\beta_1 + 24\gamma_{20}),$$
$$\gamma_2 \to \frac{1}{24}(1 + 2\beta_0 + 24\gamma_{20}),\ \alpha_{20} \to 0,\ \alpha_{11} \to \frac{1}{48}\left(-1 - 8\beta_0 + 48\beta_0^3 - 24\beta_1 - 96\beta_0\beta_1 + 48\beta_0\delta_{10} - 24\delta_{20} + 9\kappa_1 - 72\beta_0^2\kappa_1 - 144\delta_{10}\kappa_1 - 48\kappa_1^3 + 48\kappa_3\right),$$
$$\alpha_2 \to \frac{1}{8}(-1 - 4\beta_0 - 8\beta_1 + 8\delta_{10} + 8\delta_{20}),\ \delta_2 \to 0,$$
$$\delta_{11} \to \frac{1}{48}\left(-2\beta_0 + 48\beta_0^3 - 96\beta_0\beta_1 + 48\beta_0\delta_{10} - 24\delta_{20} + 9\kappa_1 - 72\beta_0^2\kappa_1 - 144\delta_{10}\kappa_1 - 48\kappa_1^3 + 48\kappa_3\right),$$
$$\omega_{11} \to \frac{1}{16}\left(-1 + 8\beta_0^2 + 16\delta_{10}\right),\ \omega_{20} \to 0,\ \omega_2 \to 0,\ \gamma_1 \to \frac{1}{6}(1 + 3\beta_0 + 6\beta_1),\ \gamma_{10} \to \frac{1}{8}(1 + 4\beta_0 + 8\beta_1),$$
$$\beta_{10} \to \frac{1}{24}(1 + 24\beta_1),\ \kappa_2 \to \frac{1}{16}\left(-1 + 8\beta_0^2 + 16\delta_{10} + 16\kappa_1^2\right),\ \alpha_{10} \to 0,\ \alpha_1 \to \frac{1}{8}(-1 - 4\beta_0 + 8\delta_{10}),$$
$$\delta_1 \to 0,\ \omega_1 \to 0,\ \omega_{10} \to 0,\ \gamma_0 \to \frac{1}{2}(1 + 2\beta_0),\ \kappa_0 \to 1,\ \alpha_0 \to 0,\ \delta_0 \to 0,\ \omega_0 \to 1\}\}$$

In[10]:= `{V0, C0} /. sol[[1]]`

Out[10]=
$$\left\{\begin{pmatrix} 1 + \frac{1}{16}c_1 c_2\left(-1 + 8\beta_0^2 + 16\delta_{10}\right)\hbar^2 + \left(\frac{1}{32}c_1^2 c_2\left(3\kappa_1 - 24\beta_0^2\kappa_1 - 48\delta_{10}\kappa_1 - 16\kappa_1^3 + 16\kappa_3\right) + \frac{1}{32}c_1 c_2^2\left(3\kappa\right.\right. \\ t[1] \\ t[2] \end{pmatrix}\right.$$

In[11]:= `indvars = Flatten[Union[Cases[Last /@ #, ϵ_k_ :> ϵk, Infinity]] & /@ sol]`

Out[11]=
$$\{\beta_0, \beta_1, \beta_{11}, \gamma_3, \gamma_{12}, \gamma_{20}, \gamma_{21}, \gamma_{30}, \delta_{10}, \delta_{12}, \delta_{20}, \delta_{21}, \delta_{30}, \kappa_1, \kappa_3\}$$

In[12]:= **V1 = V0 /. sol[[1]]**

Out[12]= $\left(1 + \frac{1}{16} c_1 c_2 \left(-1 + 8 \beta_0^2 + 16 \delta_{10}\right) \hbar^2 + \left(\frac{1}{32} c_1^2 c_2 \left(3 \kappa_1 - 24 \beta_0^2 \kappa_1 - 48 \delta_{10} \kappa_1 - 16 \kappa_1^3 + 16 \kappa_3\right) + \frac{1}{32} c_1 c_2^2 \left(3 \kappa_1\right.\right.$
t[1]
t[2]

In[13]:= **C1 = C0 /. sol[[1]]**

Out[13]= $\left(1 + c_1 \kappa_1 \hbar + \frac{1}{32} c_1^2 \left(-1 + 8 \beta_0^2 + 16 \delta_{10} + 16 \kappa_1^2\right) \hbar^2 + \frac{1}{6} c_1^3 \kappa_3 \hbar^3 + O[\hbar]^4\right)$
t[1]

In[14]:= **HardR4[V1]**

Out[14]= True

In[15]:= **TwistEq[V1]**

Out[15]= True

In[16]:= **V1 ** (V1 // dA[1] // dA[2])**

Out[16]= (1)

In[17]:= **CapEquation[V1, C1]**

Out[17]= True

In[18]:= **Φ1 = Φ[V1]**

Out[18]=
1
t[1]
t[2] $\left(\frac{c_3}{24} - c_3 \beta_0^2 + c_3 \beta_1 - c_3 \delta_{10}\right) \hbar + \left(-\frac{1}{24} c_2 c_3 - \frac{c_3^2}{48} - \frac{5}{24} c_2 c_3 \beta_0 - \frac{1}{6} c_3^2 \beta_0 + c_3^2 \beta_0^3 - c_2 c_3 \beta_1 - \frac{1}{2} c_3^2 \beta_1 - c\right.$
t[3]

In[19]:= **Pentagon[Φ1]**

Out[19]= True

In[20]:= **Hexagon[+1, Φ1]**

Out[20]= True

In[21]:= **Hexagon[-1, Φ1]**

Out[21]= True

In[22]:= **Φ1 ** (Φ1 // dP[3, 2, 1])**

Out[22]= (1)

In[23]:= **Φ1 ** (Φ1 // dS[1] // dS[2] // dS[3])**

Out[23]=
1
t[1]
t[2] $\left(-\frac{1}{12} c_2 c_3 - \frac{c_3^2}{24} - \frac{5}{12} c_2 c_3 \beta_0 - \frac{1}{3} c_3^2 \beta_0 + 2 c_3^2 \beta_0^3 - 2 c_2 c_3 \beta_1 - c_3^2 \beta_1 - 2 c_1 c_3 \beta_0 \beta_1 - 2 c_2 c_3 \beta_0 \beta_1 - 4 c\right.$
t[3]

In[24]:= **Φ1 ** (Φ1 // dS[1] // dS[2] // dS[3]) == B[1, 0]**

Out[24]= $\hbar^2 \left(\frac{1}{4} c_2 c_3 \beta_0 - 4 c_2 c_3 \beta_0^3 + 6 c_2 c_3 \beta_0 \beta_1 - 6 c_2 c_3 \beta_0 \delta_{10} + 3 c_2 c_3 \delta_{20} - \frac{3}{8} c_2 c_3 \kappa_1 + 3 c_2 c_3 \beta_0^2 \kappa_1 + 6 c_2 c_3 \delta_{10} \kappa_1 + 2 c_2 c_3 \kappa_1^3 - 2 c_2 c_3 \kappa_3\right) == 0 \&\&$

$\hbar^2 \left(-\frac{1}{12} c_2 c_3 - \frac{c_3^2}{24} - \frac{5}{12} c_2 c_3 \beta_0 - \frac{1}{3} c_3^2 \beta_0 + 2 c_3^2 \beta_0^3 - 2 c_2 c_3 \beta_1 - c_3^2 \beta_1 - 2 c_1 c_3 \beta_0 \beta_1 - 2 c_2 c_3 \beta_0 \beta_1 - 4 c_3^2 \beta_0 \beta_1 + 2 c_1 c_3 \beta_{11} + 2 c_2 c_3 \gamma_{20} + c_3^2 \gamma_{20} - 2 c_1 c_3 \beta_0 \delta_{10} - 2 c_2 c_3 \beta_0 \delta_{10} + 2 c_3^2 \beta_0 \delta_{10} + c_1 c_3 \delta_{20} + c_2 c_3 \delta_{20} - c_3^2 \delta_{20} - \frac{3}{8} c_1 c_3 \kappa_1 - \frac{3}{8} c_2 c_3 \kappa_1 + 3 c_1 c_3 \beta_0^2 \kappa_1 + 3 c_2 c_3 \beta_0^2 \kappa_1 + 6 c_1 c_3 \delta_{10} \kappa_1 + 6 c_2 c_3 \delta_{10} \kappa_1 + 2 c_1 c_3 \kappa_1^3 + 2 c_2 c_3 \kappa_1^3 - 2 c_1 c_3 \kappa_3 - 2 c_2 c_3 \kappa_3\right) == 0 \&\&$

$\hbar^2 \left(\frac{c_1 c_2}{12} - \frac{c_2 c_3}{12} + \frac{5}{12} c_1 c_2 \beta_0 - \frac{5}{12} c_2 c_3 \beta_0 + 2 c_1 c_2 \beta_1 - 2 c_2 c_3 \beta_1 + 2 c_1 c_2 \beta_{11} - 2 c_2 c_3 \beta_{11} - 2 c_1 c_2 \gamma_{20} + 2 c_2 c_3 \gamma_{20}\right) == 0 \&\&$

$\hbar^2 \left(-\frac{1}{12} c_2 c_3 - \frac{c_3^2}{24} - \frac{5}{12} c_2 c_3 \beta_0 - \frac{1}{3} c_3^2 \beta_0 + 2 c_3^2 \beta_0^3 - 2 c_2 c_3 \beta_1 - c_3^2 \beta_1 - 2 c_1 c_3 \beta_0 \beta_1 - 2 c_2 c_3 \beta_0 \beta_1 - 4 c_3^2 \beta_0 \beta_1 + 2 c_1 c_3 \beta_{11} + 2 c_2 c_3 \gamma_{20} + c_3^2 \gamma_{20} - 2 c_1 c_3 \beta_0 \delta_{10} - 2 c_2 c_3 \beta_0 \delta_{10} + 2 c_3^2 \beta_0 \delta_{10} + c_1 c_3 \delta_{20} + c_2 c_3 \delta_{20} - c_3^2 \delta_{20} - \frac{3}{8} c_1 c_3 \kappa_1 - \frac{3}{8} c_2 c_3 \kappa_1 + 3 c_1 c_3 \beta_0^2 \kappa_1 + 3 c_2 c_3 \beta_0^2 \kappa_1 + 6 c_1 c_3 \delta_{10} \kappa_1 + 6 c_2 c_3 \delta_{10} \kappa_1 + 2 c_1 c_3 \kappa_1^3 + 2 c_2 c_3 \kappa_1^3 - 2 c_1 c_3 \kappa_3 - 2 c_2 c_3 \kappa_3\right) == 0 \&\&$

$\hbar^2 \left(\frac{c_1^2}{24} + \frac{c_1 c_2}{12} + \frac{1}{3} c_1^2 \beta_0 + \frac{5}{12} c_1 c_2 \beta_0 - 2 c_1^2 \beta_0^3 + c_1^2 \beta_1 + 2 c_1 c_2 \beta_1 + 4 c_1^2 \beta_0 \beta_1 + 2 c_1 c_2 \beta_0 \beta_1 + 2 c_1 c_3 \beta_0 \beta_1 - 2 c_1 c_3 \beta_{11} - c_1^2 \gamma_{20} - 2 c_1 c_2 \gamma_{20} - 2 c_1^2 \beta_0 \delta_{10} + 2 c_1 c_2 \beta_0 \delta_{10} + 2 c_1 c_3 \beta_0 \delta_{10} + c_1^2 \delta_{20} - c_1 c_2 \delta_{20} - c_1 c_3 \delta_{20} + \frac{3}{8} c_1 c_2 \kappa_1 + \frac{3}{8} c_1 c_3 \kappa_1 - 3 c_1 c_2 \beta_0^2 \kappa_1 - 3 c_1 c_3 \beta_0^2 \kappa_1 - 6 c_1 c_2 \delta_{10} \kappa_1 - 6 c_1 c_3 \delta_{10} \kappa_1 - 2 c_1 c_2 \kappa_1^3 - 2 c_1 c_3 \kappa_1^3 + 2 c_1 c_2 \kappa_3 + 2 c_1 c_3 \kappa_3\right) == 0 \&\&$

$\hbar^2 \left(\frac{c_1 c_2}{12} - \frac{c_2 c_3}{12} + \frac{5}{12} c_1 c_2 \beta_0 - \frac{5}{12} c_2 c_3 \beta_0 + 2 c_1 c_2 \beta_1 - 2 c_2 c_3 \beta_1 + 2 c_1 c_2 \beta_{11} - 2 c_2 c_3 \beta_{11} - 2 c_1 c_2 \gamma_{20} + 2 c_2 c_3 \gamma_{20}\right) == 0 \&\&$

$\hbar^2 \left(\frac{c_1^2}{24} + \frac{c_1 c_2}{12} + \frac{1}{3} c_1^2 \beta_0 + \frac{5}{12} c_1 c_2 \beta_0 - 2 c_1^2 \beta_0^3 + c_1^2 \beta_1 + 2 c_1 c_2 \beta_1 + 4 c_1^2 \beta_0 \beta_1 + 2 c_1 c_2 \beta_0 \beta_1 + 2 c_1 c_3 \beta_0 \beta_1 - 2 c_1 c_3 \beta_{11} - c_1^2 \gamma_{20} - 2 c_1 c_2 \gamma_{20} - 2 c_1^2 \beta_0 \delta_{10} + 2 c_1 c_2 \beta_0 \delta_{10} + 2 c_1 c_3 \beta_0 \delta_{10} + c_1^2 \delta_{20} - c_1 c_2 \delta_{20} - c_1 c_3 \delta_{20} + \frac{3}{8} c_1 c_2 \kappa_1 + \frac{3}{8} c_1 c_3 \kappa_1 - 3 c_1 c_2 \beta_0^2 \kappa_1 - 3 c_1 c_3 \beta_0^2 \kappa_1 - 6 c_1 c_2 \delta_{10} \kappa_1 - 6 c_1 c_3 \delta_{10} \kappa_1 - 2 c_1 c_2 \kappa_1^3 - 2 c_1 c_3 \kappa_1^3 + 2 c_1 c_2 \kappa_3 + 2 c_1 c_3 \kappa_3\right) == 0 \&\&$

$\hbar^2 \left(-\frac{1}{4} c_1 c_2 \beta_0 + 4 c_1 c_2 \beta_0^3 - 6 c_1 c_2 \beta_0 \beta_1 + 6 c_1 c_2 \beta_0 \delta_{10} - 3 c_1 c_2 \delta_{20} + \frac{3}{8} c_1 c_2 \kappa_1 - 3 c_1 c_2 \beta_0^2 \kappa_1 - 6 c_1 c_2 \delta_{10} \kappa_1 - 2 c_1 c_2 \kappa_1^3 + 2 c_1 c_2 \kappa_3\right) == 0$

In[25]:=
```
{sol2} = SolveAlways[
  Φ1 ** (Φ1 // dS[1] // dS[2] // dS[3]) == B[1, 0],
  {ħ, c1, c2}
 ]
```

Out[25]=

$$\left\{\left\{\beta_{11} \to \frac{1}{8}\left(\beta_0 - 16\,\beta_0^3 + 32\,\beta_0\,\beta_1 - 16\,\beta_0\,\delta_{10} + 8\,\delta_{20}\right),\right.\right.$$
$$\kappa_3 \to \frac{1}{16}\left(2\,\beta_0 - 32\,\beta_0^3 + 48\,\beta_0\,\beta_1 - 48\,\beta_0\,\delta_{10} + 24\,\delta_{20} - 3\,\kappa_1 + 24\,\beta_0^2\,\kappa_1 + 48\,\delta_{10}\,\kappa_1 + 16\,\kappa_1^3\right),$$
$$\left.\left.\gamma_{20} \to \frac{1}{24}\left(1 + 8\,\beta_0 - 48\,\beta_0^3 + 24\,\beta_1 + 96\,\beta_0\,\beta_1 - 48\,\beta_0\,\delta_{10} + 24\,\delta_{20}\right)\right\}\right\}$$

In[26]:=
```
{V2, C2} = {V1, C1} /. sol2
```

Out[26]=

$$\left\{\begin{pmatrix} 1 + \frac{1}{16}\,c_1\,c_2\,\left(-1 + 8\,\beta_0^2 + 16\,\delta_{10}\right)\hbar^2 + \left(\frac{1}{32}\,c_1^2\,c_2\,\left(2\,\beta_0 - 32\,\beta_0^3 + 48\,\beta_0\,\beta_1 - 48\,\beta_0\,\delta_{10} + 24\,\delta_{20}\right) + \frac{1}{32}\,c_1\,c_2^2\,\left(2\,\ldots\right.\right. \\ t[1] \\ t[2] \end{pmatrix}\right.$$

In[27]:=
```
Φ2 = Φ[V2]
```

Out[27]=

$$\begin{pmatrix} 1 \\ t[1] \\ t[2] \quad \left(\frac{c_3}{24} - c_3\,\beta_0^2 + c_3\,\beta_1 - c_3\,\delta_{10}\right)\hbar + \left(-\frac{5}{384}\,c_1^2\,c_3 - \frac{7}{192}\,c_1\,c_2\,c_3 - \frac{29\,c_2^2\,c_3}{1152} - \frac{23\,c_1\,c_3^2}{1152} - \frac{29\,c_2\,c_3^2}{1152} - \frac{11\,c_3^3}{1152} - \frac{7}{48}\,c_1^2\,c_{\ldots}\right. \\ t[3] \end{pmatrix}$$

In[28]:=
```
Φ2 ** (Φ2 // dS[1] // dS[2] // dS[3])
```

Out[28]= (1)

In[29]:=
```
{V2, C2} >> GeneralBestSolution-120328.m
```