Pensieve header: β-calculations.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2012-03"];
<< betaCalculus.m
```

The Knot-Theoretic Equations

R2, OC, R3 and easy R4

```
{R[1, 2] Ri[3, 4],
 R[1, 2] Ri[3, 4] // dm[1, 3, 1] // dm[2, 4, 2],
 R[1, 2] Ri[3, 4] // dm[1, 3, 1] // dm[4, 2, 2]
}
```

$$\left\{\begin{pmatrix} 1 & h[2] & h[4] \\ t[1] & \frac{-1+e^{c_1}}{c_1} & 0 \\ t[3] & 0 & \frac{e^{-c_3}\left(1-e^{c_3}\right)}{c_3} \end{pmatrix}, (1), (1)\right\}$$

```
{
 R[1, 2] ** Ri[1, 2],
 R[1, 3] ** R[2, 3],
 R[1, 3] ** R[2, 3] == R[2, 3] ** R[1, 3] // Simplify,
 R[3, 1] ** R[3, 2] == R[3, 2] ** R[3, 1],
 R[1, 2] ** R[1, 3] ** R[2, 3],
 R[1, 2] ** R[1, 3] ** R[2, 3] == R[2, 3] ** R[1, 3] ** R[1, 2]
}
```

$$\left\{(1), \begin{pmatrix} 1 & h[3] \\ t[1] & \frac{-1+e^{c_1}}{c_1} \\ t[2] & \frac{-e^{c_1}+e^{c_1+c_2}}{c_2} \end{pmatrix}, \frac{\left(-1+e^{c_1}\right)\left(-1+e^{c_2}\right)}{c_1} == 0 \;\&\&\; \frac{\left(-1+e^{c_1}\right)\left(-1+e^{c_2}\right)}{c_2} == 0,\right.$$
$$\left.\text{True}, \begin{pmatrix} 1 & h[2] & h[3] \\ t[1] & \frac{-1+e^{c_1}}{c_1} & \frac{-1+e^{c_1}}{c_1} \\ t[2] & 0 & \frac{-e^{c_1}+e^{c_1+c_2}}{c_2} \end{pmatrix}, \text{True}\right\}$$

```
{
 R[3, 1] ** R[3, 2],
 R[3, 1],
 R[3, 1] // dΔ[1, 1, 2],
 R[3, 1] ** R[3, 2] == (R[3, 1] // dΔ[1, 1, 2])
}
```

$$\left\{\begin{pmatrix} 1 & h[1] & h[2] \\ t[3] & \frac{-1+e^{c_3}}{c_3} & \frac{-1+e^{c_3}}{c_3} \end{pmatrix}, \begin{pmatrix} 1 & h[1] \\ t[3] & \frac{-1+e^{c_3}}{c_3} \end{pmatrix}, \begin{pmatrix} 1 & h[1] & h[2] \\ t[3] & \frac{-1+e^{c_3}}{c_3} & \frac{-1+e^{c_3}}{c_3} \end{pmatrix}, \text{True}\right\}$$

```
R[1, 2, p1] ** R[1, 2, p2] == R[1, 2, p1 + p2] // Simplify
```

True

Hard R4

```
{
 R[1, 3] ** R[2, 3],
 R[1, 3] // dΔ[1, 1, 2],
 R[1, 3] ** R[2, 3] == (R[1, 3] // dΔ[1, 1, 2])
}
```

$$\left\{\begin{pmatrix} 1 & h[3] \\ t[1] & \frac{-1+e^{c_1}}{c_1} \\ t[2] & \frac{-e^{c_1}+e^{c_1+c_2}}{c_2} \end{pmatrix}, \begin{pmatrix} 1 & h[3] \\ t[1] & \frac{-1+e^{c_1+c_2}}{c_1+c_2} \\ t[2] & \frac{-1+e^{c_1+c_2}}{c_1+c_2} \end{pmatrix}, \frac{-1+e^{c_1}}{c_1} == \frac{-1+e^{c_1+c_2}}{c_1+c_2} \&\& \frac{-e^{c_1}+e^{c_1+c_2}}{c_2} == \frac{-1+e^{c_1+c_2}}{c_1+c_2}\right\}$$

```
{
 V0 = B[V0[c1, c2], Sum[V10 i+j[c1, c2] t[i] h[j], {i, 2}, {j, 2}]],
 R[1, 3] ** R[2, 3] ** V0,
 V0 ** (R[1, 3] // dΔ[1, 1, 2]),
 eqns1 = HardR4[V0]
}
```

$$\left\{\begin{pmatrix} V_0[c_1, c_2] & h[1] & h[2] \\ t[1] & V_{11}[c_1, c_2] & V_{12}[c_1, c_2] \\ t[2] & V_{21}[c_1, c_2] & V_{22}[c_1, c_2] \end{pmatrix}, \begin{pmatrix} V_0[c_1, c_2] & h[1] & h[2] \\ t[1] & V_{11}[c_1, c_2] & V_{12}[c_1, c_2] & \frac{-1+e^{c_1}+e^{c_1} c_1 V_{12}[c_1,c_2]-}{\ } \\ t[2] & V_{21}[c_1, c_2] & V_{22}[c_1, c_2] & \frac{-e^{c_1}+e^{c_1+c_2}-e^{c_1} c_1 V_{12}[c_1,c_2}{\ } \end{pmatrix}\right.$$

$$\frac{1-e^{c_1+c_2}}{c_1+c_2} + \frac{(-1+e^{c_1})(1+c_2 V_{21}[c_1, c_2])}{c_1} == e^{c_1}(-1+e^{c_2}) V_{12}[c_1, c_2] \&\&$$

$$\left.\frac{1-e^{c_1+c_2}}{c_1+c_2} + \frac{e^{c_1}(-1+e^{c_2})(1+c_1 V_{12}[c_1, c_2])}{c_2} == (-1+e^{c_1}) V_{21}[c_1, c_2]\right\}$$

```
sol = Solve[eqns1 && V21[c1, c2] == 0, V12[c1, c2]]
```

$$\left\{\left\{V_{12}[c_1, c_2] \to -\frac{e^{-c_1}(-e^{c_1} c_1 + e^{c_1+c_2} c_1 + c_2 - e^{c_1} c_2)}{(-1+e^{c_2}) c_1 (c_1+c_2)}\right\}\right\}$$

```
V1 = V0 /. {V21[c1, c2] → 0, V11[c1, c2] → 0, V22[c1, c2] → 0, V0[c1, c2] → 1} /. sol[[1]]
```

$$\begin{pmatrix} 1 & h[2] \\ t[1] & \frac{e^{c_1} c_1 - e^{c_1+c_2} c_1 - c_2 + e^{c_1} c_2}{-e^{c_1} c_1^2 + e^{c_1+c_2} c_1^2 - e^{c_1} c_1 c_2 + e^{c_1+c_2} c_1 c_2} \\ t[2] & 0 \end{pmatrix}$$

Φ and the Pentagon

```
Φ1 = Φ[V1]
```

$$\begin{pmatrix} 1 & h[2] \\ t[1] & \frac{-e^{c_2} c_1 c_2 + e^{c_1+c_2} c_1 c_2 + e^{c_2+c_3} c_1 c_2 - e^{c_1+c_2+c_3} c_1 c_2 - e^{c_2} c_2^2 + e^{c_1+c_2} c_2^2 + e^{c_2+c_3} c_2^2 - e^{c_1+c_2+c_3} c_2^2 + c_1 c_3 - e^{c_2} c_1 c_3 - e^{c_1+c_2+c_3} c_1 c_3 + e^{c_1+2 c_2+c_3} c_1 c_3 - \cdots}{c_1^2 c_2 - e^{c_1+c_2} c_1^2 c_2 - e^{c_2+c_3} c_1^2 c_2 + e^{c_1+2 c_2+c_3} c_1^2 c_2 + c_1 c_2^2 - e^{c_1+c_2} c_1 c_2^2 - e^{c_2+c_3} c_1 c_2^2 + e^{c_1+2 c_2+c_3} c_1 c_2^2 + c_1 c_2 c_3 - e^{c_1+c_2} c_1 c_2 c_3 - \cdots} \\ t[2] & 0 \\ t[3] & 0 \end{pmatrix}$$

```
Pentagon[Φ1] // Simplify
```

True

```
V2 = V0 /. {V21[c1, c2] → 0, V11[c1, c2] → 0, V22[c1, c2] → 0, V0[c1, c2] → 1}
```

$$\begin{pmatrix} 1 & h[2] \\ t[1] & V_{12}[c_1, c_2] \\ t[2] & 0 \end{pmatrix}$$

```
Φ2 = Φ[V2]
```

$$\begin{pmatrix} 1 & h[2] & h[3] \\ t[1] & \frac{-V_{12}[c_1,c_2]+V_{12}[c_1,c_2+c_3]}{1+c_1 V_{12}[c_1,c_2]} & \frac{V_{12}[c_1,c_2+c_3]+c_1 V_{12}[c_1,c_2] V_{12}[c_1,c_2+c_3]-V_{12}[c_1+c_2,c_3]-c_1 V_{12}[c_1,c_2] V_{12}[c_1+c_2,c_3]-c_2 V_{12}[c_1, \ldots}{1+c_1 V_{12}[c_1,c_2]+c_1 V_{12}[c_1+c_2,c_3]+c_2 V_{12}[c_1+c_2,c_3]+c_1^2 V_{12}[c_1,c_2] V_{12}[c_1+c_2,c_3 \ldots} \\ t[2] & 0 & \frac{V_{12}[c_2,c_3]+c_1 V_{12}[c_1,c_2] V_{12}[c_2,c_3]+c_1 V_{12}[c_1,c_2+c_3] V_{12}[c_2,c_3]+c_1^2 V_{12}[c_1,c_2] V_{12}[c_1,c_2+c_3] V_{12}[c \ldots}{1+c_1 V_{12}[c_1,c_2]+c_1 V_{12}[c_1+c_2,c_3]+c_2 V_{12}[c_1+c_2,c_3]+c_1^2 V_{12}[c_1,c_2] V_{12}[c_1+c_2,c_3 \ldots} \\ t[3] & 0 & 0 \end{pmatrix}$$

```
Pentagon[Φ[V2]] // Simplify
```

$$\begin{aligned}
&\Big((1 + c_1 V_{12}[c_1, c_2 + c_3 + c_4]) \Big(c_3 (1 + c_1 V_{12}[c_1, c_2]) (V_{12}[c_1, c_2 + c_3] - V_{12}[c_1 + c_2, c_3]) + \\
&\quad c_2^2 (-V_{12}[c_1, c_2] + V_{12}[c_1, c_2 + c_3]) V_{12}[c_1 + c_2, c_3] - c_2 (V_{12}[c_1, c_2] - V_{12}[c_1, c_2 + c_3]) \\
&\quad (1 + c_1 V_{12}[c_1 + c_2, c_3] + c_3 V_{12}[c_1 + c_2, c_3])\Big) V_{12}[c_1 + c_2 + c_3, c_4]\Big) \Big/ \\
&\quad ((1 + c_1 V_{12}[c_1, c_2]) (1 + c_1 V_{12}[c_1, c_2 + c_3]) (1 + c_1 V_{12}[c_1 + c_2, c_3] + c_2 V_{12}[c_1 + c_2, c_3]) \\
&\quad (1 + c_1 V_{12}[c_1 + c_2 + c_3, c_4] + c_2 V_{12}[c_1 + c_2 + c_3, c_4] + c_3 V_{12}[c_1 + c_2 + c_3, c_4])) == 0 \;\&\& \\
&((1 + c_1 V_{12}[c_1, c_2 + c_3 + c_4]) ((-(1 + c_2 V_{12}[c_2, c_3] + c_3 V_{12}[c_2, c_3]) V_{12}[c_1 + c_2 + c_3, c_4] + \\
&\quad V_{12}[c_2, c_3 + c_4] (1 + c_2 V_{12}[c_2, c_3] + c_1 V_{12}[c_1, c_2 + c_3] (1 + c_2 V_{12}[c_2, c_3]) + \\
&\quad c_3 V_{12}[c_1 + c_2 + c_3, c_4]))) / ((1 + c_1 V_{12}[c_1, c_2 + c_3]) (1 + c_2 V_{12}[c_2, c_3])) - \\
&\quad (-(1 + c_1 V_{12}[c_1 + c_2, c_3] + c_2 V_{12}[c_1 + c_2, c_3] + c_3 V_{12}[c_1 + c_2, c_3]) V_{12}[c_1 + c_2 + c_3, c_4] + \\
&\quad (1 + c_1 V_{12}[c_1, c_2]) V_{12}[c_2, c_3 + c_4] \\
&\quad (1 + c_1 V_{12}[c_1 + c_2, c_3] + c_2 V_{12}[c_1 + c_2, c_3] + c_3 V_{12}[c_1 + c_2 + c_3, c_4])) / \\
&\quad ((1 + c_1 V_{12}[c_1, c_2]) (1 + c_1 V_{12}[c_1 + c_2, c_3] + c_2 V_{12}[c_1 + c_2, c_3])))) / \\
&\quad (1 + c_1 V_{12}[c_1 + c_2 + c_3, c_4] + c_2 V_{12}[c_1 + c_2 + c_3, c_4] + c_3 V_{12}[c_1 + c_2 + c_3, c_4]) == \\
&\quad 0 \;\&\& \\
&((1 + c_1 V_{12}[c_1, c_2 + c_3 + c_4]) (1 + c_2 V_{12}[c_2, c_3 + c_4]) \\
&\quad (c_2 (V_{12}[c_2, c_3] - V_{12}[c_1 + c_2, c_3]) + \\
&\quad c_1 (V_{12}[c_1, c_2 + c_3] (1 + c_2 V_{12}[c_2, c_3]) - V_{12}[c_1 + c_2, c_3])) V_{12}[c_1 + c_2 + c_3, c_4]) / \\
&\quad ((1 + c_1 V_{12}[c_1, c_2 + c_3]) (1 + c_2 V_{12}[c_2, c_3]) (1 + c_1 V_{12}[c_1 + c_2, c_3] + c_2 V_{12}[c_1 + c_2, c_3]) \\
&\quad (1 + c_1 V_{12}[c_1 + c_2 + c_3, c_4] + c_2 V_{12}[c_1 + c_2 + c_3, c_4] + c_3 V_{12}[c_1 + c_2 + c_3, c_4])) == 0
\end{aligned}$$

Θ and the Hexagons

```
{Θ[1, 2],
 Θ[2, 1] == Θ[1, 2] // Simplify,
 (R[1, 2] // dP[1, 23]) ** R[2, 3] == R[2, 3] ** (R[1, 2] // dP[1, 23]),
 (R[2, 1] // dP[1, 23]) ** R[2, 3] == R[2, 3] ** (R[2, 1] // dP[1, 23]) // Simplify,
 (R[2, 1] // dP[1, 23]) ** Θ[2, 3] == Θ[2, 3] ** (R[2, 1] // dP[1, 23])  // Simplify
}
```

$$\left\{\begin{pmatrix} 1 & h[1] & h[2] \\ t[1] & \frac{-c_1+e^{\frac{c_2}{2}} c_1-e^{\frac{c_1}{2}+\frac{c_2}{2}} c_2+e^{\frac{c_2}{2}} c_2}{c_1^2+c_1 c_2} & \frac{-1+e^{\frac{c_1}{2}+\frac{c_2}{2}}}{c_1+c_2} \\ t[2] & \frac{-1+e^{\frac{c_1}{2}+\frac{c_2}{2}}}{c_1+c_2} & \frac{e^{\frac{c_1}{2}} c_1-e^{\frac{c_1}{2}+\frac{c_2}{2}} c_1-c_2+e^{\frac{c_1}{2}} c_2}{c_1 c_2+c_2^2} \end{pmatrix}, \text{True},\right.$$

$$\left. \text{True}, \frac{(-1 + e^{c_2}) (-1 + e^{c_2+c_3}) c_3}{c_2 (c_2 + c_3)} == 0 \;\&\&\; \frac{(-1 + e^{c_2}) (-1 + e^{c_2+c_3})}{c_2 + c_3} == 0, \text{True}\right\}$$

```
t1 = Hexagon[+1, Φ[V1]]
```

A very large output was generated. Here is a sample of it:

<<1>>

Show Less | Show More | Show Full Output | Set Size Limit...

```
Simplify[t1]
```

A very large output was generated. Here is a sample of it:

<<1>>

Show Less | Show More | Show Full Output | Set Size Limit...

The Twist Equation

```
V0
```

$$\begin{pmatrix} V_0[c_1, c_2] & h[1] & h[2] \\ t[1] & V_{11}[c_1, c_2] & V_{12}[c_1, c_2] \\ t[2] & V_{21}[c_1, c_2] & V_{22}[c_1, c_2] \end{pmatrix}$$

```
eqns2 = Simplify[(V0 // dP[2, 1]) ** Θ[1, 2] == R[1, 2] ** V0]
```

$$V_0[c_1, c_2] == V_0[c_2, c_1] \;\&\&\; \frac{1}{c_1 (c_1 + c_2)} \left(-e^{\frac{c_2}{2}} \left(-1 + e^{\frac{c_1}{2}}\right) c_2 \left(1 + c_2 V_{12}[c_2, c_1]\right) + c_1 \left(-1 + e^{\frac{c_2}{2}} - e^{\frac{c_2}{2}} c_2 \left(\left(-1 + e^{\frac{c_1}{2}}\right) V_{12}[c_2, c_1] - V_{22}[c_2, c_1]\right)\right) + e^{\frac{c_2}{2}} c_1^2 V_{22}[c_2, c_1]\right) ==$$

$$V_{11}[c_1, c_2] \;\&\&\; \frac{1}{c_1 + c_2} \left(-1 + e^{\frac{1}{2}(c_1+c_2)} + e^{\frac{1}{2}(c_1+c_2)} c_1 V_{21}[c_2, c_1] + e^{\frac{1}{2}(c_1+c_2)} c_2 V_{21}[c_2, c_1]\right) ==$$

$$\frac{1}{c_1} \left(e^{c_1} c_1 V_{12}[c_1, c_2] + \left(-1 + e^{c_1}\right) \left(1 + c_2 V_{21}[c_1, c_2]\right)\right) \;\&\&$$

$$\frac{1}{c_1 + c_2} \left(-1 + e^{\frac{1}{2}(c_1+c_2)} + e^{\frac{1}{2}(c_1+c_2)} c_1 V_{12}[c_2, c_1] + e^{\frac{1}{2}(c_1+c_2)} c_2 V_{12}[c_2, c_1]\right) == V_{21}[c_1, c_2] \;\&\&$$

$$\left(-1 + e^{c_1}\right) V_{21}[c_1, c_2] + \frac{1}{c_2 (c_1 + c_2)} \left(c_2 \left(-1 + e^{\frac{c_1}{2}} + e^{\frac{c_1}{2}} c_2 V_{11}[c_2, c_1]\right) - e^{\frac{c_1}{2}} \left(-1 + e^{\frac{c_2}{2}}\right) c_1^2 V_{21}[c_2, c_1] - e^{\frac{c_1}{2}} c_1 \left(-1 + e^{\frac{c_2}{2}} - c_2 \left(V_{11}[c_2, c_1] - \left(-1 + e^{\frac{c_2}{2}}\right) V_{21}[c_2, c_1]\right)\right)\right) == e^{c_1} V_{22}[c_1, c_2]$$

The Non-Degeneracy Equations

```
eqns3 = Simplify[dη[1][V0] == B[1, 0] && dη[2][V0] == B[1, 0]]
```

$$V_0[0, c_2] == 1 \;\&\&\; V_{22}[0, c_2] == 0 \;\&\&\; V_0[c_1, 0] == 1 \;\&\&\; V_{11}[c_1, 0] == 0$$

Solving 1-3

eqns13 = eqns1 && eqns2 && eqns3

$$\frac{1-e^{c_1+c_2}}{c_1+c_2}+\frac{(-1+e^{c_1})\,(1+c_2\,V_{21}[c_1,\,c_2])}{c_1} == e^{c_1}\,(-1+e^{c_2})\,V_{12}[c_1,\,c_2]\ \&\&$$

$$\frac{1-e^{c_1+c_2}}{c_1+c_2}+\frac{e^{c_1}\,(-1+e^{c_2})\,(1+c_1\,V_{12}[c_1,\,c_2])}{c_2} == (-1+e^{c_1})\,V_{21}[c_1,\,c_2]\ \&\&$$

$$V_0[c_1,\,c_2] == V_0[c_2,\,c_1]\ \&\&\ \frac{1}{c_1\,(c_1+c_2)}\left(-e^{\frac{c_2}{2}}\left(-1+e^{\frac{c_1}{2}}\right)c_2\,(1+c_2\,V_{12}[c_2,\,c_1])+\right.$$

$$\left.c_1\left(-1+e^{\frac{c_2}{2}}-e^{\frac{c_2}{2}}\,c_2\left(\left(-1+e^{\frac{c_1}{2}}\right)V_{12}[c_2,\,c_1]-V_{22}[c_2,\,c_1]\right)\right)+e^{\frac{c_2}{2}}\,c_1^2\,V_{22}[c_2,\,c_1]\right) ==$$

$$V_{11}[c_1,\,c_2]\ \&\&\ \frac{1}{c_1+c_2}\left(-1+e^{\frac{1}{2}(c_1+c_2)}+e^{\frac{1}{2}(c_1+c_2)}\,c_1\,V_{21}[c_2,\,c_1]+e^{\frac{1}{2}(c_1+c_2)}\,c_2\,V_{21}[c_2,\,c_1]\right) ==$$

$$\frac{1}{c_1}\,(e^{c_1}\,c_1\,V_{12}[c_1,\,c_2]+(-1+e^{c_1})\,(1+c_2\,V_{21}[c_1,\,c_2]))\ \&\&$$

$$\frac{1}{c_1+c_2}\left(-1+e^{\frac{1}{2}(c_1+c_2)}+e^{\frac{1}{2}(c_1+c_2)}\,c_1\,V_{12}[c_2,\,c_1]+e^{\frac{1}{2}(c_1+c_2)}\,c_2\,V_{12}[c_2,\,c_1]\right) == V_{21}[c_1,\,c_2]\ \&\&$$

$$(-1+e^{c_1})\,V_{21}[c_1,\,c_2]+\frac{1}{c_2\,(c_1+c_2)}\left(c_2\left(-1+e^{\frac{c_1}{2}}+e^{\frac{c_1}{2}}\,c_2\,V_{11}[c_2,\,c_1]\right)-\right.$$

$$\left.e^{\frac{c_1}{2}}\left(-1+e^{\frac{c_2}{2}}\right)c_1^2\,V_{21}[c_2,\,c_1]-e^{\frac{c_1}{2}}\,c_1\left(-1+e^{\frac{c_2}{2}}-c_2\left(V_{11}[c_2,\,c_1]-\left(-1+e^{\frac{c_2}{2}}\right)V_{21}[c_2,\,c_1]\right)\right)\right) ==$$

$$e^{c_1}\,V_{22}[c_1,\,c_2]\ \&\&\ V_0[0,\,c_2] == 1\ \&\&\ V_{22}[0,\,c_2] == 0\ \&\&\ V_0[c_1,\,0] == 1\ \&\&\ V_{11}[c_1,\,0] == 0$$

Print /@ Solve[

$$\frac{1-e^{c_1+c_2}}{c_1+c_2}+\frac{(-1+e^{c_1})\,(1+c_2\,V_{21}[c_1,\,c_2])}{c_1} == e^{c_1}\,(-1+e^{c_2})\,V_{12}[c_1,\,c_2]\ \&\&$$

$$\frac{1-e^{c_1+c_2}}{c_1+c_2}+\frac{e^{c_1}\,(-1+e^{c_2})\,(1+c_1\,V_{12}[c_1,\,c_2])}{c_2} == (-1+e^{c_1})\,V_{21}[c_1,\,c_2]\ \&\&$$

$$\frac{1}{c_1\,(c_1+c_2)}\left(-e^{\frac{c_2}{2}}\left(-1+e^{\frac{c_1}{2}}\right)c_2\,(1+c_2\,V_{12}[c_2,\,c_1])+\right.$$

$$\left.c_1\left(-1+e^{\frac{c_2}{2}}-e^{\frac{c_2}{2}}\,c_2\left(\left(-1+e^{\frac{c_1}{2}}\right)V_{12}[c_2,\,c_1]-V_{22}[c_2,\,c_1]\right)\right)+e^{\frac{c_2}{2}}\,c_1^2\,V_{22}[c_2,\,c_1]\right) ==$$

$$V_{11}[c_1,\,c_2]\ \&\&\ \frac{1}{c_1+c_2}\left(-1+e^{\frac{1}{2}(c_1+c_2)}+e^{\frac{1}{2}(c_1+c_2)}\,c_1\,V_{21}[c_2,\,c_1]+e^{\frac{1}{2}(c_1+c_2)}\,c_2\,V_{21}[c_2,\,c_1]\right) ==$$

$$\frac{1}{c_1}\,(e^{c_1}\,c_1\,V_{12}[c_1,\,c_2]+(-1+e^{c_1})\,(1+c_2\,V_{21}[c_1,\,c_2]))\ \&\&$$

$$\frac{1}{c_1+c_2}\left(-1+e^{\frac{1}{2}(c_1+c_2)}+e^{\frac{1}{2}(c_1+c_2)}\,c_1\,V_{12}[c_2,\,c_1]+e^{\frac{1}{2}(c_1+c_2)}\,c_2\,V_{12}[c_2,\,c_1]\right) == V_{21}[c_1,\,c_2]\ \&\&$$

$$(-1+e^{c_1})\,V_{21}[c_1,\,c_2]+\frac{1}{c_2\,(c_1+c_2)}\left(c_2\left(-1+e^{\frac{c_1}{2}}+e^{\frac{c_1}{2}}\,c_2\,V_{11}[c_2,\,c_1]\right)-e^{\frac{c_1}{2}}\left(-1+e^{\frac{c_2}{2}}\right)c_1^2\right.$$

$$\left.V_{21}[c_2,\,c_1]-e^{\frac{c_1}{2}}\,c_1\left(-1+e^{\frac{c_2}{2}}-c_2\left(V_{11}[c_2,\,c_1]-\left(-1+e^{\frac{c_2}{2}}\right)V_{21}[c_2,\,c_1]\right)\right)\right) ==$$

$$e^{c_1}\,V_{22}[c_1,\,c_2],\ \{V_{11}[c_2,\,c_1],\ V_{12}[c_2,\,c_1],\ V_{21}[c_2,\,c_1],\ V_{22}[c_2,\,c_1]\}\Big][[1]];$$

$$V_{11}[c_2, c_1] \to \frac{1}{c_2\,(c_1 + c_2)} \left(e^{\frac{c_1}{2}} c_1 - e^{\frac{c_1}{2}+\frac{c_2}{2}} c_1 - c_2 + e^{\frac{c_1}{2}} c_2 + e^{\frac{c_1}{2}} c_1^2 V_{12}[c_1, c_2] - e^{\frac{c_1}{2}+\frac{c_2}{2}} c_1^2 V_{12}[c_1, c_2] + e^{\frac{c_1}{2}} c_1 c_2 V_{12}[c_1, c_2] - e^{\frac{c_1}{2}+\frac{c_2}{2}} c_1 c_2 V_{12}[c_1, c_2] + e^{\frac{c_1}{2}} c_1 c_2 V_{22}[c_1, c_2] + e^{\frac{c_1}{2}} c_2^2 V_{22}[c_1, c_2]\right)$$

$$V_{12}[c_2, c_1] \to -\frac{1}{c_1 + c_2} e^{-\frac{c_1}{2}-\frac{c_2}{2}} \left(-1 + e^{\frac{c_1}{2}+\frac{c_2}{2}} - c_1 V_{21}[c_1, c_2] - c_2 V_{21}[c_1, c_2]\right)$$

$$V_{21}[c_2, c_1] \to \frac{1}{c_1 + c_2} \left(-1 + e^{\frac{c_1}{2}+\frac{c_2}{2}} + e^{\frac{c_1}{2}+\frac{c_2}{2}} c_1 V_{12}[c_1, c_2] + e^{\frac{c_1}{2}+\frac{c_2}{2}} c_2 V_{12}[c_1, c_2]\right)$$

$$V_{22}[c_2, c_1] \to -\frac{1}{c_1\,(c_1 + c_2)} e^{-\frac{c_1}{2}-\frac{c_2}{2}} \left(-e^{\frac{c_1}{2}} c_1 + e^{\frac{c_1}{2}+\frac{c_2}{2}} c_1 + c_2 - e^{\frac{c_1}{2}} c_2 - e^{\frac{c_1}{2}} c_1^2 V_{11}[c_1, c_2] - e^{\frac{c_1}{2}} c_1 c_2 V_{11}[c_1, c_2] + c_1 c_2 V_{21}[c_1, c_2] - e^{\frac{c_1}{2}} c_1 c_2 V_{21}[c_1, c_2] + c_2^2 V_{21}[c_1, c_2] - e^{\frac{c_1}{2}} c_2^2 V_{21}[c_1, c_2]\right)$$

$$\textbf{eqns} = \frac{1 - e^{c_1+c_2}}{c_1 + c_2} + \frac{(-1 + e^{c_1})\,(1 + c_2 V_{21}[c_1, c_2])}{c_1} == e^{c_1}\,(-1 + e^{c_2})\, V_{12}[c_1, c_2] \;\&\&$$

$$\frac{1 - e^{c_1+c_2}}{c_1 + c_2} + \frac{e^{c_1}\,(-1 + e^{c_2})\,(1 + c_1 V_{12}[c_1, c_2])}{c_2} == (-1 + e^{c_1})\, V_{21}[c_1, c_2] \;\&\&$$

$$\frac{1}{c_1\,(c_1 + c_2)} \left(-e^{\frac{c_2}{2}} \left(-1 + e^{\frac{c_1}{2}}\right) c_2\,(1 + c_2 V_{12}[c_2, c_1]) + c_1 \left(-1 + e^{\frac{c_2}{2}} - e^{\frac{c_2}{2}} c_2 \left(\left(-1 + e^{\frac{c_1}{2}}\right) V_{12}[c_2, c_1] - V_{22}[c_2, c_1]\right)\right) + e^{\frac{c_2}{2}} c_1^2 V_{22}[c_2, c_1]\right) ==$$

$$V_{11}[c_1, c_2] \;\&\&\; \frac{1}{c_1 + c_2} \left(-1 + e^{\frac{1}{2}(c_1+c_2)} + e^{\frac{1}{2}(c_1+c_2)} c_1 V_{21}[c_2, c_1] + e^{\frac{1}{2}(c_1+c_2)} c_2 V_{21}[c_2, c_1]\right) ==$$

$$\frac{1}{c_1} \left(e^{c_1} c_1 V_{12}[c_1, c_2] + (-1 + e^{c_1})\,(1 + c_2 V_{21}[c_1, c_2])\right) \;\&\&$$

$$\frac{1}{c_1 + c_2} \left(-1 + e^{\frac{1}{2}(c_1+c_2)} + e^{\frac{1}{2}(c_1+c_2)} c_1 V_{12}[c_2, c_1] + e^{\frac{1}{2}(c_1+c_2)} c_2 V_{12}[c_2, c_1]\right) == V_{21}[c_1, c_2] \;\&\&$$

$$(-1 + e^{c_1})\, V_{21}[c_1, c_2] + \frac{1}{c_2\,(c_1 + c_2)} \left(c_2 \left(-1 + e^{\frac{c_1}{2}} + e^{\frac{c_1}{2}} c_2 V_{11}[c_2, c_1]\right) - e^{\frac{c_1}{2}} \left(-1 + e^{\frac{c_2}{2}}\right) c_1^2 V_{21}[c_2, c_1] - e^{\frac{c_1}{2}} c_1 \left(-1 + e^{\frac{c_2}{2}} - c_2 \left(V_{11}[c_2, c_1] - \left(-1 + e^{\frac{c_2}{2}}\right) V_{21}[c_2, c_1]\right)\right)\right) == e^{c_1} V_{22}[c_1, c_2];$$

$$\texttt{Solve}\left[\texttt{eqns}\ \&\&\ (\texttt{eqns}\ /.\ \{c_1 \to c_2,\ c_2 \to c_1\})\ \&\&\ V_{12}[c_2, c_1] == -V_{12}[c_1, c_2],\ \{V_{11}[c_1, c_2],\ V_{12}[c_1, c_2],\ V_{21}[c_1, c_2],\ V_{22}[c_1, c_2],\ V_{11}[c_2, c_1],\ V_{12}[c_2, c_1],\ V_{21}[c_2, c_1],\ V_{22}[c_2, c_1]\}\right][[1]]$$

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

$$\Big\{V_{12}[c_1, c_2] \to \left(e^{\frac{c_1}{2}} c_1 - e^{\frac{c_1}{2}+c_2} c_1 + e^{c_1+\frac{c_2}{2}} c_2 - e^{\frac{c_2}{2}} c_2\right) \Big/ \left((c_1 + c_2) \left(-e^{\frac{c_1}{2}} c_1 + e^{\frac{c_1}{2}+c_2} c_1 + e^{c_1+\frac{c_2}{2}} c_2 - e^{\frac{c_2}{2}} c_2\right)\right),$$

$$V_{21}[c_1, c_2] \to \left(e^{\frac{c_1}{2}} c_1 - 2\, e^{c_1+\frac{c_2}{2}} c_1 - e^{\frac{c_1}{2}+c_2} c_1 + 2\, e^{c_1+\frac{3 c_2}{2}} c_1 - e^{c_1+\frac{c_2}{2}} c_2 + e^{\frac{c_2}{2}} c_2\right) \Big/ \left((c_1 + c_2) \left(-e^{\frac{c_1}{2}} c_1 + e^{\frac{c_1}{2}+c_2} c_1 + e^{c_1+\frac{c_2}{2}} c_2 - e^{\frac{c_2}{2}} c_2\right)\right),\ V_{11}[c_2, c_1] \to$$

$$\left(\left(-1 + e^{\frac{c_1}{2}}\right) \left(-e^{\frac{c_1}{2}} c_1 + 2\, e^{\frac{c_1}{2}+\frac{c_2}{2}} c_1 + 2\, e^{c_1+\frac{c_2}{2}} c_1 - e^{\frac{c_1}{2}+c_2} c_1 - 2\, e^{c_1+c_2} c_1 + e^{c_1+\frac{c_2}{2}} c_2 - e^{\frac{c_2}{2}} c_2\right)\right) \Big/ \left((c_1 + c_2) \left(-e^{\frac{c_1}{2}} c_1 + e^{\frac{c_1}{2}+c_2} c_1 + e^{c_1+\frac{c_2}{2}} c_2 - e^{\frac{c_2}{2}} c_2\right)\right) + e^{\frac{c_1}{2}} V_{22}[c_1, c_2],\ V_{12}[c_2, c_1] \to$$

$$-\left(e^{\frac{c_1}{2}} c_1 - e^{\frac{c_1}{2}+c_2} c_1 + e^{c_1+\frac{c_2}{2}} c_2 - e^{\frac{c_2}{2}} c_2\right) \Big/ \left((c_1 + c_2) \left(-e^{\frac{c_1}{2}} c_1 + e^{\frac{c_1}{2}+c_2} c_1 + e^{c_1+\frac{c_2}{2}} c_2 - e^{\frac{c_2}{2}} c_2\right)\right),$$

$$V_{21}[c_2, c_1] \to -\left(-e^{\frac{c_1}{2}} c_1 + e^{\frac{c_1}{2}+c_2} c_1 + e^{c_1+\frac{c_2}{2}} c_2 - e^{\frac{c_2}{2}} c_2 + 2\, e^{\frac{c_1}{2}+c_2} c_2 - 2\, e^{\frac{3 c_1}{2}+c_2} c_2\right) \Big/ \left((c_1 + c_2) \left(-e^{\frac{c_1}{2}} c_1 + e^{\frac{c_1}{2}+c_2} c_1 + e^{c_1+\frac{c_2}{2}} c_2 - e^{\frac{c_2}{2}} c_2\right)\right),\ V_{22}[c_2, c_1] \to$$

$$\left(e^{-\frac{c_2}{2}} \left(-1 + e^{\frac{c_2}{2}}\right) \left(e^{\frac{c_1}{2}} c_1 - e^{\frac{c_1}{2}+c_2} c_1 - 2\, e^{\frac{c_1}{2}+\frac{c_2}{2}} c_2 + e^{c_1+\frac{c_2}{2}} c_2 + e^{\frac{c_2}{2}} c_2 - 2\, e^{\frac{c_1}{2}+c_2} c_2 + 2\, e^{c_1+c_2} c_2\right)\right) \Big/ \left((c_1 + c_2) \left(-e^{\frac{c_1}{2}} c_1 + e^{\frac{c_1}{2}+c_2} c_1 + e^{c_1+\frac{c_2}{2}} c_2 - e^{\frac{c_2}{2}} c_2\right)\right) + e^{-\frac{c_2}{2}} V_{11}[c_1, c_2]\Big\}$$

$$\left(\left(e^{\frac{c_1}{2}} c_1 - e^{\frac{c_1}{2}+c_2} c_1 + e^{c_1+\frac{c_2}{2}} c_2 - e^{\frac{c_2}{2}} c_2\right) \Big/ \left((c_1 + c_2) \left(-e^{\frac{c_1}{2}} c_1 + e^{\frac{c_1}{2}+c_2} c_1 + e^{c_1+\frac{c_2}{2}} c_2 - e^{\frac{c_2}{2}} c_2\right)\right)\ /.\ \{c_{i\_} :\to c_{3-i}\}\right) == -\left(e^{\frac{c_1}{2}} c_1 - e^{\frac{c_1}{2}+c_2} c_1 + e^{c_1+\frac{c_2}{2}} c_2 - e^{\frac{c_2}{2}} c_2\right) \Big/ \left((c_1 + c_2) \left(-e^{\frac{c_1}{2}} c_1 + e^{\frac{c_1}{2}+c_2} c_1 + e^{c_1+\frac{c_2}{2}} c_2 - e^{\frac{c_2}{2}} c_2\right)\right)\ //\ \texttt{Simplify}$$

True

$$\texttt{Series}\left[\left(e^{\frac{c_1}{2}} c_1 - e^{\frac{c_1}{2}+c_2} c_1 + e^{c_1+\frac{c_2}{2}} c_2 - e^{\frac{c_2}{2}} c_2\right) \Big/ \left((c_1 + c_2) \left(-e^{\frac{c_1}{2}} c_1 + e^{\frac{c_1}{2}+c_2} c_1 + e^{c_1+\frac{c_2}{2}} c_2 - e^{\frac{c_2}{2}} c_2\right)\right),\ \{c_1, 0, 2\},\ \{c_2, 0, 2\}\right]$$

$$\left(-\frac{c_2}{48} + O[c_2]^3\right) + \left(\frac{1}{48} - \frac{c_2^2}{5760} + O[c_2]^3\right) c_1 + \left(\frac{c_2}{5760} + O[c_2]^3\right) c_1^2 + O[c_1]^3$$

```
sol2 = Solve[eqns && (eqns /. {c1 → c2, c2 → c1}) && V12[c2, c1] == -V12[c1, c2] &&
    V22[c1, c2] == 0 && V22[c2, c1] == 0, {V11[c1, c2], V12[c1, c2], V21[c1, c2],
    V22[c1, c2], V11[c2, c1], V12[c2, c1], V21[c2, c1], V22[c2, c1]}][[1]]
```

$$\Big\{V_{11}[c_1, c_2] \to -\Big(\big(-1 + e^{\frac{c_2}{2}}\big)\big(e^{\frac{c_1}{2}} c_1 - e^{\frac{c_1}{2}+c_2} c_1 - 2 e^{\frac{c_1}{2}+\frac{c_2}{2}} c_2 + e^{c_1+\frac{c_2}{2}} c_2 + e^{\frac{c_2}{2}} c_2 - 2 e^{\frac{c_1}{2}+c_2} c_2 + 2 e^{c_1+c_2} c_2\big)\Big) \Big/ \Big((c_1 + c_2)\big(-e^{\frac{c_1}{2}} c_1 + e^{\frac{c_1}{2}+c_2} c_1 + e^{c_1+\frac{c_2}{2}} c_2 - e^{\frac{c_2}{2}} c_2\big)\Big),$$

$$V_{12}[c_1, c_2] \to \big(e^{\frac{c_1}{2}} c_1 - e^{\frac{c_1}{2}+c_2} c_1 + e^{c_1+\frac{c_2}{2}} c_2 - e^{\frac{c_2}{2}} c_2\big) \Big/ \Big((c_1 + c_2)\big(-e^{\frac{c_1}{2}} c_1 + e^{\frac{c_1}{2}+c_2} c_1 + e^{c_1+\frac{c_2}{2}} c_2 - e^{\frac{c_2}{2}} c_2\big)\Big),$$

$$V_{21}[c_1, c_2] \to \big(e^{\frac{c_1}{2}} c_1 - 2 e^{c_1+\frac{c_2}{2}} c_1 - e^{\frac{c_1}{2}+c_2} c_1 + 2 e^{c_1+\frac{3 c_2}{2}} c_1 - e^{c_1+\frac{c_2}{2}} c_2 + e^{\frac{c_2}{2}} c_2\big) \Big/ \Big((c_1 + c_2)\big(-e^{\frac{c_1}{2}} c_1 + e^{\frac{c_1}{2}+c_2} c_1 + e^{c_1+\frac{c_2}{2}} c_2 - e^{\frac{c_2}{2}} c_2\big)\Big), V_{22}[c_1, c_2] \to 0,$$

$$V_{11}[c_2, c_1] \to \Big(\big(-1 + e^{\frac{c_1}{2}}\big)\big(-e^{\frac{c_1}{2}} c_1 + 2 e^{\frac{c_1}{2}+\frac{c_2}{2}} c_1 + 2 e^{c_1+\frac{c_2}{2}} c_1 - e^{\frac{c_1}{2}+c_2} c_1 - 2 e^{c_1+c_2} c_1 + e^{c_1+\frac{c_2}{2}} c_2 - e^{\frac{c_2}{2}} c_2\big)\Big) \Big/ \Big((c_1 + c_2)\big(-e^{\frac{c_1}{2}} c_1 + e^{\frac{c_1}{2}+c_2} c_1 + e^{c_1+\frac{c_2}{2}} c_2 - e^{\frac{c_2}{2}} c_2\big)\Big),$$

$$V_{12}[c_2, c_1] \to -\big(e^{\frac{c_1}{2}} c_1 - e^{\frac{c_1}{2}+c_2} c_1 + e^{c_1+\frac{c_2}{2}} c_2 - e^{\frac{c_2}{2}} c_2\big) \Big/ \Big((c_1 + c_2)\big(-e^{\frac{c_1}{2}} c_1 + e^{\frac{c_1}{2}+c_2} c_1 + e^{c_1+\frac{c_2}{2}} c_2 - e^{\frac{c_2}{2}} c_2\big)\Big),$$

$$V_{21}[c_2, c_1] \to -\big(-e^{\frac{c_1}{2}} c_1 + e^{\frac{c_1}{2}+c_2} c_1 + e^{c_1+\frac{c_2}{2}} c_2 - e^{\frac{c_2}{2}} c_2 + 2 e^{\frac{c_1}{2}+c_2} c_2 - 2 e^{\frac{3 c_1}{2}+c_2} c_2\big) \Big/ \Big((c_1 + c_2)\big(-e^{\frac{c_1}{2}} c_1 + e^{\frac{c_1}{2}+c_2} c_1 + e^{c_1+\frac{c_2}{2}} c_2 - e^{\frac{c_2}{2}} c_2\big)\Big), V_{22}[c_2, c_1] \to 0\Big\}$$

```
V2 = V0 /. sol2 /. V0[c1, c2] → 1
```

	1	h[1]	
t[1]		$\frac{e^{\frac{c_1}{2}} c_1 - e^{\frac{c_1}{2}+\frac{c_2}{2}} c_1 - e^{\frac{c_1}{2}+c_2} c_1 + e^{\frac{c_1}{2}+\frac{3 c_2}{2}} c_1 - 2 e^{\frac{c_1}{2}+\frac{c_2}{2}} c_2 + e^{c_1+\frac{c_2}{2}} c_2 + e^{\frac{c_2}{2}} c_2 - e^{c_2} c_2 + e^{c_1+c_2} c_2 + 2 e^{\frac{c_1}{2}+\frac{3 c_2}{2}} c_2 - 2 e^{c_1+\frac{3 c_2}{2}} c_2}{-e^{\frac{c_1}{2}} c_1^2 + e^{\frac{c_1}{2}+c_2} c_1^2 - e^{\frac{c_1}{2}} c_1 c_2 + e^{c_1+\frac{c_2}{2}} c_1 c_2 - e^{\frac{c_2}{2}} c_1 c_2 + e^{\frac{c_1}{2}+c_2} c_1 c_2 + e^{c_1+\frac{c_2}{2}} c_2^2 - e^{\frac{c_2}{2}} c_2^2}$	$\frac{}{-e^{\frac{c_1}{2}} c_1^2 + e^{\frac{c_1}{2}+c_2} c_1^2 - e^{\frac{c_1}{2}} c}$
t[2]		$\frac{e^{\frac{c_1}{2}} c_1 - 2 e^{c_1+\frac{c_2}{2}} c_1 - e^{\frac{c_1}{2}+c_2} c_1 + 2 e^{c_1+\frac{3 c_2}{2}} c_1 - e^{c_1+\frac{c_2}{2}} c_2 + e^{\frac{c_2}{2}} c_2}{-e^{\frac{c_1}{2}} c_1^2 + e^{\frac{c_1}{2}+c_2} c_1^2 - e^{\frac{c_1}{2}} c_1 c_2 + e^{c_1+\frac{c_2}{2}} c_1 c_2 - e^{\frac{c_2}{2}} c_1 c_2 + e^{\frac{c_1}{2}+c_2} c_1 c_2 + e^{c_1+\frac{c_2}{2}} c_2^2 - e^{\frac{c_2}{2}} c_2^2}$	

```
V3 = Series[# /. c_i_ :> x c_i, {x, 0, 3}] & /@ V2
```

	1	h[1]	h[2]
t[1]		$\frac{1}{384}\left(-48 x c_2 - 4 x^2 c_1 c_2 - 20 x^2 c_2^2 - 2 x^3 c_1 c_2^2 - 5 x^3 c_2^3\right)$	$\frac{120 x c_1 - x^3 c_1^3 - 120 x c_2 + x^3 c_1^2 c_2 - x^3 c}{5760}$
t[2]		$\frac{2880 + 600 x c_1 + 60 x^2 c_1^2 + x^3 c_1^3 + 840 x c_2 + 240 x^2 c_1 c_2 + 29 x^3 c_1^2 c_2 + 180 x^2 c_2^2 + 61 x^3 c_1 c_2^2 + 29 x^3 c_2^3}{5760}$	0

```
HardR4[V2]
```

True

```
Simplify[(V2 // dP[2, 1]) ** Θ[1, 2] == R[1, 2] ** V2]
```

True

```
Φ2 = Φ[V2]
```

A very large output was generated. Here is a sample of it:

(<<1>>)

Show Less | Show More | Show Full Output | Set Size Limit...

```
t1 = Hexagon[+1, Φ2]
```

```
Simplify[t1]
V3 /. x → 0
```

$$\begin{pmatrix} 1 & h[1] \\ t[2] & \frac{1}{2} \end{pmatrix}$$

The Cap Equations

```
{
  V0 = B[V0[c1, c2], Sum[V10 i+j[c1, c2] t[i] h[j], {i, 2}, {j, 2}]],
  C0 = B[κ[c1], 0],
  C0 // dP[12],
  V0 ** (C0 // dP[12]),
  V0 ** (C0 // dP[12]) // dcap[1] // dcap[2],
  C0 (C0 // dP[2]),
  C0 (C0 // dP[2]) // dcap[1] // dcap[2],
  (V0 ** (C0 // dP[12]) // dcap[1] // dcap[2]) ==
   (C0 (C0 // dP[2]) // dcap[1] // dcap[2]),
  C0 // tη[1]
 } // ColumnForm
```

$$\begin{pmatrix} V_0[c_1, c_2] & h[1] & h[2] \\ t[1] & V_{11}[c_1, c_2] & V_{12}[c_1, c_2] \\ t[2] & V_{21}[c_1, c_2] & V_{22}[c_1, c_2] \end{pmatrix}$$

$$\begin{pmatrix} \kappa[c_1] \\ t[1] \end{pmatrix}$$

$$\begin{pmatrix} \kappa[c_1 + c_2] \\ t[1] \\ t[2] \end{pmatrix}$$

$$\begin{pmatrix} \kappa[c_1 + c_2]\, V_0[c_1, c_2] & h[1] & h[2] \\ t[1] & V_{11}[c_1, c_2] & V_{12}[c_1, c_2] \\ t[2] & V_{21}[c_1, c_2] & V_{22}[c_1, c_2] \end{pmatrix}$$

$$\begin{pmatrix} \frac{\kappa[c_1+c_2]\, V_0[c_1,c_2] + c_1\, \kappa[c_1+c_2]\, V_0[c_1,c_2]\, V_{12}[c_1,c_2] + c_2\, \kappa[c_1+c_2]\, V_0[c_1,c_2]\, V_{21}[c_1,c_2]}{1 + c_1 V_{11}[c_1,c_2] + c_1 V_{12}[c_1,c_2] + c_1^2 V_{11}[c_1,c_2]\, V_{12}[c_1,c_2] + c_2 V_{21}[c_1,c_2] + c_1 c_2 V_{12}[c_1,c_2]\, V_{21}[c_1,c_2] + c_2 V_{22}[c_1,c_2] + c_1 c_2 V_{11}[c_1,c_2]\, V_{22}[c_1,c} \\ t[1] \\ t[2] \end{pmatrix}$$

$$\begin{pmatrix} \kappa[c_1]\, \kappa[c_2] \\ t[1] \\ t[2] \end{pmatrix}$$

$$\begin{pmatrix} \kappa[c_1]\, \kappa[c_2] \\ t[1] \\ t[2] \end{pmatrix}$$

$$\frac{\kappa[c_1+c_2]\, V_0[c_1,c_2] + c_1\, \kappa[c_1+c_2]\, V_0[c_1,c_2]\, V_{12}[c_1,c_2] + c_2\, \kappa[c_1+c_2]\, V_0[c_1,c_2]\, V_{21}[c_1,c_2]}{1 + c_1 V_{11}[c_1,c_2] + c_1 V_{12}[c_1,c_2] + c_1^2 V_{11}[c_1,c_2]\, V_{12}[c_1,c_2] + c_2 V_{21}[c_1,c_2] + c_1 c_2 V_{12}[c_1,c_2]\, V_{21}[c_1,c_2] + c_2 V_{22}[c_1,c_2] + c_1 c_2 V_{11}[c_1,c_2]\, V_{22}[c_1,c_2]}$$

$$(\ \kappa[0]\ )$$