

Pensieve Header: Switching to the Burau presentation; continues pensieve://2012-02/.

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βSimplify = Simplify;
SetAttributes[βCollect, Listable];
βCollect[B[ω_, μ_]] := B[
  βSimplify[ω],
  Collect[μ, _h, Collect[#, _t, βSimplify] &]
];
(* "L" for "Labels" *)
hL[β_] := Union[Cases[β, h[s_] => s, Infinity]];
tL[β_] := Union[Cases[β, t[s_] | T_s_ => s, Infinity]];
dL[β_] := Union[hL[β], tL[β]];
βForm[B[ω_, μ_]] := Module[
  {tails, heads, mat},
  tails = tL[B[ω, μ]]; heads = hL[B[ω, μ]];
  mat = Outer[βSimplify[Coefficient[μ, h[#1] t[#2]]] &, heads, tails];
  PrependTo[mat, t /@ tails];
  mat = Prepend[Transpose[mat], Prepend[h /@ heads, ω]];
  MatrixForm[mat]
];
βForm[else_] := else /. {β_B => βForm[β], β_Bu => βForm[β]};
tm[x_, y_, z_][β_] := β /. {t[x] → t[z], t[y] → t[z], T_x → T_z, T_y → T_z};
hm[x_, y_, z_][B[ω_, μ_]] := Module[
  {γx = D[μ, h[x]], γy = D[μ, h[y]], M = μ /. h[x] | h[y] → 0},
  B[ω, M + h[z] (γx + γy + (γx /. t[i_] => 1) γy)] // βCollect
];
thswap[y_, x_][B[ω_, μ_]] := Module[
  {α, β, γ, δ, ε},
  α = Coefficient[μ, h[x] t[y]];
  β = D[μ, t[y]] /. h[x] → 0;
  γ = D[μ, h[x]] /. t[y] → 0;
  δ = μ /. h[x] | t[y] → 0;
  ε = 1 + α;
  B[ω * ε, Plus[
    α (1 + (γ /. t[i_] => 1) / ε) h[x] t[y],
    β (1 + (γ /. t[i_] => 1) / ε) t[y],
    γ / ε h[x],
    δ - 1 / ε γ * β
  ]] // βCollect
];
dm[x_, y_, z_][β_] := β // thswap[x, y] // hm[x, y, z] // tm[x, y, z];
B /: B[ω1_, μ1_] B[ω2_, μ2_] := B[ω1 * ω2, μ1 + μ2];
R[x_, y_] := B[1, (T_x - 1) * t[x] h[y]];
Ri[x_, y_] := B[1, (1 / T_x - 1) * t[x] h[y]];

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Unprotect[NonCommutativeMultiply];
β_ ** ν_ := Module[
  {ρ, σ, labels},
  ρ = β * (ν /. {h[s_] => h[σ[s]], t[s_] => t[σ[s]], T[s_] => T[σ[s]]});
  labels = Union[Cases[{β, ν}, h[s_] | t[s_] | T[s_] => s, Infinity]];
  Do[
    ρ = ρ // dm[s, σ[s], s],
    {s, labels}
  ];
  ρ
];

βCollect[Bu[ω_, λ_, μ_]] := Bu[
  βSimplify[ω],
  Collect[λ, _h, βSimplify],
  Collect[μ, _h, Collect[#, _t, βSimplify] &]
];

βForm[Bu[ω_, λ_, μ_]] := Module[
  {tails, heads, mat},
  tails = tL[B[ω, λ, μ]];
  heads = hL[B[ω, λ, μ]];
  mat = Outer[βSimplify[Coefficient[μ, h[#1] t[#2]]] &, heads, tails];
  PrependTo[mat, t /@ tails];
  mat = Prepend[Transpose[mat], Prepend[λ, ω]];
  MatrixForm[mat]
];

Bu[n_Integer, β_B] := Bu[h /@ Range[n], β];
Bu[ηs_List, B[ω_, μ_]] := Module[{λ},
  λ = (1 + Coefficient[μ, #] /. t[i_] -> 1) & /@ ηs;
  Bu[ω,
    Thread[ηs -> λ],
    -μ + (ηs /. h[j_] => t[j] h[j]).λ
  ] // βCollect
];

B[Bu[ω_, λ_, μ_]] := βCollect[B[ω,
  -μ + Total[λ /. (h[j_] -> λj_) => t[j] h[j] λj]
]];

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{R[1, 2],
  R[1, 2] //  $\beta$ Form,
  Bu[2, R[1, 2]],
  Bu[2, R[1, 2]] //  $\beta$ Form,
  B[Bu[2, R[1, 2]]],
  B[Bu[2, R[1, 2]]] //  $\beta$ Form
} // ColumnForm

B[1, h[2] (-1 + T1) t[1]]

$$\begin{pmatrix} 1 & h[2] \\ t[1] & -1 + T_1 \end{pmatrix}$$

Bu[1, {h[1] → 1, h[2] → T1}, h[1] t[1] + h[2] ((1 - T1) t[1] + T1 t[2])]

$$\begin{pmatrix} 1 & h[1] \rightarrow 1 & h[2] \rightarrow T_1 \\ t[1] & 1 & 1 - T_1 \\ t[2] & 0 & T_1 \end{pmatrix}$$

B[1, h[2] (-1 + T1) t[1]]

$$\begin{pmatrix} 1 & h[2] \\ t[1] & -1 + T_1 \end{pmatrix}$$


n = 3;
{
   $\beta$ 1 = B[1, Sum[ $\alpha_{10 i+j}$  t[i] h[j], {i, n}, {j, n}]], Bu[n,  $\beta$ 1],
   $\beta$ 2 = B[1, Sum[ $\beta_{10 i+j}$  t[i] h[j], {i, n}, {j, n}]], Bu[n,  $\beta$ 2],
   $\beta$ 1 **  $\beta$ 2,
  Bu[n,  $\beta$ 1 **  $\beta$ 2]
} //  $\beta$ Form // ColumnForm


$$\begin{pmatrix} 1 & h[1] & h[2] & h[3] \\ t[1] & \alpha_{11} & \alpha_{12} & \alpha_{13} \\ t[2] & \alpha_{21} & \alpha_{22} & \alpha_{23} \\ t[3] & \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix}$$


$$\begin{pmatrix} 1 & h[1] \rightarrow 1 + \alpha_{11} + \alpha_{21} + \alpha_{31} & h[2] \rightarrow 1 + \alpha_{12} + \alpha_{22} + \alpha_{32} & h[3] \rightarrow 1 + \alpha_{13} + \alpha_{23} + \alpha_{33} \\ t[1] & 1 + \alpha_{21} + \alpha_{31} & -\alpha_{12} & -\alpha_{13} \\ t[2] & -\alpha_{21} & 1 + \alpha_{12} + \alpha_{32} & -\alpha_{23} \\ t[3] & -\alpha_{31} & -\alpha_{32} & 1 + \alpha_{13} + \alpha_{23} \end{pmatrix}$$


$$\begin{pmatrix} 1 & h[1] & h[2] & h[3] \\ t[1] & \beta_{11} & \beta_{12} & \beta_{13} \\ t[2] & \beta_{21} & \beta_{22} & \beta_{23} \\ t[3] & \beta_{31} & \beta_{32} & \beta_{33} \end{pmatrix}$$


$$\begin{pmatrix} 1 & h[1] \rightarrow 1 + \beta_{11} + \beta_{21} + \beta_{31} & h[2] \rightarrow 1 + \beta_{12} + \beta_{22} + \beta_{32} & h[3] \rightarrow 1 + \beta_{13} + \beta_{23} + \beta_{33} \\ t[1] & 1 + \beta_{21} + \beta_{31} & -\beta_{12} & -\beta_{13} \\ t[2] & -\beta_{21} & 1 + \beta_{12} + \beta_{32} & -\beta_{23} \\ t[3] & -\beta_{31} & -\beta_{32} & 1 + \beta_{13} + \beta_{23} \end{pmatrix}$$


$$\begin{matrix} 1 & & h[1] & & & & & & h[2] \\ t[1] & (1 + \alpha_{21} + \alpha_{31}) \beta_{11} - \alpha_{21} \beta_{12} - \alpha_{31} \beta_{13} + \alpha_{11} (1 + \beta_{11} + \beta_{21} + \beta_{31}) & & & & (1 + \alpha_{32}) \beta_{12} - \alpha_{32} \beta_{13} + \alpha_{12} & & & \\ t[2] & (1 + \alpha_{31}) \beta_{21} - \alpha_{31} \beta_{23} + \alpha_{21} (1 + \beta_{12} + \beta_{21} + \beta_{32}) & & & & (1 + \alpha_{32}) \beta_{22} + \alpha_{12} (-\beta_{21} + \beta_{22}) - \alpha_{32} & & & \\ t[3] & (1 + \alpha_{21}) \beta_{31} + \alpha_{31} (1 + \beta_{13} + \beta_{23} + \beta_{31}) - \alpha_{21} \beta_{32} & & & & \beta_{32} + \alpha_{32} (1 + \beta_{13} + \beta_{23} + \beta_{33}) & & & \end{matrix}$$


$$\begin{matrix} 1 & & h[1] \rightarrow (1 + \alpha_{11} + \alpha_{21} + \alpha_{31}) (1 + \beta_{11} + \beta_{21} + \beta_{31}) & & & & & & h[2] \rightarrow (1 + \alpha_{12} + \alpha_{22} + \alpha_{32}) \\ t[1] & 1 + \beta_{21} + \beta_{31} + \alpha_{21} (1 + \beta_{12} + \beta_{21} + \beta_{31}) + \alpha_{31} (1 + \beta_{13} + \beta_{21} + \beta_{31}) & & & & - (1 + \alpha_{32}) \beta_{12} + \alpha_{32} \beta_{13} - \alpha_{12} & & & \\ t[2] & - (1 + \alpha_{31}) \beta_{21} + \alpha_{31} \beta_{23} - \alpha_{21} (1 + \beta_{12} + \beta_{21} + \beta_{32}) & & & & 1 + \beta_{12} + \beta_{32} + \alpha_{12} (1 + \beta_{12} + \beta_{21} + \beta_{32}) & & & \\ t[3] & - (1 + \alpha_{21}) \beta_{31} - \alpha_{31} (1 + \beta_{13} + \beta_{23} + \beta_{31}) + \alpha_{21} \beta_{32} & & & & \alpha_{12} (\beta_{31} - \beta_{32}) - \beta_{32} - \alpha_{32} & & & \end{matrix}$$


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n = 3;
{
  β1 = Bu[ω1, Table[h[j] → λ1j, {j, n}], Sum[α110 i+j t[i] h[j], {i, n}, {j, n}]], B[β1],
  β2 = Bu[ω2, Table[h[j] → λ2j, {j, n}], Sum[α210 i+j t[i] h[j], {i, n}, {j, n}]], B[β2],
  B[β1] ** B[β2],
  Bu[n, B[β1] ** B[β2]]
} // βForm // ColumnForm

( ω1  h[1] → λ11  h[2] → λ12  h[3] → λ13 )
( t[1]  α111  α112  α113 )
( t[2]  α121  α122  α123 )
( t[3]  α131  α132  α133 )
( ω1  h[1]  h[2]  h[3] )
( t[1] -α111 + λ11  -α112  -α113 )
( t[2]  -α121  -α122 + λ12  -α123 )
( t[3]  -α131  -α132  -α133 + λ13 )
( ω2  h[1] → λ21  h[2] → λ22  h[3] → λ23 )
( t[1]  α211  α212  α213 )
( t[2]  α221  α222  α223 )
( t[3]  α231  α232  α233 )
( ω2  h[1]  h[2]  h[3] )
( t[1] -α211 + λ21  -α212  -α213 )
( t[2]  -α221  -α222 + λ22  -α223 )
( t[3]  -α231  -α232  -α233 + λ23 )
( ω1 ω2  h[1] )
( t[1]  -α121 α212 - α131 α213 + (-1 + α121 + α131) (α211 - λ21) + (α111 - λ11) (-1 + α211 + α221 + α231 - λ1 ) )
( t[2]  (-1 + α131) α221 - α131 α223 + α121 (-1 + α212 + α221 + α232) )
( t[3]  (-1 + α121) α231 + α131 (-1 + α213 + α223 + α231) - α121 α232 )
( ω1 ω2  h[1] → (-1 + α111 + α121 + α131 - λ11) (-1 + α211 + α221 + α231 - λ21)  h[2] → (-1 + α1 )
( t[1]  1 - α221 - α231 + α121 (-1 + α212 + α221 + α231) + α131 (-1 + α213 + α221 + α231)  - (-1 + α1 )
( t[2]  - (-1 + α131) α221 + α131 α223 - α121 (-1 + α212 + α221 + α232)  1 - α212 - α232 + α1 )
( t[3]  - (-1 + α121) α231 - α131 (-1 + α213 + α223 + α231) + α121 α232  α112 (α1 )

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n = 3;
{
  β1 = Bu[ω1, Table[h[j] → λ1j, {j, n}],
    Sum[α110 i+j t[i] h[j] + (1/n - α110 i+j) t[j] h[j], {i, n}, {j, n}]
  ], B[β1],
  β2 = Bu[ω2, Table[h[j] → λ2j, {j, n}],
    Sum[α210 i+j t[i] h[j] + (1/n - α210 i+j) t[j] h[j], {i, n}, {j, n}]
  ], B[β2],
  B[β1] ** B[β2],
  Bu[n, B[β1] ** B[β2]]
} // βForm // ColumnForm

( ω1   h[1] → λ11   h[2] → λ12   h[3] → λ13 )
( t[1] 1 - α121 - α131   α112   α113 )
( t[2]   α121   1 - α112 - α132   α123 )
( t[3]   α131   α132   1 - α113 - α123 )

( ω1   h[1]   h[2]   h[3] )
( t[1] -1 + α121 + α131 + λ11   -α112   -α113 )
( t[2]   -α121   -1 + α112 + α132 + λ12   -α123 )
( t[3]   -α131   -α132   -1 + α113 + α123 + λ13 )

( ω2   h[1] → λ21   h[2] → λ22   h[3] → λ23 )
( t[1] 1 - α221 - α231   α212   α213 )
( t[2]   α221   1 - α212 - α232   α223 )
( t[3]   α231   α232   1 - α213 - α223 )

( ω2   h[1]   h[2]   h[3] )
( t[1] -1 + α221 + α231 + λ21   -α212   -α213 )
( t[2]   -α221   -1 + α212 + α232 + λ22   -α223 )
( t[3]   -α231   -α232   -1 + α213 + α223 + λ23 )

( ω1 ω2   h[1] )
( t[1] -1 + α221 + α231 - α121 (-1 + α212 + α221 + α231) - α131 (-1 + α213 + α221 + α231) + λ11 λ21 )
( t[2]   (-1 + α131) α221 - α131 α223 + α121 (-1 + α212 + α221 + α232)   -1 + α2 )
( t[3]   (-1 + α121) α231 + α131 (-1 + α213 + α223 + α231) - α121 α232 )

( ω1 ω2   h[1] → λ11 λ21 )
( t[1] 1 - α221 - α231 + α121 (-1 + α212 + α221 + α231) + α131 (-1 + α213 + α221 + α231)   - (-1 + α1 )
( t[2]   - (-1 + α131) α221 + α131 α223 - α121 (-1 + α212 + α221 + α232)   1 - α212 - α232 + α1 )
( t[3]   - (-1 + α121) α231 - α131 (-1 + α213 + α223 + α231) + α121 α232   α112 (α )

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n = 3;
{
  β1 = Bu[ω, Table[h[j] → λj, {j, n}],
    Sum[α10 i+j t[i] h[j] + (1/n - α10 i+j) t[j] h[j], {i, n}, {j, n}]
  ], B[β1],
  B[β1] // dm[1, 2, 1],
  Bu[n, B[β1] // dm[1, 2, 1]]
} // βForm // ColumnForm

(
  ω      h[1] → λ1   h[2] → λ2   h[3] → λ3
  t[1]  1 - α21 - α31   α12         α13
  t[2]      α21       1 - α12 - α32   α23
  t[3]      α31         α32       1 - α13 - α23
)

(
  ω      h[1]           h[2]           h[3]
  t[1]  -1 + α21 + α31 + λ1   -α12           -α13
  t[2]      -α21           -1 + α12 + α32 + λ2   -α23
  t[3]      -α31           -α32           -1 + α13 + α23 + λ3
)

(
  ω - ω α12           h[1]           h[3]
  t[1]   $\frac{1 + \alpha_{31}(-1 + \alpha_{32}) - \alpha_{32} + \alpha_{21} \alpha_{32} - \lambda_1 \lambda_2 + \alpha_{12}(-1 + \alpha_{31} + \lambda_1 \lambda_2)}{-1 + \alpha_{12}}$     $-\frac{(-1 + \alpha_{12}) \alpha_{23} + \alpha_{13}(-1 + \alpha_{12} + \alpha_{32})}{-1 + \alpha_{12}}$ 
  t[3]   $-\frac{(-1 + \alpha_{21}) \alpha_{32} + \alpha_{31}(-1 + \alpha_{12} + \alpha_{32})}{-1 + \alpha_{12}}$     $-1 + \alpha_{23} + \alpha_{13} \left(1 + \frac{\alpha_{32}}{-1 + \alpha_{12}}\right) + \lambda_3$ 
)

(
  ω - ω α12           h[1] → λ1 λ2           h[2] → 1           h[3] → λ3
  t[1]   $1 - \frac{(-1 + \alpha_{21}) \alpha_{32} + \alpha_{31}(-1 + \alpha_{12} + \alpha_{32})}{-1 + \alpha_{12}}$    0    $\frac{(-1 + \alpha_{12}) \alpha_{23} + \alpha_{13}(-1 + \alpha_{12} + \alpha_{32})}{-1 + \alpha_{12}}$ 
  t[2]  0   1   0
  t[3]   $\frac{(-1 + \alpha_{21}) \alpha_{32} + \alpha_{31}(-1 + \alpha_{12} + \alpha_{32})}{-1 + \alpha_{12}}$    0    $\frac{-1 + \alpha_{23} - \alpha_{12}(-1 + \alpha_{13} + \alpha_{23}) - \alpha_{13}(-1 + \alpha_{32})}{-1 + \alpha_{12}}$ 
)

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```

n = 4;
{
  β1 = Bu[ω, Table[h[j] → λj, {j, n}],
    Sum[α10 i+j t[i] h[j] + (1/n - α10 i+j) t[j] h[j], {i, n}, {j, n}]
  ], B[β1],
  B[β1] // dm[1, 2, 1],
  Bu[n, B[β1] // dm[1, 2, 1]]
} // βForm // ColumnForm

(
  ω      h[1] → λ1      h[2] → λ2      h[3] → λ3      h[4] → λ4
  t[1]  1 - α21 - α31 - α41      α12      α13      α14
  t[2]      α21      1 - α12 - α32 - α42      α23      α24
  t[3]      α31      α32      1 - α13 - α23 - α43      α34
  t[4]      α41      α42      α43      1 - α14 - α24 - α34
)

(
  ω      h[1]      h[2]      h[3]      h[4]
  t[1]  -1 + α21 + α31 + α41 + λ1      -α12      -α13      -α14
  t[2]      -α21      -1 + α12 + α32 + α42 + λ2      -α23      -α24
  t[3]      -α31      -α32      -1 + α13 + α23 + α43 + λ3      -α34
  t[4]      -α41      -α42      -α43      -1 + α14 + α24 + α34 + λ4
)

(
  ω - ω α12      h[1]      h[3]      h[4]
  t[1]   $\frac{1 - \alpha_{32} + \alpha_{21} \alpha_{32} - \alpha_{41} + \alpha_{32} \alpha_{41} - \alpha_{42} + \alpha_{21} \alpha_{42} + \alpha_{41} \alpha_{42} + \alpha_{31} (-1 + \alpha_{32} + \alpha_{42}) - \lambda_1 \lambda_2 + \alpha_{12} (-1 + \alpha_{31} + \alpha_{41} + \lambda_1 \lambda_2)}{-1 + \alpha_{12}}$       -  $\frac{(-1 + \alpha_{12}) \alpha_{23} + \alpha_{13} (-1 + \alpha_{12})}{-1 + \alpha_{12}}$ 
  t[3]      -  $\frac{\alpha_{31} (-1 + \alpha_{12} + \alpha_{32}) + \alpha_{32} (-1 + \alpha_{21} + \alpha_{41})}{-1 + \alpha_{12}}$       -1 + α23 + α13  $\left(1 + \frac{\alpha_{32}}{-1 + \alpha_{12}}\right)$ 
  t[4]      -  $\frac{(-1 + \alpha_{21} + \alpha_{31}) \alpha_{42} + \alpha_{41} (-1 + \alpha_{12} + \alpha_{42})}{-1 + \alpha_{12}}$        $\frac{\alpha_{13} \alpha_{42}}{-1 + \alpha_{12}} - \alpha_{43}$ 
)

(
  ω - ω α12      h[1] → λ1 λ2      h[2] → 1      h[3] → λ3
  t[1]  1 -  $\frac{\alpha_{31} (-1 + \alpha_{12} + \alpha_{32}) + \alpha_{32} (-1 + \alpha_{21} + \alpha_{41})}{-1 + \alpha_{12}}$  -  $\frac{(-1 + \alpha_{21} + \alpha_{31}) \alpha_{42} + \alpha_{41} (-1 + \alpha_{12} + \alpha_{42})}{-1 + \alpha_{12}}$       0       $\frac{(-1 + \alpha_{12}) \alpha_{23} + \alpha_{13} (-1 + \alpha_{12})}{-1 + \alpha_{12}}$ 
  t[2]      0      1      0
  t[3]       $\frac{\alpha_{31} (-1 + \alpha_{12} + \alpha_{32}) + \alpha_{32} (-1 + \alpha_{21} + \alpha_{41})}{-1 + \alpha_{12}}$       0       $\frac{-1 + \alpha_{23} - \alpha_{13} (-1 + \alpha_{32}) + \alpha_{43} - \alpha_{12} (-1 + \alpha_{12})}{-1 + \alpha_{12}}$ 
  t[4]       $\frac{(-1 + \alpha_{21} + \alpha_{31}) \alpha_{42} + \alpha_{41} (-1 + \alpha_{12} + \alpha_{42})}{-1 + \alpha_{12}}$       0      -  $\frac{\alpha_{13} \alpha_{42}}{-1 + \alpha_{12}} + \alpha_{43}$ 
)

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```

n = 5;
{
  β1 = Bu[ω, Table[h[j] → λj, {j, n}],
    Sum[α10 i+j t[i] h[j] + (1/n - α10 i+j) t[j] h[j], {i, n}, {j, n}]
  ],
  B[β1],
  (Expand /@ Bu[n, B[β1] // dm[1, 2, 1]]) /. t[i_] h[i_] → 0
} // βForm // ColumnForm

(
  ω          h[1] → λ1          h[2] → λ2          h[3] → λ3          h[4] → λ4
t[1] 1 - α21 - α31 - α41 - α51      α12          α13          α14
t[2]          α21          1 - α12 - α32 - α42 - α52      α23          α24
t[3]          α31          α32          1 - α13 - α23 - α43 - α53      α34
t[4]          α41          α42          α43          1 - α14 - α24 - α34 - α54
t[5]          α51          α52          α53          α54          1 -

(
  ω          h[1]          h[2]          h[3]
t[1] -1 + α21 + α31 + α41 + α51 + λ1      -α12          -α13
t[2]          -α21          -1 + α12 + α32 + α42 + α52 + λ2      -α23
t[3]          -α31          -α32          -1 + α13 + α23 + α43 + α53 + λ3
t[4]          -α41          -α42          -α43          -1 + α14 +
t[5]          -α51          -α52          -α53

(
  ω - ω α12          h[1] → λ1 λ2          h[2] → 1          h[3] → λ3          h[4] → λ4
t[1]          0          0           $\frac{(-1+\alpha_{12})\alpha_{23}+\alpha_{13}(-1+\alpha_{12}+\alpha_{32}+\alpha_{42}+\alpha_{52})}{-1+\alpha_{12}}$            $\frac{(-1+\alpha_{12})\alpha_{24}+\alpha_{14}(-1+\alpha_{12}+\alpha_{32})}{-1+\alpha_{12}}$ 
t[3]           $\frac{\alpha_{31}(-1+\alpha_{12}+\alpha_{32})+\alpha_{32}(-1+\alpha_{21}+\alpha_{41}+\alpha_{51})}{-1+\alpha_{12}}$           0          0           $-\frac{\alpha_{14}\alpha_{32}}{-1+\alpha_{12}} + \alpha_{34}$ 
t[4]           $\frac{\alpha_{41}(-1+\alpha_{12}+\alpha_{42})+\alpha_{42}(-1+\alpha_{21}+\alpha_{31}+\alpha_{51})}{-1+\alpha_{12}}$           0           $-\frac{\alpha_{13}\alpha_{42}}{-1+\alpha_{12}} + \alpha_{43}$           0
t[5]           $\frac{(-1+\alpha_{21}+\alpha_{31}+\alpha_{41})\alpha_{52}+\alpha_{51}(-1+\alpha_{12}+\alpha_{52})}{-1+\alpha_{12}}$           0           $-\frac{\alpha_{13}\alpha_{52}}{-1+\alpha_{12}} + \alpha_{53}$            $-\frac{\alpha_{14}\alpha_{52}}{-1+\alpha_{12}} + \alpha_{54}$ 
)
)
)

```