

Simplify[eqns13 /. {c1|2 -> c}]

$$V_0[0, c] = 1 \ \&\& \ V_0[c, 0] = 1 \ \&\& \ V_{11}[c, 0] = 0 \ \&\& \\ \frac{-1 + e^c + 2 c e^c V_{12}[c, c]}{2 c} = V_{21}[c, c] \ \&\& \ \frac{-1 + e^c + 2 c e^c V_{12}[c, c] - 2 c V_{21}[c, c]}{c} = 0 \ \&\& \\ \frac{(-1 + e^c) (-1 + e^c + 2 c e^c V_{12}[c, c] - 2 c V_{21}[c, c])}{c} = 0 \ \&\&$$

$$\frac{2 c e^{c/2} V_{11}[c, c] - (-1 + e^{c/2}) (-1 + e^{c/2} - 2 c V_{21}[c, c])}{2 c} = e^c V_{22}[c, c] \ \&\&$$

$$V_{22}[0, c] = 0 \ \&\& \ - \frac{(-1 + e^{c/2})^2 + 2 c (-e^{c/2} + e^c) V_{12}[c, c] - 2 c e^{c/2} V_{22}[c, c]}{2 c} = V_{11}[c, c]$$

$$\text{Solve} \left[\frac{-1 + e^c + 2 c e^c V_{12}[c, c]}{2 c} = V_{21}[c, c] \ \&\& \ \frac{-1 + e^c + 2 c e^c V_{12}[c, c] - 2 c V_{21}[c, c]}{c} = 0 \ \&\& \right. \\ \left. \frac{(-1 + e^c) (-1 + e^c + 2 c e^c V_{12}[c, c] - 2 c V_{21}[c, c])}{c} = 0 \ \&\& \right.$$

$$\left. \frac{2 c e^{c/2} V_{11}[c, c] - (-1 + e^{c/2}) (-1 + e^{c/2} - 2 c V_{21}[c, c])}{2 c} = e^c V_{22}[c, c] \ \&\& \ V_{22}[0, c] = 0 \ \&\& \right.$$

$$\left. - \frac{(-1 + e^{c/2})^2 + 2 c (-e^{c/2} + e^c) V_{12}[c, c] - 2 c e^{c/2} V_{22}[c, c]}{2 c} = V_{11}[c, c] \ \&\& \right]$$

$$V_{11}[c, c] = 0 \ \&\& \ V_{22}[c, c] = 0, \{V_{11}[c, c], V_{12}[c, c], V_{21}[c, c], V_{22}[c, c]\}$$

$$\left\{ \left\{ V_{11}[c, c] \rightarrow 0, V_{12}[c, c] \rightarrow - \frac{e^{-c/2} (-1 + e^{c/2})}{2 c}, V_{21}[c, c] \rightarrow \frac{-1 + e^{c/2}}{2 c}, V_{22}[c, c] \rightarrow 0 \right\} \right\}$$

Solve[eqns, {V11[c1, c2], V12[c1, c2], V21[c1, c2],

V22[c1, c2], V11[c2, c1], V12[c2, c1], V21[c2, c1], V22[c2, c1]}][[1]]

Solve::svars: Equations may not give solutions for all "solve" variables. >>

$$\{V_{11}[c_1, c_2] \rightarrow$$

$$\frac{e^{-\frac{c_1}{2}} \left(-e^{\frac{c_1}{2}} c_1 + e^{\frac{c_1}{2} + \frac{c_2}{2}} c_1 + c_2 - e^{\frac{c_1}{2}} c_2 \right) - \frac{e^{-\frac{c_1}{2}} (-1 + e^{\frac{c_1}{2}}) c_2 V_{21}[c_1, c_2]}{c_1}}{c_1 (c_1 + c_2)} + e^{\frac{c_2}{2}} V_{22}[c_2, c_1],$$

$$V_{12}[c_1, c_2] \rightarrow \frac{e^{-c_1} \left(\frac{-1 + e^{c_1}}{c_1} + \frac{1 - e^{c_1 + c_2}}{c_1 + c_2} \right)}{-1 + e^{c_2}} + \frac{e^{-c_1} (-1 + e^{c_1}) c_2 V_{21}[c_1, c_2]}{(-1 + e^{c_2}) c_1},$$

$$V_{22}[c_1, c_2] \rightarrow - \frac{e^{-c_1} (-1 + e^{\frac{c_1}{2}}) (-1 + e^{\frac{c_1}{2} + \frac{c_2}{2}})}{(1 + e^{\frac{c_2}{2}}) (c_1 + c_2)} + e^{-\frac{c_1}{2}} V_{11}[c_2, c_1] + \frac{e^{-c_1} (-1 + e^{c_1}) V_{21}[c_1, c_2]}{1 + e^{\frac{c_2}{2}}},$$

$$V_{12}[c_2, c_1] \rightarrow - \frac{e^{-\frac{c_1}{2} - \frac{c_2}{2}} (-1 + e^{\frac{c_1}{2} + \frac{c_2}{2}})}{c_1 + c_2} + e^{-\frac{c_1}{2} - \frac{c_2}{2}} V_{21}[c_1, c_2],$$

$$V_{21}[c_2, c_1] \rightarrow - \frac{e^{-\frac{c_1}{2}} \left(-e^{\frac{c_1}{2}} c_1 + e^{\frac{c_1}{2} + c_2} c_1 - e^{c_1 + \frac{c_2}{2}} c_2 + e^{\frac{c_2}{2}} c_2 \right)}{(-1 + e^{c_2}) c_1 (c_1 + c_2)} + \frac{e^{-\frac{c_1}{2} + \frac{c_2}{2}} (-1 + e^{c_1}) c_2 V_{21}[c_1, c_2]}{(-1 + e^{c_2}) c_1} \}$$

Solve[eqns , {V11[c2, c1], V12[c2, c1], V21[c2, c1], V22[c2, c1]}]

$$\left\{ \left\{ \begin{aligned} V_{11}[c_2, c_1] &\rightarrow \frac{1}{c_2 (c_1 + c_2)} \left(e^{\frac{c_1}{2}} c_1 - e^{\frac{c_1+c_2}{2}} c_1 - c_2 + e^{\frac{c_1}{2}} c_2 + e^{\frac{c_1}{2}} c_1^2 V_{12}[c_1, c_2] - e^{\frac{c_1+c_2}{2}} c_1^2 V_{12}[c_1, c_2] + \right. \\ &\quad \left. e^{\frac{c_1}{2}} c_1 c_2 V_{12}[c_1, c_2] - e^{\frac{c_1+c_2}{2}} c_1 c_2 V_{12}[c_1, c_2] + e^{\frac{c_1}{2}} c_1 c_2 V_{22}[c_1, c_2] + e^{\frac{c_1}{2}} c_2^2 V_{22}[c_1, c_2] \right), \\ V_{12}[c_2, c_1] &\rightarrow - \frac{e^{-\frac{c_1}{2} - \frac{c_2}{2}} \left(-1 + e^{\frac{c_1+c_2}{2}} - c_1 V_{21}[c_1, c_2] - c_2 V_{21}[c_1, c_2] \right)}{c_1 + c_2}, \\ V_{21}[c_2, c_1] &\rightarrow \frac{-1 + e^{\frac{c_1+c_2}{2}} + e^{\frac{c_1+c_2}{2}} c_1 V_{12}[c_1, c_2] + e^{\frac{c_1+c_2}{2}} c_2 V_{12}[c_1, c_2]}{c_1 + c_2}, V_{22}[c_2, c_1] \rightarrow \\ &\quad - \frac{1}{c_1 (c_1 + c_2)} e^{-\frac{c_1}{2} - \frac{c_2}{2}} \left(-e^{\frac{c_1}{2}} c_1 + e^{\frac{c_1+c_2}{2}} c_1 + c_2 - e^{\frac{c_1}{2}} c_2 - e^{\frac{c_1}{2}} c_1^2 V_{11}[c_1, c_2] - e^{\frac{c_1}{2}} c_1 c_2 V_{11}[c_1, c_2] + \right. \\ &\quad \left. c_1 c_2 V_{21}[c_1, c_2] - e^{\frac{c_1}{2}} c_1 c_2 V_{21}[c_1, c_2] + c_2^2 V_{21}[c_1, c_2] - e^{\frac{c_1}{2}} c_2^2 V_{21}[c_1, c_2] \right) \} \} \end{aligned} \right.$$