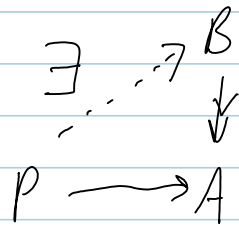
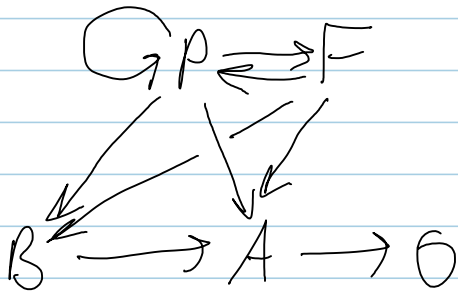


# Projective Modules

September-24-08  
1:34 PM



True if  $P$  is free



True if

$$G \rightleftharpoons F \quad (F \text{ free})$$

converse?

Claim IF  $G \rightleftharpoons_{\pi} F$  then

$$F = P \oplus (\ker \pi).$$

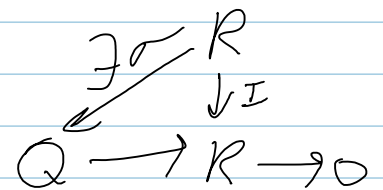
Q. What is the fundamental reason why this is significant?

Claim IF  $0 \rightarrow P \rightarrow Q \rightarrow R \rightarrow 0$  is exact,

then  $Q = P \oplus R$ .

PF By projectivity of  $R$ ,

the sequence splits:



$$0 \longrightarrow P \longrightarrow Q \xleftarrow{\sigma} R \longrightarrow 0$$

We've only used the projectivity of  $R$  here;

in fact, we've only used that every surjection on  $R$  has a section. Converse?

Claim If every surjection on  $P$  has a section, then  $P$  is projective.

$$\begin{array}{ccc} & P & \\ & \downarrow & \\ B & \longrightarrow & A \longrightarrow 0 \end{array} \quad \begin{array}{c} ? \\ \downarrow \\ ? \end{array}$$