Staudacher Lecture 3

July-18-11
9:24 AM

Ask him about the relation w/ YB!

Algebraic Bethe Ansatz

(The Leningrad school ~70s
Baxter, Faddeev)

Start w/ an "R-matrix":

$$R_{ab}(u) = u\,\mathbb{1}_{ab} + i\,\mathbb{P}_{ab}$$

u: the "spectral parameter"

acting on $\mathbb{C}^2 \otimes \mathbb{C}^2$:

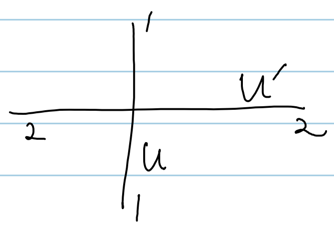
$$R_{ab}(u) = \begin{pmatrix} u+i & & & \\ & u & i & \\ & i & u & \\ & & & u+i \end{pmatrix}$$

Satisfies Yang-Baxter:

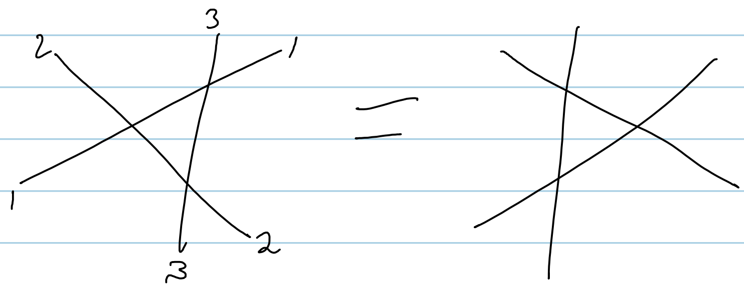
$$R_{12}(u-v)R_{13}(u)R_{23}(v) = R_{23}(v)R_{12}(u)R_{12}(u-v)$$

Graphical notation:

$$R_{12}(u-u') =$$



YB becomes:



The Quantum Lax operator:

$$L_{n,a} = R_{na}(u - i/2)$$

$\mathcal{L}_{al}$

auxilary

$\mathbb{C}^2$ at site l of spin chain

YBE: $1 = a =$ 1st aux space

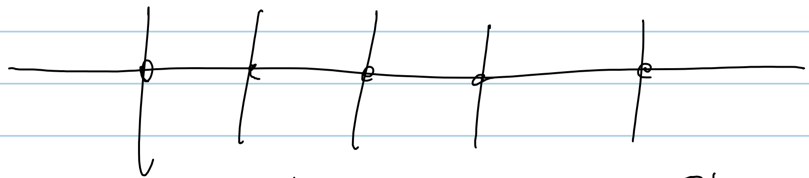
$2 = b = 2$ aux space

$3 =$ site l

$$\Rightarrow \quad R_{ab}(u-u')\mathcal{L}_{al}(u)\mathcal{L}_{bl}(u') = \text{Transpose}$$

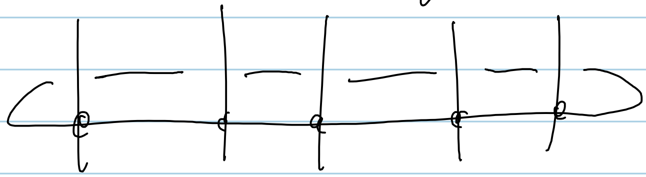
Monodromy matrix, on $\mathbb{C}^2 \otimes (\mathbb{C}^2)^{\otimes L}$

$$M_a(u) = \mathcal{L}_{a,L}(u)\mathcal{L}_{a,L-1} \cdots \mathcal{L}_{a,1}(u)$$



Transfer matrix, on $(\mathbb{C}^2)^{\otimes L}$:

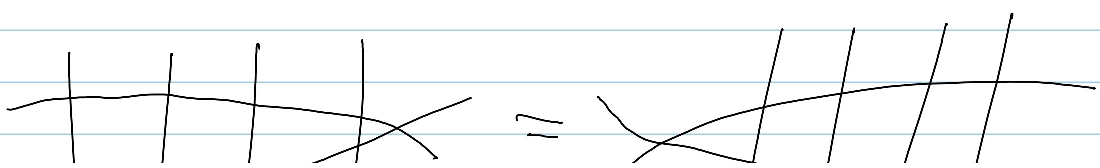
$$\hat{T}(u) = \mathrm{Tr}_a M_a(u)$$

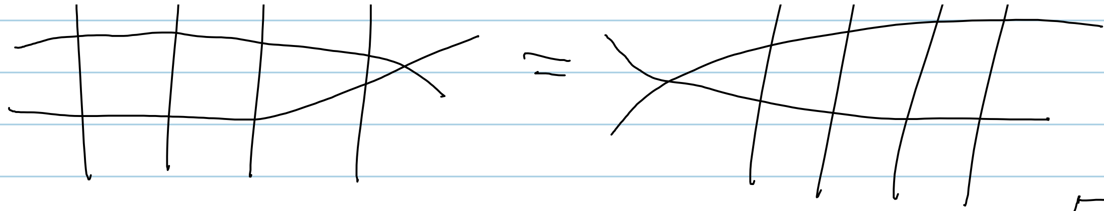


Then

$$R_{ab}(u-u')M_a(u)M_b(u') = \text{Transpose}$$

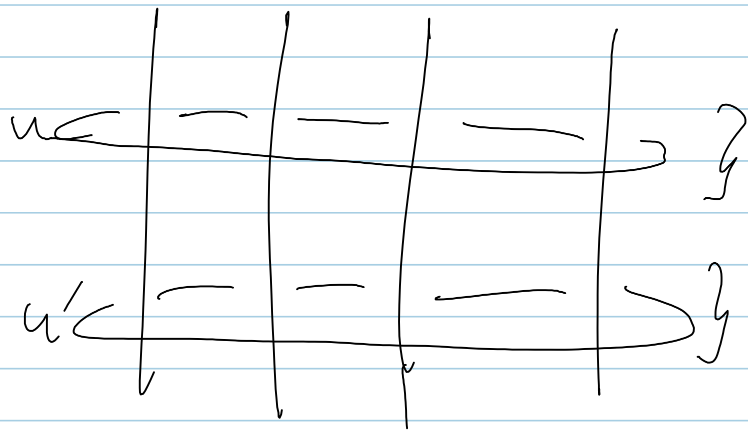
Pf





$\square$

$\Rightarrow$

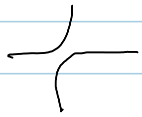


commute

$$\hat{T}(u)\,\hat{T}(u') = \hat{T}(u')\,\hat{T}(u)$$

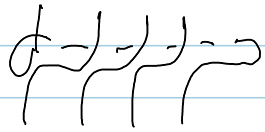
Note

$$R_{al}(0) = i\,\mathbb{P}_{al} \qquad \frac{1}{i} R_{al}(0) =$$



$$\mathcal{L}_{al}\left(\frac{i}{2}\right) = i\,\mathbb{P}_{al}$$

$$\frac{1}{i^L}\hat{T}\left(\frac{i}{2}\right) =$$



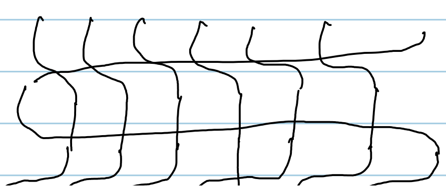
$$= \hat{U} = e^{iP_{tot}}$$

$$i^L\,\hat{T}^{-1}\left(\frac{i}{2}\right) = \hat{U}^{-1}$$

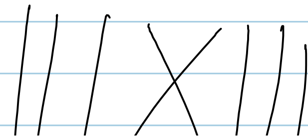
set $\hat{T}'(u) = \frac{d}{du}\hat{T}(u)$

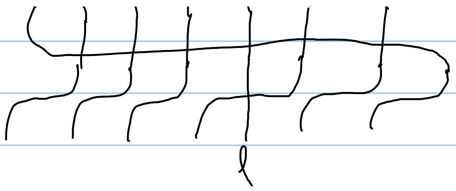
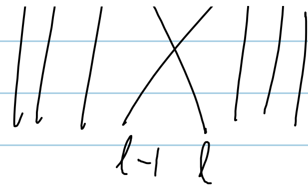
$$i\,\hat{T}^{-1}\left(\frac{i}{2}\right)\hat{T}'\left(\frac{i}{2}\right) =$$

$$= \sum_{l=1}^{L}$$



$$= \sum_{l=1}^{L}$$



$= \sum_{l=1}$  $= \sum_{l=1}$ 

$$= \sum_{l=1}^{L} \mathbb{P}_{l-1,l}$$

$$\Longrightarrow \hat{H} = 2L - 2i \frac{d}{du} \hat{T}(u)\Big|_{u=i/2}$$

in general,

$$\hat{T}(u) = e^{iP_{tot} + \frac{i}{2}(u - i/2)(\hat{H} - 2L) + (u - i/2)^2 \hat{Q}_3 + \dots}$$

where $\hat{Q}_1 \sim P_{tot}$ $\quad \hat{Q}_2 \sim \hat{H} - 2L$, ...

All these charges $\hat{Q}_m$ commute:

$$[\hat{Q}_m, \hat{Q}_n] = 0$$

So solve

$$\hat{T}(u)|\Psi\rangle = T(u)|\Psi\rangle \qquad (*)$$

write

$$M_a(u) = \begin{pmatrix} \hat{A}(u) & \hat{B}(u) \\ \hat{C}(u) & \hat{D}(u) \end{pmatrix}$$

Bethe:

$$\hat{C}(u)|0\rangle = 0 \qquad |0\rangle = |\uparrow\uparrow\uparrow\uparrow \dots \uparrow\rangle$$

$$|\Psi\rangle = \hat{B}(u_1)\,\hat{B}(u_2) \dots \hat{B}(u_M)|0\rangle$$

(*) is satisfied iff $\{u_k\}$ satisfy the BA equations

$$T(u) = A(u) + D(u)$$

$$\Rightarrow T(u) = \left(u+\frac{i}{2}\right)^L \prod_{j=1}^{M} \frac{u-u_j-i}{u-u_j+i} + \left(u-\frac{i}{2}\right)^L \prod_{j=1}^{M} \frac{u-u_j+i}{u-u_j-i}$$