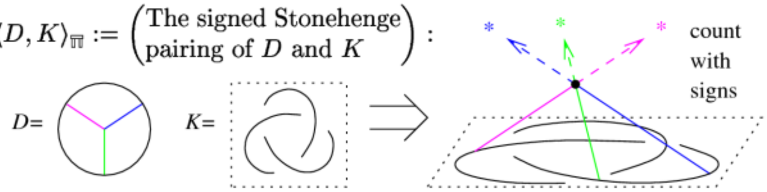
Lecture 5 Extras

Review Material (mostly)

Dror Bar-Natan at Villa de Leyva, July 2011, http://www.math.toronto.edu/~drorbn/Talks/Colombia-1107

$\langle D, K\rangle_{\overline{\sqcap}} := \left(\begin{array}{l}\text{The signed Stonehenge}\\ \text{pairing of } D \text{ and } K\end{array}\right)$:



Thus we consider the generating function of all stellar coincidences:

$$Z(K) := \lim_{N\to\infty} \sum_{\text{3-valent } D} \frac{1}{2^c c! \binom{N}{e}} \langle D, K\rangle_{\overline{\sqcap}} D \cdot \left(\begin{array}{c}\text{framing-}\\ \text{dependent}\\ \text{counter-term}\end{array}\right) \in \mathcal{A}(\circlearrowleft)$$

Theorem. Modulo Relations, $Z(K)$ is a knot invariant!

When deforming, catastrophes occur when:

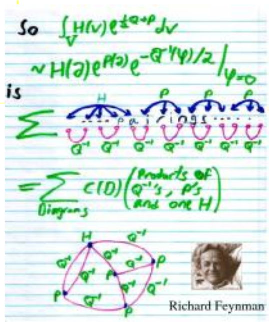
A plane moves over an intersection point – Solution: Impose IHX,	An intersection line cuts through the knot – Solution: Impose STU,	The Gauss curve slides over a star – Solution: Multiply by

(see below) (similar argument)

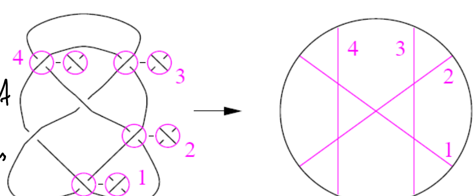
It all is perturbative Chern–Simons–Witten theory:

$$\int_{\mathfrak{g}\text{-connections}} \mathcal{D}A\, hol_K(A) \exp\left[\frac{ik}{4\pi}\int_{\mathbb{R}^3} \operatorname{tr}\left(A\wedge dA + \frac{2}{3}A\wedge A\wedge A\right)\right]$$

$$\to \sum_{\substack{D:\ \text{Feynman}\\ \text{diagram}}} W_{\mathfrak{g}}(D) \cancel{\sum} \mathcal{E}(D) \to \sum_{\substack{D:\ \text{Feynman}\\ \text{diagram}}} D \cancel{\sum} \mathcal{E}(D)$$



Definition. Any $V: \{\text{knots}\} \to$ Abelian Group A can be extended to "knots w/ double points" using $V(\times) = V(\nearrow) - V(\nwarrow)$. (Think "differentiation")



Definition. V is of type m if always

$V(\underbrace{\times\times \ldots\ldots \times}_{m+1}) = 0$ (think "polynomial")

Conjecture. Finite type invariants separate knots.

Theorem. If $C(K) = \sum_{m=0}^{\infty} V_m(K) z^m$ then V_m is of type m.

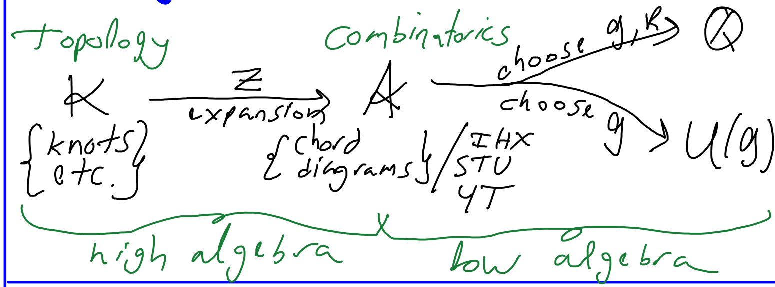
Proof. $C(\times) = C(\nearrow) - C(\nwarrow) = zC(\uparrow\uparrow)$

Proposition. The Fundamental thm holds iff there exists an expansion

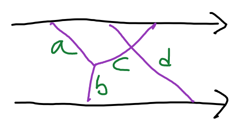
$Z: \mathcal{K} \to \hat{\mathcal{A}}$ s.t. if K is m-singular, then

$Z(K) = D_K + \text{higher degrees}$

The big picture, "u" case.



Low algebra. $\mathcal{A}(\uparrow\uparrow) \to U(\mathfrak{g})^{\otimes 2}$ via

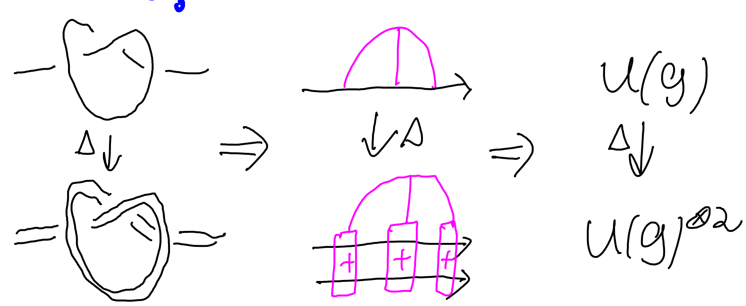


$\longrightarrow \sum_{a\text{-}d} f_{abc} \begin{pmatrix} x_a x_d x_c \\ \otimes \\ x_b x_d \end{pmatrix}$

& likewise, $\mathcal{A}(\uparrow_n) \to U(\mathfrak{g})^{\otimes n} \Rightarrow$

$\mathcal{A}(\uparrow_n)$ is "universal universal rep. theory"!

What's Δ?

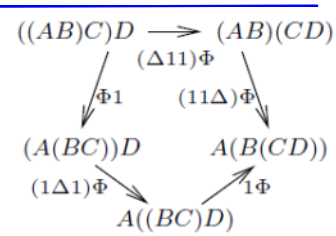


A "Homomorphic Expansion" $Z: \mathcal{K} \to \mathcal{A}$ is an expansion that intertwines all relevant algebraic ops. If $\mathcal{K}$ is finitely presented, finding Z is **High Algebra.**

An Associator:
Quantum Algebra's "root object"

$(AB)C \xrightarrow{\Phi \in \mathcal{U}(\mathfrak{g})^{\otimes 3}} A(BC)$

satisfying the "pentagon",



$\Phi 1 \cdot (1\Delta 1)\Phi \cdot 1\Phi = (\Delta 11)\Phi \cdot (11\Delta)\Phi$

V.G. Drinfel'd

The hexagon? Never heard of it.

See Also. B-N & Dancso, arXiv:1103.1896