

(A = Table[a[i, j], {i, 2}, {j, 2}]) // MatrixForm

$$\begin{pmatrix} a[1, 1] & a[1, 2] \\ a[2, 1] & a[2, 2] \end{pmatrix}$$

(B = Table[b[i, j], {i, 2}, {j, 2}]) // MatrixForm

$$\begin{pmatrix} b[1, 1] & b[1, 2] \\ b[2, 1] & b[2, 2] \end{pmatrix}$$

ArrayFlatten[Outer[Times, A, B]] // MatrixForm

$$\begin{pmatrix} a[1, 1] b[1, 1] & a[1, 1] b[1, 2] & a[1, 2] b[1, 1] & a[1, 2] b[1, 2] \\ a[1, 1] b[2, 1] & a[1, 1] b[2, 2] & a[1, 2] b[2, 1] & a[1, 2] b[2, 2] \\ a[2, 1] b[1, 1] & a[2, 1] b[1, 2] & a[2, 2] b[1, 1] & a[2, 2] b[1, 2] \\ a[2, 1] b[2, 1] & a[2, 1] b[2, 2] & a[2, 2] b[2, 1] & a[2, 2] b[2, 2] \end{pmatrix}$$

Unprotect[NonCommutativeMultiply];

A_ ** B_ := ArrayFlatten[Outer[Times, A, B]];

A ** B // MatrixForm

$$\begin{pmatrix} a[1, 1] b[1, 1] & a[1, 1] b[1, 2] & a[1, 2] b[1, 1] & a[1, 2] b[1, 2] \\ a[1, 1] b[2, 1] & a[1, 1] b[2, 2] & a[1, 2] b[2, 1] & a[1, 2] b[2, 2] \\ a[2, 1] b[1, 1] & a[2, 1] b[1, 2] & a[2, 2] b[1, 1] & a[2, 2] b[1, 2] \\ a[2, 1] b[2, 1] & a[2, 1] b[2, 2] & a[2, 2] b[2, 1] & a[2, 2] b[2, 2] \end{pmatrix}$$

MatrixForm /@ ({Id, s1, s2, s3} = { $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\frac{1}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\frac{1}{2} \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}$, $\frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ })

$$\left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}, \begin{pmatrix} 0 & -\frac{i}{2} \\ \frac{i}{2} & 0 \end{pmatrix}, \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix} \right\}$$

b[x_, y_] := x.y - y.x;

MatrixForm /@ {b[s1, s2], b[s2, s3], b[s3, s1]}

$$\left\{ \begin{pmatrix} \frac{i}{2} & 0 \\ 0 & -\frac{i}{2} \end{pmatrix}, \begin{pmatrix} 0 & \frac{i}{2} \\ \frac{i}{2} & 0 \end{pmatrix}, \begin{pmatrix} 0 & \frac{1}{2} \\ -\frac{1}{2} & 0 \end{pmatrix} \right\}$$

(H = 4 $\left(\frac{1}{4} \text{Id} ** \text{Id} - s1 ** s1 - s2 ** s2 - s3 ** s3\right)$) // MatrixForm

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & -2 & 0 \\ 0 & -2 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

MatrixForm /@ {b[H, Id ** s1 + s1 ** Id], b[H, Id ** s2 + s2 ** Id], b[H, Id ** s3 + s3 ** Id]}

$$\left\{ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right\}$$

(H2 = 2 H) // MatrixForm

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 4 & -4 & 0 \\ 0 & -4 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigensystem[H2]

{{8, 0, 0, 0}, {{0, -1, 1, 0}, {0, 0, 0, 1}, {0, 1, 1, 0}, {1, 0, 0, 0}}}

MatrixForm /@ Eigensystem[H4]

$$\left\{ \begin{array}{l} 12 \\ 8 \\ 8 \\ 8 \\ 4 \\ 4 \\ 4 \\ 4 \\ 4 \\ 4 \\ 4 \\ 4 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \right\}, \left(\begin{array}{cccccccccccccccc} 0 & 0 & 0 & 1 & 0 & -2 & 1 & 0 & 0 & 1 & -2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

Time this took: about 30 minutes.