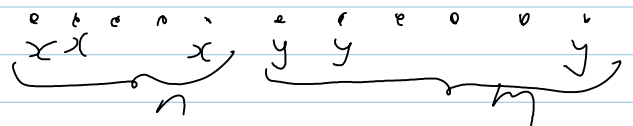


$$x^n y^m = \sum_{k=0}^m \binom{m}{k} h^k y^{m-k} x^n$$

verified in U(20).nb



Better with $[x, y] = h x$. Then
 $x^n y^m = \sum_{k=0}^m \binom{m}{k} (h x)^k y^{m-k} x^n$

$$\begin{aligned} e^x e^y &= \sum_{n,m} \frac{1}{n!} \frac{1}{m!} x^n y^m = \sum_{n,m} \sum_{k=0}^m \frac{\binom{m}{k} h^k}{n! m!} y^{m-k} x^n \\ &= \sum_{n,p} \sum_k \frac{\binom{p+k}{k} h^k}{n! (p+k)!} y^p x^n \quad \left| \sum_k \frac{h^k}{p! k!} = e^{h/p} \right. \\ &= \sum_{n,p} \frac{h^n}{n! p!} y^p x^n = e^y e^{hx} \end{aligned}$$

So $e^x e^y = e^y e^{hx}$ ← "the spirits of the dead".

$\frac{\partial}{\partial h}$: $0 = e^y e^{hx} e^{hx}$ ✗ In the presence of $[x, y] = hx$ differentiation w.r.t. h makes no sense.

E: $e^x (x+y) e^y = e^y (y + h e^{hx} x + e^{hx} x) e^x$ $E e^a = (E_a) e^a$

$$\begin{aligned} e^{-y} x e^y &= e^{-y} \sum_{m=0}^{\infty} \frac{1}{m!} \sum_{k=0}^m \binom{m}{k} h^k y^{m-k} x = \\ &= e^{-y} \sum_{p=0}^{\infty} \sum_k \frac{1}{(p+k)!} \binom{p+k}{k} h^k y^p x = e^{-y} \sum_{p,k} \frac{1}{k! p!} h^k y^p x \\ &= e^{hx} x \end{aligned}$$

$\Rightarrow \forall F(x), \quad e^{-y} F(x) e^y = F(e^{hx} x)$