recycled June 19

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June-19-11
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$$x^n y^m = \sum_{k=0}^{m} \binom{m}{k} n^k y^{m-k} x^n \qquad \underbrace{x\,x \cdots x}_{n}\,\underbrace{y\,y \cdots y}_{m}$$

Verified in U(2D).nb

$$e^x e^y = \sum \frac{1}{n!}\frac{1}{m!} x^n y^m = \sum_{n,m} \sum_{k=0}^{m} \frac{\binom{m}{k} n^k}{n!\,m!} y^{\overbrace{m-k}^{p}} x^n$$

$$= \sum_{n,p} \sum_{k} \frac{\binom{p+k}{k} n^k}{n!\,(p+k)!} y^p x^n \qquad \sum_k \frac{n^k}{p!\,k!} = e^n/p!$$

$$= \sum_{n,p} \frac{e^n}{n!\,p!} y^p x^n = e^y e^{ex}$$

Better with $[x,y] = \hbar x$. Then

$$x^n y^m = \sum_{k=0}^{m} \binom{m}{k} (\hbar n)^k y^{m-k} x^n$$

So

$$e^x e^y = e^y e^{e^{\hbar} x}$$

"The spirits of the dead".

$$\frac{\partial}{\partial \hbar}: \quad 0 = e^y e^{\hbar} x e^{e^{\hbar} x}$$

✗ In the presence of $[x,y]=\hbar x$ differentiation w.r.t. $\hbar$ makes no sense.

$$E: \quad e^x (x+y) e^y = e^y (y + \hbar e^{\hbar} x + e^{\hbar} x) e^x \qquad E e^a = (Ea) e^a$$

$$e^{-y} x e^y = e^{-y} \sum_{m=0}^{\infty} \frac{1}{m!} \sum_{k=0}^{m} \binom{m}{k} \hbar^k y^{\overbrace{m-k}^{p}} x =$$

$$= e^{-y} \sum_{p=0}^{\infty} \sum_{k} \frac{1}{(p+k)!} \binom{p+k}{k} \hbar^k y^p x = e^{-y} \sum_{p,k} \frac{1}{k!\,p!} \hbar^k y^p x$$

$$= e^{\hbar} x$$

$$\Rightarrow \forall f(x), \qquad e^{-y} f(x) e^y = f(e^{\hbar} x)$$