Dylan: A Question by Mrowka on Pseudo-Diagrams

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Subject: Pseudo-diagrams

Hello Dror,

In a talk of Tom Mrowka, I just heard a knot theory question that seems up our alley.

Definition. A _pseudo-diagram_ for K in S^3 is an embedded 4-valent graph G in Y with a distinguished choice of 3 resolutions 0, +, infty for each 4-valent vertex, so that

Presumably $\{0, \infty\} = \{)(, \asymp\}$ and $+ \in \{\times, \times\}$

(a) The three resolutions look locally like the standard 3 resolutions;
(b) If you resolve G with all + resolutions, you get K;
(c) If you resolve G with all 0 or infty resolutions (in any combination), you get an unlink.

Two pseudo-diagrams D, D' are related by _forgetting_ a crossing if D' is obtained from D by doing a + resolution at one crossing.

Presumably a "forgetting" op is allowed only if the resulting embedded graph is again a pseudo-diagram.

Question: Are any two pseudo-diagrams for K related by a sequence of (un)forgetting operations?

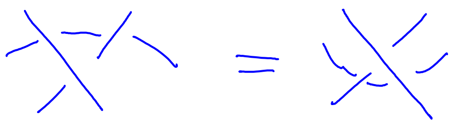
Every standard diagram gives a pseudo-diagram in an obvious way, and the resulting pseudo-diagrams are related by (un)forgetting operations. (Tom claimed that, in a standard diagram, you can always forget two of the three crossings near a Reidemeister III move. You can then do an isotopy of the graph and unforget two new crossings to get to the new diagram.)

A. below.

The question comes from understanding a Seiberg-Witten theory related to Khovanov homology.

You could also ask similar questions for null-homologous knots in an arbitrary 3-manifold. (You need null-homologous, since otherwise there's clearly never a pseudo-diagram.) If we could answer this positively, I think it would allow defining a Khovanov homology theory for null-homologous knots in general.

Best,
Dylan

A. 

Mrowka: In any specific completion of , at least two of the crossings are forgettable.

Proof The only non-unlink that can be formed with just two crossings is the Hopf link, and for both the red pair on the right and the green pair there, no planar completion could be a Hopf link.