

June-05-11
10:15 PM

Date: Sun, 5 Jun 2011 23:39:31 +0400 (MSD)
From: duzhin@...
To: Dror Bar-Natan <drorbn@math.toronto.edu>
Subject: Re: kleinian weight systems

Dror,
I've been thinking a lot about your writing. Something does not work out for me. Let's take a simple example, a wheel with 3 spokes, leaving aside the obvious fact that it is 0. Just applying your construction, pairing off 2 of the 3 trivalent vertices, what we get? We'll get 3 lines with a lambda-box in between, but then 2 of those lines will be connected into circles, and besides, one of those circles will have an extra leg on it. How do you prove that the w_D evaluates to 0 on this object?

Best,
Sergei.

The diagram shows the evaluation of a wheel with 3 spokes. It starts with a wheel with 3 spokes, which is equal to a lambda-box with 3 legs. This is then transformed into a sum of terms:

- id : A lambda-box with 3 legs, one of which is connected to a circle.
- 231 : A lambda-box with 3 legs, two of which are connected to a circle.
- 312 : A lambda-box with 3 legs, two of which are connected to a circle.
- (12) : A lambda-box with 3 legs, one of which is connected to a circle.
- (23) : A lambda-box with 3 legs, one of which is connected to a circle.
- (13) : A lambda-box with 3 legs, one of which is connected to a circle.

A note in green says: "vanish by the anti-symmetry of the vertex, once the anti-symmetry of the vertex is established."

Below this, there are more diagrams showing the evaluation of a lambda-box with 3 legs. One diagram shows a lambda-box with 3 legs, one of which is connected to a circle, and another diagram shows a lambda-box with 3 legs, one of which is connected to a circle with a dot. The evaluation is given as:

$$I - 6 \cdot (12) + 8 \cdot (123) - 6 \cdot (1231) + 3 \cdot (1231)$$

$$c_1^4 - 6c_1^2c_2 + 8c_1c_3 - 6c_4 + 3c_2^2 \xrightarrow{\text{classical}} 81 - 162 + 72 - 18 + 27 = 0 \checkmark$$

Another diagram shows a lambda-box with 3 legs, one of which is connected to a circle, and another diagram shows a lambda-box with 3 legs, one of which is connected to a circle with a dot. The evaluation is given as:

$$27 - 27 + 6 = 6$$

What $w/4?$

The diagram shows the evaluation of a lambda-box with 3 legs, one of which is connected to a circle. It is shown to be equal to 0. The diagram is circled in red.

$$\Rightarrow \text{lambda-box with 3 legs, one connected to a circle} = 0 \cdot \epsilon$$

$$\text{lambda-box with 3 legs, one connected to a circle with a dot} = 0$$

Sergei,

The purpose of the "rerouting relation" from one of my previous notes was to ensure that all ways of "replacing a pair of trivalent vertices" by a "Lambda" are equivalent. I was careless - the rerouting relation is sufficient for that purpose if there is an even number of trivalent vertices to start with, but not if there is an odd number. For the odd

(and greater than 1) case I think you'd need to add the relation circled in red above; I'm not sure what the consequences of that are. Yet note that it implies the AS relation for diagrams with more than one internal vertex.

Best,

Dror.