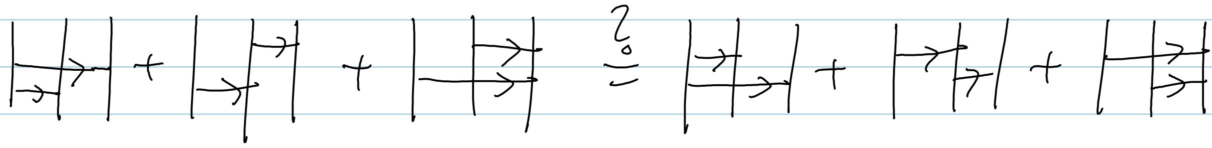
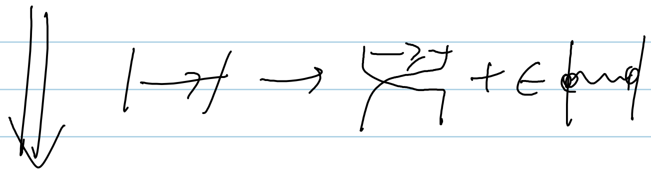
6T for Perturbed gl(N)

June-21-11
2:42 PM

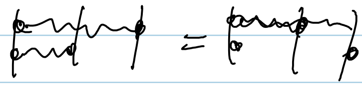
Continues 2009-01/6T For gl(N)
& BBS/BCHKW-110621-111441

$$r \longmapsto r + \epsilon \sum_{i,j} a_{ij} E_{ii} \wedge E_{jj} = r + \epsilon \delta r$$

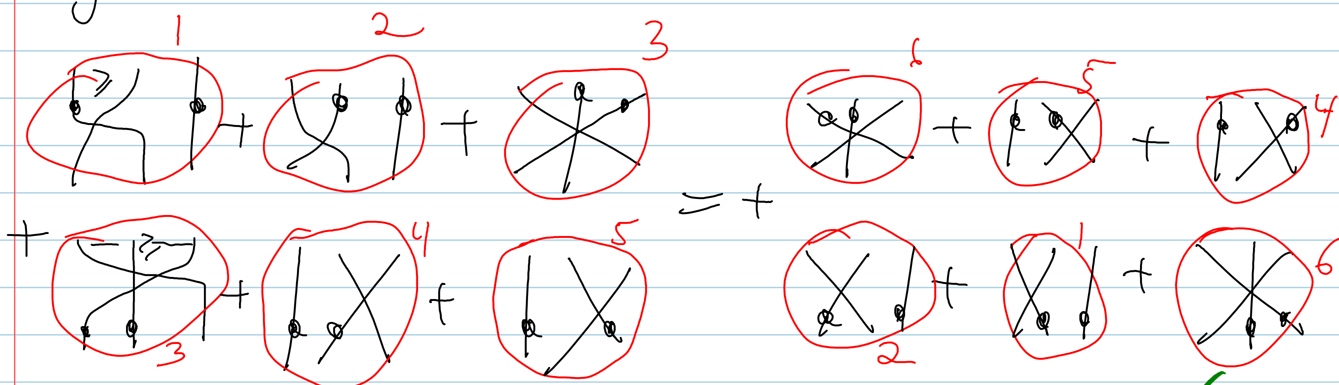




deg 0 in ϵ: checked in 2009-01. ✓

deg 2 in ϵ: obvious:  etc. ✓

deg 1 in ϵ:



3,6 use AS of δr! ✓

This seems to reduce to Louis' weight system w/ an extra copy of H. If this is indeed so, it would be worthwhile to understand the mechanism by which

$$\begin{pmatrix}\text{a Lie bialgebra} \\ \text{on } \mathfrak{g} \oplus \mathfrak{h}\end{pmatrix} \sim \begin{pmatrix}\text{an } \wedge^2 \mathfrak{h}\text{-parameter} \\ \text{family of Lie bialgebras} \\ \text{on } \mathfrak{g}\end{pmatrix}$$

$\{$ on $g\otimes \check{h}$ $\}$ $\sim$ $\{$Family of Lie bi-algebras on $g\}$

This mechanism may generalize, and may simplify Belavin-Drinfel'd.

Formulas by Emily and Iva:

- $[E_{ij}, E_{kl}] = \delta_{jk} E_{il} - \delta_{il} E_{kj}$
- $r = \underbrace{\frac{1}{2}\sum_{i=1}^{n} E_{ii}\otimes E_{ii} + \sum_{i<j} a_{ij} E_{ii}\wedge E_{jj}}_{r^0} + \sum_{i<j} E_{ji}\otimes E_{ij}.$

$\delta(X) : X.r$

$$\delta(E_{ab}) = \begin{cases} \frac{1}{2} E_{ab}\wedge(E_{bb}-E_{aa}) + \sum_{j=a+1}^{b-1} E_{aj}\wedge E_{jb} + \sum_{j=1}^{n} (\tilde{a}_{bj}-\tilde{a}_{aj}) E_{ab}\wedge E_{jj} & \text{if } a\le b. \\ -\frac{1}{2} E_{ab}\wedge(E_{bb}-E_{aa}) - \sum_{j=b+1}^{a-1} E_{aj}\wedge E_{jb} + \sum_{j=1}^{n} (\tilde{a}_{bj}-\tilde{a}_{aj}) E_{ab}\wedge E_{jj} & \text{if } a> b. \end{cases}$$

convention : $\tilde{a}_{ij} = \begin{cases} a_{ij} & \text{if } i<j \\ 0 & \text{if } i=j \\ -a_{ji} & \text{if } i>j \end{cases}$

One ring to rule the all: (GL(n)-Prototypes.nb)

```
(* "TS" for "TensorSum", "T" is "Tensor" *)
TS[
 params_List,
 order_List, (* this list is an ordered list which contains params;
   The variables in it that are not in params are "summation variables" *)
 T[E[s_, i_, j_] ...] (* "s" is strand number, "i" and "j" are indices in 1..n *)
 ];
r = Plus[
  1/2 TS[{}, {i}, T[E[1, i, i], E[2, i, i]]],
  TS[{}, {i, j},
   a[i, j] Wedge[E[i, i], E[j, j]] + T[E[1, j, i], E[2, i, j]]
   ]
  ];
delta[E[a_, b_]] := Plus[
 ... + TS[{a, b}, {a, j, b}, Wedge[E[a, j], E[j, b]]] + ...,
 ... + TS[{a, b}, {b, j, a}, ...] + ...
```

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