

$$P \subset \mathbb{R}^2 \quad D(P) = \{ |P-Q| : P \neq Q \in P \} \quad |P| = n$$

Examples 0. Generic: $|D(P)| = \binom{n}{2}$

1. Along a line, equal spacing: $|D(P)| = n-1$

2. Square grid: $|D(P)| \sim n(\log n)^{-1/2}$

Conjecture. Always, $|D(P)| \geq C n(\log n)^{-1/2}$

Thm. $|D(P)| \geq C \cdot n \cdot (\log n)^{-1}$