Severa's GRT Question

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In Drinfeld's Gal(\bar Q/Q) paper there is a bijection between associators and elements of the Lie algebra gt with the numerical component equal to 1 (gt(K) is composed of pairs (s,psi), where s \in K and psi is from the completed free Lie algebra with 2 generators, satisfying some (5gon&6gon) equations; in the bijection we demand s=1). The bijection psi<->Phi ((1,psi) in gt, Phi an associator) is given by the equation (saying that (1,psi) acts on Phi by rescaling)

Phi(x,y)^-1 d/dt Phi(tx,ty)|_{t=1} = psi(x, Phi^-1 y Phi). (*)

The question is: do we know some psi "explicitly"? For example, what are the coefficients of psi_KZ?

Maybe a better formulation of the question: Let's just suppose that Phi is a group-like element of K<<x,y>>, but let's forget about 5gon and 6gon. Then (*) defines a Lie series psi. Since Phi is group-like, we can express the product of any of its coefficients as a linear combination of some other coefficients. As a result, the coefficients of psi are linear combinations of coefficients of Phi. Is there some intelligent way how to write these linear combinations?

(the reason for asking is that the 5gon & 6gon relations for psi are (inhomogeneous) linear, unlike the 5gon & 6gon for Psi; seeing that they are just the double shuffle equations would be nice. Anyway, psi is "the right log of Phi" (log Phi being "the wrong log of Phi"))

$$\tilde{E}\Phi := \Phi^{-1} E \Phi = \psi(x, \Phi^{-1} y \Phi)$$

$$\begin{tikzcd} B \arrow[r, "Z_\Phi"] & C \arrow[loop below, "E"] \end{tikzcd}$$

From Drinfeld's $Gal(\bar{\mathbb{Q}}/\mathbb{Q})$ paper:

if $M_1(k) \neq \varnothing$, then the sequence $1 \to \mathrm{GT}_1(k) \to \mathrm{GT}(k) \xrightarrow{\nu} k^* \to 1$, where $\nu(\lambda, f) = \lambda$, is exact and to every $\varphi \in M_1(k)$ corresponds a homomorphism $\theta_\varphi\colon k^* \to \mathrm{GT}(k)$ such that $\nu \circ \theta_\varphi = \mathrm{id}$, while $\theta_\varphi(k^*)$ is the stabilizer of φ in $\mathrm{GT}(k)$.

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If $M_1(k) \neq \varnothing$, then the sequence

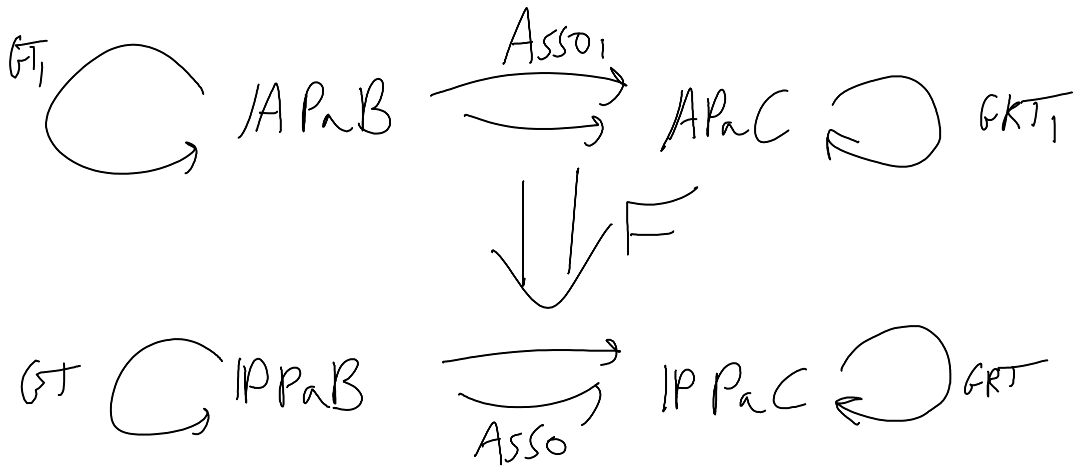
$$0 \to \mathfrak{gt}_1(k) \to \mathfrak{gt}(k) \xrightarrow{\nu_*} k \to 0, \qquad \nu_*(s, \psi) = s, \tag{5.8}$$

is exact, and to every $\varphi \in M_1(k)$ corresponds a splitting, defined by the Lie algebra of the stabilizer of φ in $\mathrm{GT}(k)$.

Proposition 5.2. *The mapping $M_1(k) \to \{$splittings of the sequence (5.8)$\}$ is bijective. In particular, exactness of (5.8) implies that $M_1(k) \neq \varnothing$.*

Proof. The mapping takes $\varphi \in M_1(k)$ into the splitting defined by the element $(1, \psi) \in \mathfrak{gt}(k)$, where ψ is found from the condition

$$\varphi(A, B)^{-1} \cdot \frac{d}{dt}\varphi(tA, tB)\Big|_{t=1} = \psi(A, \varphi(A, B)^{-1} B \varphi(A, B)). \tag{5.9}$$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