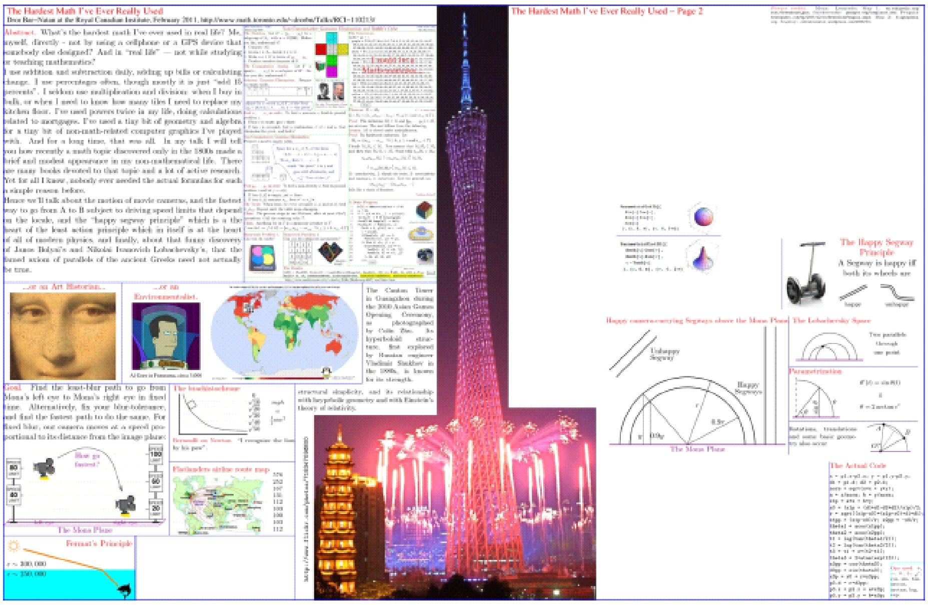
RCI Lecture as of February 6

February-06-11
3:21 PM



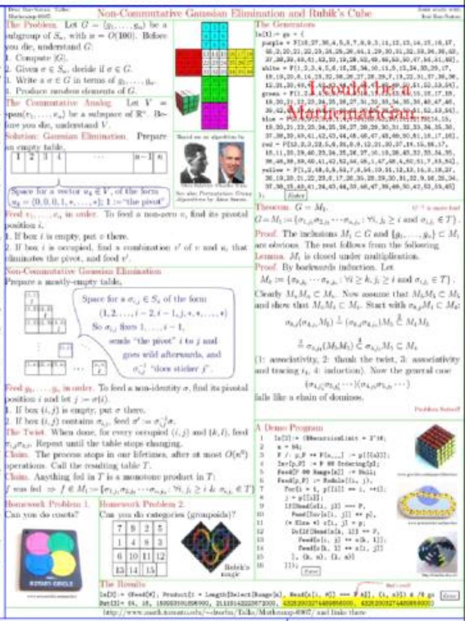
The Hardest Math I've Ever Really Used

Dror Bar–Natan at the Royal Canadian Institute, February 2011, http://www.math.toronto.edu/~drorbn/Talks/RCI–110213/

Abstract. What's the hardest math I've ever used in real life? Me, myself, directly - not by using a cellphone or a GPS device that somebody else designed? And in "real life" — not while studying or teaching mathematics?

I use addition and subtraction daily, adding up bills or calculating change. I use percentages often, though mostly it is just "add 15 percents". I seldom use multiplication and division: when I buy in bulk, or when I need to know how many tiles I need to replace my kitchen floor. I've used powers twice in my life, doing calculations related to mortgages. I've used a tiny bit of geometry and algebra for a tiny bit of non-math-related computer graphics I've played with. And for a long time, that was all. In my talk I will tell you how recently a math topic discovered only in the 1800s made a brief and modest appearance in my non-mathematical life. There are many books devoted to that topic and a lot of active research. Yet for all I know, nobody ever needed the actual formulas for such a simple reason before.

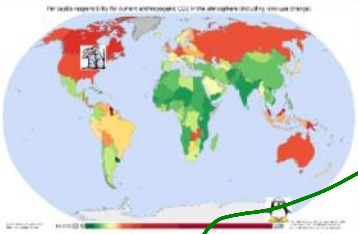
Hence we'll talk about the motion of movie cameras, and the fastest way to go from A to B subject to driving speed limits that depend on the locale, and the "happy segway principle" which is a the heart of the least action principle which in itself is at the heart of all of modern physics, and finally, about that funny discovery of Janos Bolyai's and Nikolai Ivanovich Lobachevsky's, that the famed axiom of parallels of the ancient Greeks need not actually be true.



...or an Art Historian...

...or an Environmentalist.

Al Gore in Futurama, circa 3,000

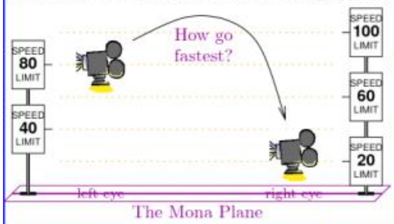


The Canton Tower in Guangzhou during the 2010 Asian Games Opening Ceremony, as photographed by Colin Zhu. Its hyperboloid structure, first explored by Russian engineer Vladimir Shukhov in the 1880s, is known for its strength, structural simplicity, and its relationship with heyprbolic geometry and with Einstein's theory of relativity.

this would be better on the other side.

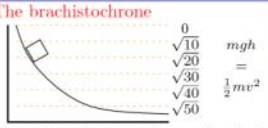
http://www.flickr.com/photos/71834709@N00

Goal. Find the least-blur path to go from Mona's left eye to Mona's right eye in fixed time. Alternatively, fix your blur-tolerance, and find the fastest path to do the same. For fixed blur, our camera moves at a speed proportional to its distance from the image plane:



The Mona Plane

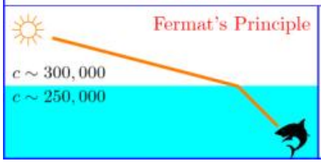
The brachistochrone



Bernoulli on Newton. "I recognize the lion by his paw".

Flatlanders airline route map

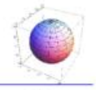
Fermat's Principle



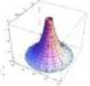
The principle of least action

Picture credits. Mona: Leonrado; Map 1: en.wikipedia.org/wiki/Greenhouse_gas; Smokestacks: gbuapcd.org/complaint.htm; Penguin: brentpabst.com/bp/2007/12/15/BrentGoesPenguin.aspx; Map 2: flightpedia.org; Segway: co2calculator.wordpress.com/2008/10;

```
ParametricPlot3D[{
  Sin[u] Cos[v],
  Sin[u] Sin[v],
  Cos[u]
 }, {u, 0, π}, {v, 0, 2 π}]
```

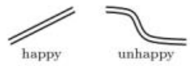


```
ParametricPlot3D[{
  Sech[u] Cos[v],
  Sech[u] Sin[v],
  u - Tanh[u]
 }, {u, 0, e}, {v, 0, 2 π}
```

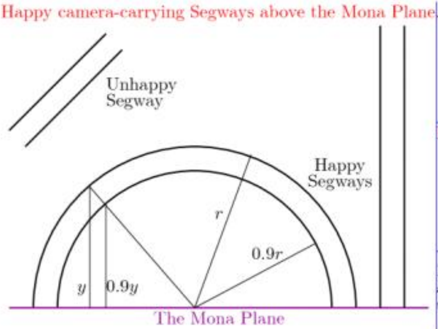


The Happy Segway Principle

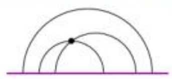
A Segway is happy iff both its wheels are



Happy camera-carrying Segways above the Mona Plane

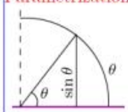


The Lobachevsky Space



Two parallels through one point

Parametrization

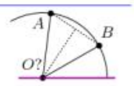


$$\theta'(t) = \sin\theta(t)$$

$$\Downarrow$$

$$\theta = 2\arctan e^t$$

Rotations, translations and some basic geometry also occur



The Actual Code

```
x = p1.x-p2.x; y = p1.y-p2.y;
d1 = p1.d; d2 = p2.d;
norm = sqrt(x*x + y*y);
a = x/norm; b = y/norm;
x1p = a*x + b*y;
x0 = (x1p + (d1*d1-d2*d2)/x1p)/2;
r = sqrt((x1p-x0)*(x1p-x0)+d1*d1);
x1pp = (x1p-x0)/r; x2pp = -x0/r;
theta1 = acos(x1pp);
theta2 = acos(x2pp);
t1 = log(tan(theta1/2));
t2 = log(tan(theta2/2));
t3 = t1 + s*(t2-t1);
theta3 = 2*atan(exp(t3));
x3pp = cos(theta3);
d3pp = sin(theta3);
x3p = x0 + r*x3pp;
p3.d = r*d3pp;
p3.x = p2.x + a*x3p;
p3.y = p2.y + b*x3p;
```

Ops used. $+$, $-$, $\times$, $\div$, $\sqrt{\cdot}$, cos, sin, tan, arccos, arctan, log, exp.