Oberwolfach: Rational Homotopy Theory in Mathematics and Physics

February-10-11
11:48 AM

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17. **Chen's Iterated Integrals and higher Hochschild chain complex**

K. T. Chen's theory of iterated integrals is a geometric way to obtain information about the rational homotopy type of manifolds and simplicial sets. In particular, this point of view is well-suited to the study of spaces of loops, see [Che73] or [Mer04]. This talk is concerned with a generalization to the higher Hochschild chain complex (introduced in [Pir00]) as it appears in Section 2 of [GTZ10].

[Che73] Kuo-tsai Chen, *Iterated integrals of differential forms and loop space homology*, Ann. of Math. (2) **97** (1973), 217–246. MR 0380859 (52 #1756)

[Mer98] Sergei Merkulov, *Formality of canonical symplectic complexes and Frobenius manifolds*, Internat. Math. Res. Notices **14** (1998), 727–733. MR MR1637093 (99j:58078)

[Mer04] ______, *De Rham model for string topology*, Int. Math. Res. Not. (2004), no. 55, 2955–2981. MR 2099178 (2005g:57055)

[Pir00] Teimuraz Pirashvili, *Hodge decomposition for higher order Hochschild homology*, Ann. Sci. École Norm. Sup. (4) **33** (2000), no. 2, 151–179. MR 1755114 (2001e:19006)

[GTZ10] Grégor Ginot, Thomas Tradler, and Mahmoud Zeinalian, *A Chen model for mapping spaces and the surface product*, Ann. Scient. c. Norm. Sup. **43** (2010), no. 5, 811–881.

$G \mapsto EG \mapsto BG \to LBG \to \Lambda^*(LBG)$

BG is the classifying space of G-bundles. What's LBG?

From Chen's 1977 paper:

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2.3. A theorem on loop space cohomology. Recall that k is the field of real (or complex) numbers.

THEOREM 2.3.1. *Let M be a topological differentiable space with a base point x_0 and having the following properties*:

(a) *The underlying topological space $_TM$ is simply connected with homology of finite type.*

(b) *The inclusion $\Delta(M)_{x_0} \subset \Delta(_TM)$ induces an isomorphism $H(\Delta(M)_{x_0}) \approx H_*(_TM)$.*

If A is a differential graded subalgebra of $\Lambda(M)$ such that $H(A) \approx H^*(_TM; k)$

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If A is a differential graded subalgebra of $\Delta(M)$ such that $H(A) \approx H^*(_TM;k)$ via integration over $\Delta(M)_{x_0}$, then there is an isomorphism

$$H\left(A'_{x_0}\right) \approx H^*(\Omega_TM;k). \tag{2.3.1}$$

<u>Question</u>. What in the non-simply connected case?