Kamnitzer: Categorification and Geometric Representation Theory

October-05-10
2:09 PM

$\mathfrak{g}$: Semisimple Lie algebra $\longrightarrow U_q\mathfrak{g}$

$\underline{\lambda} = (\lambda_1 \dots \lambda_n) \in X_+^n \quad V(\underline{\lambda}) = V(\lambda_1) \otimes \dots \otimes V(\lambda_n)$

$V(\underline{\lambda})_\mu$: μ-wt space, for $\mu \in X$

① $\mathfrak{g}$ acts $\quad E_i, F_i$:

$$E_i : V(\underline{\lambda})_\mu \longrightarrow V(\underline{\lambda})_{\mu+\alpha_i}$$

$$F_i : V(\underline{\lambda})_\mu \longrightarrow V(\underline{\lambda})_{\mu-\alpha_i}$$

② B_n acts :

$$\sigma_i : V(\underline{\lambda})_\mu \longrightarrow V(s_i(\underline{\lambda}))_\mu$$

(tangles act too...)

<u>Goal of categorification</u> Find categories $D(\underline{\lambda})_\mu$

& functors
$$E_i, F_i : D(\underline{\lambda})_\mu \longrightarrow D(\underline{\lambda})_{\mu\pm\alpha_i}$$
$$\sigma_i : D(\underline{\lambda})_\mu \longrightarrow D(s_i(\underline{\lambda}))_\mu$$

s.t.
$$K(D(\underline{\lambda})_\mu) = V(\underline{\lambda})$$ [with compatibility for all induced maps]

Motivation. 1. Homological knot invariants.

2. Produce special bases.

Combinatorial/pictorial approach: Khovanov-Lauda, Webster.

Geometric approach: Find varieties & study assoc. categories.

$Gr(\underline{\lambda})_\mu$ $\mathcal{M}(\underline{\lambda})_\mu$.

constructed using affine Grassmannian

constructed using quiver varieties.

$H_\cdot(Gr(\underline{\lambda})_\mu) \cong V(\underline{\lambda})_\mu \cong H(\mathcal{M}(\underline{\lambda})_\mu)$

$\mathfrak{g}$ ... $\mathfrak{g}, B_n$ ...

eg $\mathfrak{g} = \mathfrak{sl}_2$, $\underline{\lambda} = (\underbrace{\omega_1, \dots, \omega_1}_{n=2m})$ $V(\omega_1) = \mathbb{C}^2$

$\mu = 0$.

$$Gr(\underline{\lambda})_\mu = \left\{ (X, V_\cdot) : 0 \subset V_1 \subset V_2 \subset \dots \subset V_{2m} = \mathbb{C}^{2m},\ X = \begin{bmatrix} * & I & & 0 \\ \vdots & 0 & \ddots & \\ \vdots & & \ddots & I \\ * & 0 & & 0 \end{bmatrix}, XV_i \subset V_{i-1} \right\}$$

$$\mathcal{M}(\underline{\lambda})_\mu = T^* G(m, \mathbb{C}^{2m}) = \left\{ (X, V) : V \subset \mathbb{C}^{2m}, \dim V = m, X\mathbb{C}^{2m} \subset V, XV = 0 \right\}$$